Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 13: Fundamentals of Thermodynamics and Heat Engines

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Plant Equipment

2.2 Closed System

2.2.1 Non-Flow Energy Equation

Theory

The Non-Flow Energy Equation is given as Eq.2.1, no flow means that there is no kinetic or potential energy. The assumptions when calculating a closed system are:

- The fluid is compressible
- The system is insulated – meaning that heat is not lost to the environment over time
- The fluid is a "perfect" gas – the implication of this term will be discussed in Section 2.1.3

2.2.2 Applying the Non-Flow Energy Equation

Theory

Most problems are simplified into defining one aspect of the system as constant. Table.2.1 shows the equations used to calculate work, heat and internal energy for the Non-Flow Energy Equation. Using the equations defined in workbook 1 will help find the temperature, pressure and volume.

The term C_v seen in the table is the specific heat capacity of the fluid, there is also the specific heat capacity at constant pressure, C_p. The ratio $C_v/C_p = \gamma$ is used in the calculations mentioned in workbook 1. The specific heat capacities will change with temperature, and so the assumption of "a perfect gas" is used, which means that C_v, C_p and γ are constant at all temperatures.

Table.2.1: Equations used to calculate heat, work and internal energy for a closed system

Process	Q	W	Δu
Isobaric	$\Delta u + W$	$\int pdv$	$C_v(T_2 - T_1)$
Isochoric	$C_v(T_1 - T_2)$	0	$C_v(T_2 - T_1)$
Isothermal	$RT\ln\left(\frac{v_2}{v_1}\right)$	$RT\ln\left(\frac{v_2}{v_1}\right)$	0
Adiabatic	0	$-C_v(T_2 - T_1)$	$C_v(T_2 - T_1)$
Isentropic	0	$-C_v(T_2 - T_1)$	$C_v(T_2 - T_1)$

Example 2

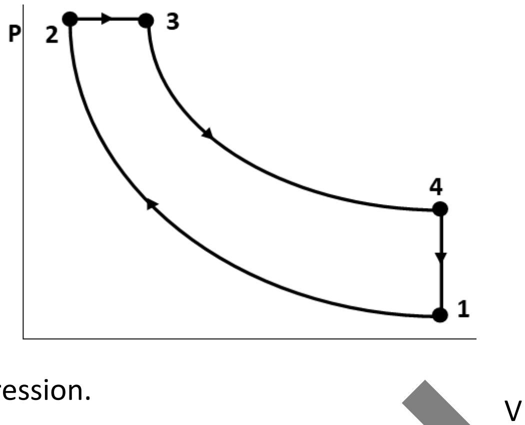
A closed system has a four-stage process. The working fluid's original state is $293K$ at $0.1MPa$. The system then shrinks from $0.3m^3$ down to $0.15m^3$ through isentropic compression. The fluid then undergoes isobaric heating and expands to $0.18m^3$. The system then undergoes isentropic expansion back to $0.3m^3$, before isochoric cooling to its original state. Calculate:

a) the work, heat and internal energy change of each stage
b) the overall work, heat and internal energy change of the system
c) and calculate the overall efficiency of the system.

Assume the working fluid is air acting as a perfect gas, with $C_V = 0.718kJ \cdot kg^{-1} \cdot K^{-1}$ and $\gamma = 1.4$.

Answer:

a) First, we draw the P-V diagram.



Stage 1-2: Isentropic Compression.

The new pressure is: $\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{P_2}{P_1} \therefore P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 0.1 \cdot 10^6\left(\frac{0.3}{0.15}\right)^{1.4} = 0.2639MPa$

With a temperature: $\frac{T_2}{T_1} = \frac{V_1}{V_2}^{\gamma-1} \therefore T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293\left(\frac{0.3}{0.15}\right)^{1.4-1} = 386.7K$

Heat: $Q_{21} = 0$ (Isentropic)

Work: $W_{12} = C_v(T_2 - T_1) = 0.718(386.7 - 293) = -67.3kJ \cdot kg^{-1}$

Internal Energy: $\Delta U_{12} = -W = 67.3kJ \cdot kg^{-1}$

Stage 2-3: Isobaric Heating

The new temperature is: $\frac{V_2}{T_2} = \frac{V_3}{T_3} \therefore T_3 = \frac{V_3 T_2}{V_2} = \frac{0.18 \cdot 386.7}{0.15} = 464.0K$

