Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 13: Fundamentals of Thermodynamics and Heat Engines

Unit Workbook 3

in a series of 4 for this unit

Learning Outcome 3

Heat Transfer for Industrial Applications

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Approved Centre

3.1 Methods of Heat Transfer

3.1.1 Conduction

Theory The conduction of heat is the transfer of energy through particle vibrations. In solids this is through the lattice, with adjacent molecules transferring energy in packets (phonons) to neighbouring molecules. In the case of fluids, the energy is transferred through collisions and diffusion.

The rate of heat loss, $\dot{Q}$ $[W]$ through a solid material (or a fluid with no flow) of length dx is described as Eq.3.1, where k $[W \cdot m^{-1} \cdot K^{-1}]$ is the thermal conductivity of the material, A is the area of the face.

$$\dot{Q} = -kA\frac{dT}{dx} \qquad \text{(Eq.3.1)}$$

Since k will always be a positive, the negative sign indicates that heat flows in the direction of decreasing temperature. With Eq.3.1 we can see that the temperature drop through the wall is linear. Demonstrated in Fig.3.1.
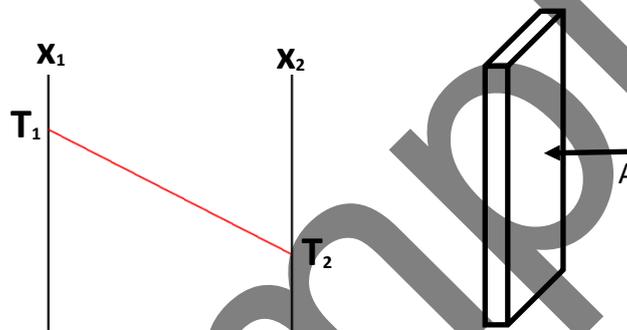

Fig.3.1: 1-D steady heat conduction through a planar wall.

Table.3.1 gives the thermal conductivity for various materials

Table.3.1: Thermal conductivity values for materials

Material	k
Diamond	2300
Copper	401
Aluminium	237
Iron	80
Glass	0.78
Water	0.61
Air	0.026

3.1.2 Convection

Theory Convection is the transfer of heat between flowing fluid and solid boundaries. When the fluid flow is driven by the Earth's atmosphere this is known as *natural convection*. Many engineering applications will use a fluid flow generated by the motion of an object or with an imposed pressure difference, known as *forced convection*.

Due to the presence of a viscous boundary layer close to the wall, shown by Fig.3.2, the flow velocity at the wall is zero, this means that heat is transferred through conduction at the surface of the solid. The role of convection is to make the thermal boundary layer thin (its thickness is related to thickness of the velocity boundary layer). This leads to large temperature gradients and higher overall heat transfer rates.
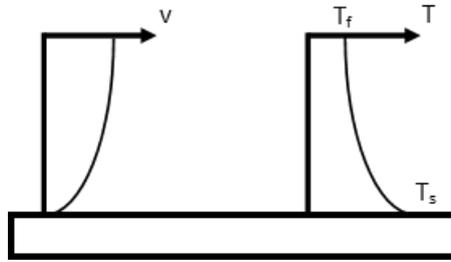
Fig.3.2: Velocity and temperature change with convective heat transfer

In engineering, the rate of heat transfer is given as Eq.3.2, where $h\ [W \cdot m^{-2} \cdot K^{-1}]$ is the convective heat transfer coefficient, T_f is the average temperature of the fluid and T_s is the temperature of the surface of the solid.

$$\dot{Q} = hA\left(T_s - T_f\right)$$ (Eq.3.2)

To evaluate how much more effective convection is than conduction of a certain length L, the dimensionless Nusselt number (Nu) is defined using Eq.3.3

$$\frac{\dot{Q}_{conv}}{\dot{Q}_{cond}} = \frac{UL}{k} = Nu$$ (Eq.3.3)

3.1.3 Radiation

Theory Radiation is the weakest of the three forms of heat transfer, and in most heat transfer simulations is ignored. The only time radiation is realistically considered is when there is no convection or conduction possible (in space). There are two important aspects of radiative heat transfer: emission, and absorption of the radiation.

Black bodies: A black body is an ideal material which absorbs all radiation at all wavelengths, nothing is reflected off it. A black body is also a perfect emitter of radiation, and it emits radiation uniformly in all direction, so it is described as a diffuse emitter. If the body itself did not emit radiation, they would appear black. An example of a real black body would be stars.

The predicted electromagnetic spectrum emitted by a black body is shown by Fig.3.3, integrating the rate of emission at each wavelength gives the total rate of radiation emission.
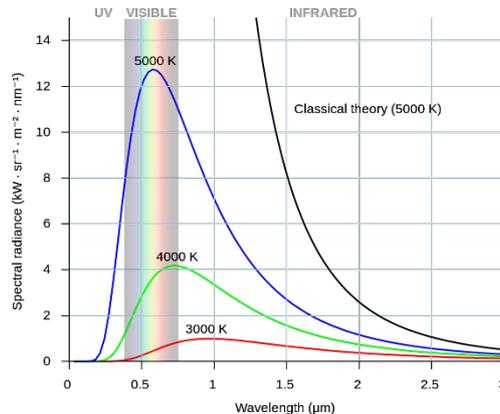

Fig.3.3: Radiation spectrum for a black body

For a black body of area A at temperature T, the rate at which energy is emitted is calculated using Eq.3.4. This is also known as Stefan's law, where σ is the Stefan-Boltzmann constant $(5.67 \cdot 10^8 Wm^{-2}K^{-1})$

$$\dot{Q} = \sigma A T^4 \qquad \text{(Eq.3.4)}$$

Grey bodies: Grey bodies are the emission from a real object, this is dependent on the material's emissivity, ϵ, in practice $0 \leq \epsilon \leq 1$, meaning Eq.3.4 becomes Eq.3.5.

$$\dot{Q} = \epsilon \sigma A T^4 \qquad \text{(Eq.3.5)}$$

The fraction of incident radiation that is reflected is known as the reflectance (ρ), the fraction that passes through the object without being absorbed is the transmissivity (τ), and the fraction that heats the object is the absorptivity (α). This accounts for all the incident radiation so $\rho + \tau + \alpha = 1$. For solid materials, the transmissivity is usually zero, and $\alpha = \epsilon$. Therefore, if a grey body at temperature T has surroundings that behave as a black body at temperature T_s, the net heat transfer *from* the grey body (power emitted minus absorbed power) will be demonstrated by Eq.3.6.

$$\dot{Q}_{net} = \epsilon \sigma A (T^4 - T_s^4) \qquad \text{(Eq.3.6)}$$

The rate of heat transfer also depends on the visible area of the object.

3.2.1 Resistance Networks and Heat Transfer Through a Planar Wall

Theory One of the best ways to model heat transfer is to consider the system as an electrical circuit. Looking at the potential difference drop across a resistor with a current, I, we can model Ohm's law as Eq.3.7.

$$I = \frac{V_1 - V_2}{R} \qquad \text{(Eq.3.7)}$$

And considering Eq.3.1, we can define it as:

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2)$$

And define thermal resistance for 1-D conduction as Eq.3.8:

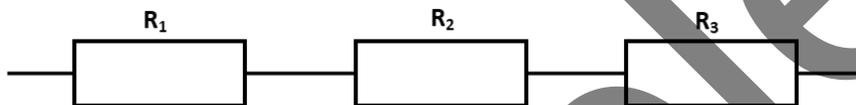$$R_{cond} = \frac{L}{kA_s} \qquad \text{(Eq.3.8)}$$

The same approach can be used for convective and radiative shown in Eq.3.9 and Eq.3.10, respectively:

$$R_{conv} = \frac{1}{hA_s} \qquad \text{(Eq.3.9)}$$

$$R_{rad} = \frac{1}{\sigma \epsilon A_s (T_1^2 + T_S^2)(T_1 + T_S)} \qquad \text{(Eq.3.10)}$$
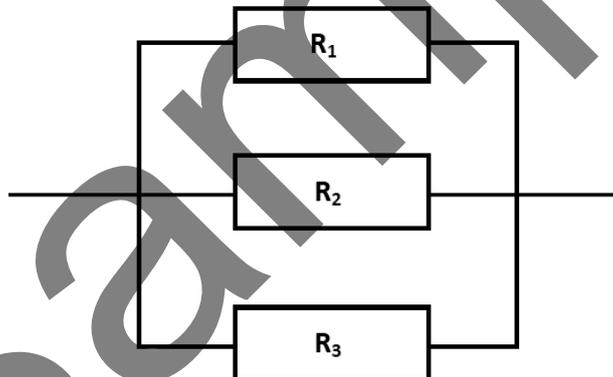
We can use this analogy to build a resistance circuit to construct series and parallel thermal circuits. As a quick reminder, resistance in series is calculated using Eq.3.11.