Work: $W_{23} = \int P\,dV = 0.18P - 0.15P = 0.03P = 7.917kJ \cdot kg^{-1}$

Internal energy change: $\Delta U_{23} = C_v(T_3 - T_2) = 55.5kJ \cdot kg^{-1}$

Heat: $Q_{23} = \Delta U + W = 55.5 + 7.917 = 63.4kJ \cdot kg^{-1}$

Stage 3-4: Isentropic expansion

New pressure: $\left(\frac{V_3}{V_4}\right)^{\gamma} = \frac{P_4}{P_3} \therefore P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = 0.2639 \cdot 10^6\left(\frac{0.18}{0.3}\right)^{1.4} = 0.1291MPa$

New temperature: $\frac{T_4}{T_3} = \frac{V_3}{V_4}^{\gamma-1} \therefore T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = 464.0\left(\frac{0.18}{0.3}\right)^{1.4-1} = 378.2K$

Heat: $Q_{34} = 0$ (Isentropic)

Work: $W_{34} = -C_v(T_4 - T_3) = -0.718(378.2 - 464.0) = 61.6kJ \cdot kg^{-1}$

Internal energy change: $\Delta U_{34} = -W_{34} = -61.6kJ \cdot kg^{-1}$

Stage 4-1: Isochoric cooling

Returns to original state, so we know all temperatures, pressures and volumes

Work: $W_{41} = 0 \; (dV = 0)$

Heat: $Q_{41} = C_v(T_1 - T_2) = 0.718(293 - 378.2) = -61.2kJ \cdot kg^{-1}$

Internal energy change $\Delta U_{41} = Q = -61.2kJ \cdot kg^{-1}$

b) Overall work: $W_{net} = -67.3 + 7.9 + 61.6 = 2.2kJ \cdot kg^{-1}$
Overall heat: $Q_{net} = 0 + 63.4 + 0 - 61.2 = 2.2kJ \cdot kg^{-1}$
Overall internal energy change: $\Delta U = 67.3 + 55.5 - 61.6 - 61.2 = 0kJ \cdot kg^{-1}$

This complies with the non-flow energy equation $Q - W = \Delta U$

c) The efficiency of the system is:

$$\eta_O = \frac{W_{net}}{Q_{in}} = \frac{W_{net}}{Q_{23}} = \frac{2.2}{63.4} = 0.03$$

The engine has an efficiency of 3%, which is extremely low.

2.3 Open Systems

2.3.1 The Steady Flow Energy Equation

Theory

Remembering the overall equation of the first law, we have Eq.2.2 below

$$Q - W = \left(U_2 + \tfrac{1}{2}mc_2^2 + mgz_2\right) - \left(U_1 + \tfrac{1}{2}mc_1^2 + mgz_1\right) \qquad \text{(Eq.2.2)}$$

Many mechanical engineering devices involve open systems where the flow can be analysed as being in steady state, such as a gas turbine or a refrigeration cycle. For steady flows undergoing changes in volume, the enthalpy, represented by Eq.2.3 is the most convenient variable to represent that static energy of the flow.

$$h = U + PV \qquad \text{(Eq.2.3)}$$

For these applications the first law of thermodynamics is expressed as Eq.2.4, known as the Steady Flow Energy Equation

$$\dot{Q} - \dot{W}_x = \dot{m}_2\left(h_2 + \tfrac{1}{2}c_2^2 + gz_2\right) - \dot{m}_1\left(h_1 + \tfrac{1}{2}c_2^2 + gz_1\right) \qquad \text{(Eq.2.4)}$$

An important aspect is the introduction of mass flow, the system must follow the conservation of mass. If there is a change of area, speed or density then the mass flow will change. Shown by Eq.2.5, where ρ is the density of the fluid, A is the cross-sectional area of the pipe, and c is the velocity of the fluid.

$$\dot{m} = \rho A c \qquad \text{(Eq.2.5)}$$

Since the flow can change, or flows can be added or diverted from the pipes, we build Eq.2.6 for mass flow continuity.

$$\sum \dot{m}_{IN} = \sum \dot{m}_{OUT} \qquad \text{(Eq.2.6)}$$

Example 3

Two water pipes are converging into one large pipe, the first pipe has an area of $0.01m^2$ and its water is flowing at $160ms^{-1}$. The second pipe has an area of $0.03m^2$ and its water is flowing at $20ms^{-1}$. The larger pipe has an area of $0.45m^2$. Calculate:

a) The mass flow rate of the larger pipe.
b) The velocity of the larger pipe.

Answers:

a) The continuity of mass equation gives:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \rho A_1 c_1 + \rho A_2 c_2 = 1000(0.01 \cdot 160 + 0.03 \cdot 20) = 2200kg \cdot s^{-1}$$

b) Using Eq.2.5, we can find velocity as

$$\dot{m}_3 = \rho A_3 c_3 \therefore c_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{2200}{1000 \cdot 0.45} = 4.89ms^{-1}$$

2.3.2 Plant Compression/Expansion Equipment

Theory

Much like a non-flow system, we can break down an open system into several sub-systems. Each one has a different assumption that makes a thermodynamic analysis much simpler to calculate.

Compression and expansion devices are used to do work on, or extract work from a fluid. This unit will mostly look at turbo machinery called compressors and turbines, but we will also look at pumps. Turbines are used to drive the generators that provide most of the world's electricity. The detailed analysis of this machinery is more based around fluid mechanics, so for the time being we will only consider the inflow and outflow conditions. The mass flow rate and the overall isentropic efficiency of the system. These machines generate a lot of heat, and would require cooling in some applications. But this will be neglected in this unit.

The changes in potential energy will be negligible in these machines, but the velocity change can be significant, and cannot be immediately ignored.

Compressor: Compressors will compress the working fluid to increase the pressures, which causes an increase in temperature, it also allows increases the effectiveness of heating in the boiler, as there will be less empty space. Compressors are assumed to be an adiabatic, or isentropic compression, its effect on a P-V diagram will look like Fig.2.1.

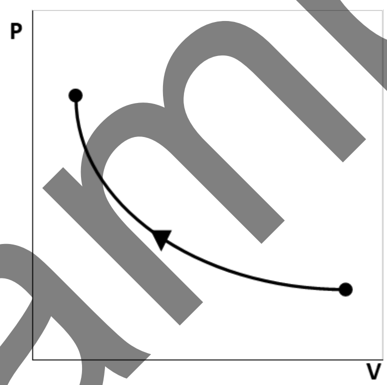


Fig.2.1: A compressor's effect on the P-V diagram

Turbine: Turbines are the work output of a system, and are used to power the shaft that drives the generator, and typically the compressor's shaft. Turbines can be assumed to be an adiabatic, or isentropic expansion, its effect on a P-V diagram will be the reverse of Fig.2.1.

Pumps: Pumps are a more complicated form of compression, they are considered to be isentropic, and are used in refrigeration cycles, which are covered in Unit 38: Further Thermodynamics.

2.3.3 Plant Heating/Cooling Equipment

Theory

Most heating and cooling systems in a plant are constant volume systems, as it is difficult to set a constant pressure.

Boiler: A boiler is assumed to be a constant pressure heat addition, and will affect a P-V diagram as shown on Fig.2.2

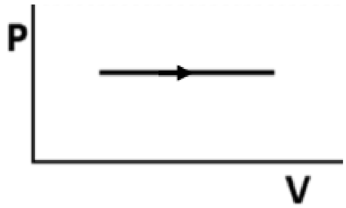


Fig.2.2: A boiler's impact to a P-V diagram

Condenser: A constant pressure heat rejection, and will be the reverse of Fig.2.2.

Super-heaters: Super-heaters are, essentially, a more powerful boiling system, and are discussed further in Unit 38: Further Thermodynamics.

2.3.4 Plant Equipment Equations

Theory

In this unit, there are no systems that diverge the mass flow, and so $\dot{m}$ will be constant throughout the system. More complicated plant systems that involve a change in mass flow rate will be discussed in Unit 38: Further Thermodynamics.

The typical plant cycle is known as the "Brayton", or in some cases "Joule", cycle starting at the compressor where there is a temperature and pressure increase, then to the boiler for a constant pressure heat addition, the fluid is moved onto the turbine where the work output is produced at the expense of temperature and pressure, before moving to the condenser to return to its original state Shown by Fig.2.3.

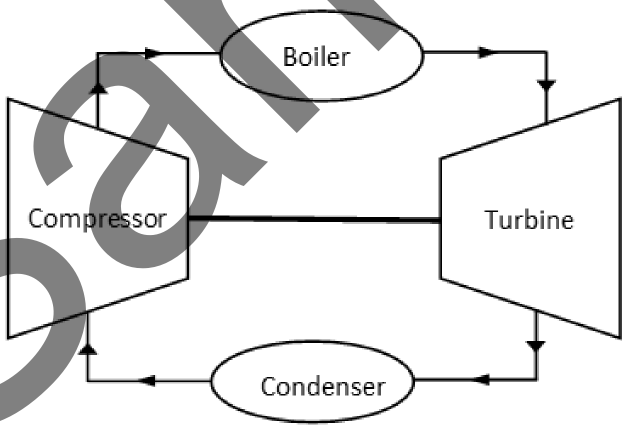


Fig.2.3: Brayton cycle

The thermal efficiency of the system is given by Eq.2.7, where r_t is the isentropic pressure ratio $\left(r_t = T_2/T_1 = rp^{(\gamma-1)/\gamma}\right)$.

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r_t} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \qquad \text{(Eq.2.7)}$$