$$R = R_1 + R_2 + R_3 + \cdots + R_n \qquad \text{(Eq.3.11)}$$

And in parallel using Eq.3.12:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \qquad \text{(Eq.3.12)}$$

Or we can use the overall heat transfer coefficient, U, which is essentially just Eq.3.13.

$$U = 1/R \qquad \text{(Eq.3.13)}$$

We can find out the rate of heat transfer through a system with Eq.3.14.

$$\dot{Q} = A\frac{T_1 - T_2}{R} = UA(T_1 - T_2) \qquad \text{(Eq.3.14)}$$

Example 1

The inside temperature of the air in a room is $18°C$, and the temperature of the air outside the room is $13°C$. There is only a brick wall separating the room from the outside. The wall is 3m high and 2m wide, and the brick is 100 mm thick. The thermal conductivity of the brick is $0.6 Wm^{-1}K^{-1}$, and the coefficient of the air inside and outside the room is $5Wm^{-2}K^{-1}$ and $10Wm^{-2}K^{-1}$. Calculate:

a. The total heat loss of the environment
b. The temperature on the outside surface of the brick.

Answer:

The first thing we need to do is create the resistance circuit through the wall. There is the convection of the air inside the room $R_{conv,I}$, then there is the conduction through the brick R_{cond}, and finally the convection on the outside of the wall $R_{conv,O}$. The resistance map looks like the one below.
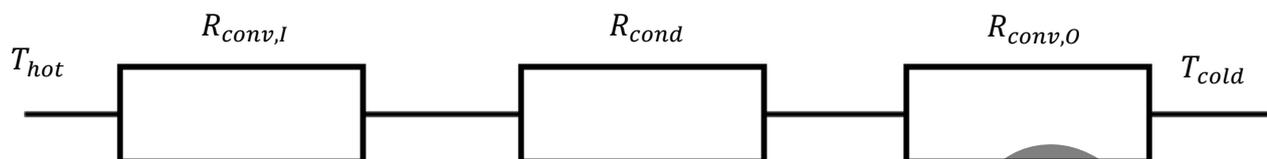
The total resistance for the system is:

$$R = R_{conv,O} + R_{cond} + R_{conv,O} = \frac{1}{h_I A}\frac{L}{k_B A} + \frac{1}{h_O A} = \frac{1}{5(3 \cdot 2)} + \frac{0.1}{0.8(3 \cdot 2)} + \frac{1}{10(3 \cdot 2)} = 0.0708 W K^{-1}$$

The overall heat transfer is therefore:

$$\dot{Q} = \frac{T_{hot} - T_{cold}}{R} = \frac{5}{0.0708} = 70.6 W$$

With the overall heat transfer, we can find the temperature around throughout the brick.

$$T_{b,out} = T_{cold} + \dot{Q} R_{conv,O} = \frac{70.6}{10(3 \cdot 2)} = 14.2°C$$

3.2 Water Pipes

Pipes are defined into two categories, either lagged or unlagged. An unlagged pipe is one that is not insulated. Pipe lagging is a special type of insulation fitted around water pipes that prevents condensation on cold pipes, insulating pipes is important as it not only saves on energy, it also prevents pipes from freezing and bursting, which can cause lots of damage and halt production.

3.2.1 Unlagged Pipes

Theory The parameters for the equation for heat loss in a pipe are defined in Fig.3.4, T_I and T_O are the temperature on the inside and outside of the pipe, respectively; r_I and r_O are the radii between the centre to the inside pipe, and the centre to the outside of the pipe, respectively; and L is the length of the pipe.
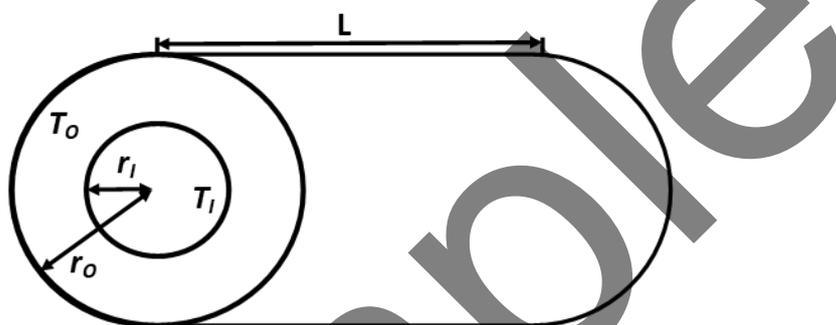

Fig.3.4: An unlagged pipe

We can also measure the rate of heat loss across the pipe as Eq.3.15:

$$\dot{Q} = \frac{T_I - T_O}{[ln(r_O/r_I)/2\pi k L]} \qquad \text{(Eq.3.15)}$$

Once we have worked out the rate of heat loss, we can work out the temperature across the pipe as a function of r by rearranging Eq.3.15 to get Eq.3.16, where $r_I \leq r \leq r_O$.

$$T = T_I - \frac{\dot{Q} ln(r_O/r_I)}{2\pi k L} \qquad \text{(Eq.3.16)}$$

Example 2

A $1m$ copper pipe with an overall diameter of $25mm$, with thickness $5mm$ is moving water at $80°C$. The outside temperature of the room is $25°C$ and the thermal conductivity of the pipe is $401 W m^{-1} K^{-1}$.

a) Calculate the rate of heat loss
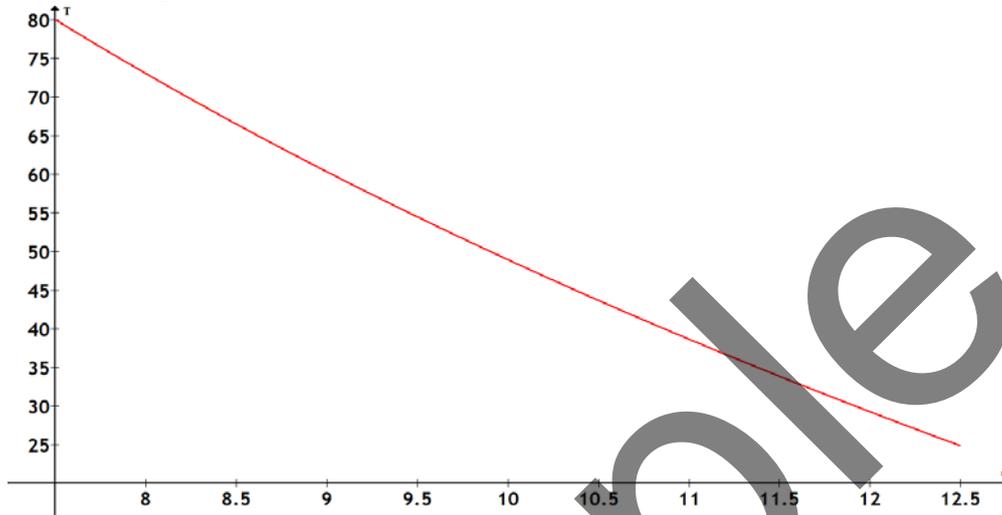b) Plot the temperature profile across the pipe

Answer:

a) The rate of heat loss is calculated with Eq.3.15 and Eq.3.16

$$\dot{Q} = \frac{T_I - T_O}{[ln(r_O/r_I)/2\pi kL]} = \frac{80 - 25}{[ln(12.5/7.5)/2\pi \cdot 401 \cdot 1]} = 271kW$$

b) The temperature profile is defined as Eq.3.17

$$T = T_I - \frac{\dot{Q}ln(r/r_I)}{2\pi kL} = 80 - \frac{271 \cdot 10^3 \cdot ln(r/7.5)}{2\pi \cdot 401 \cdot 1}$$

Which will give the graph below,

3.2.2 Lagged Pipes

Theory The parameters of a lagged pipe are shown below in Fig.3.5, this time including r_S and T_S, which are the radius and temperature on the surface of the insulation.
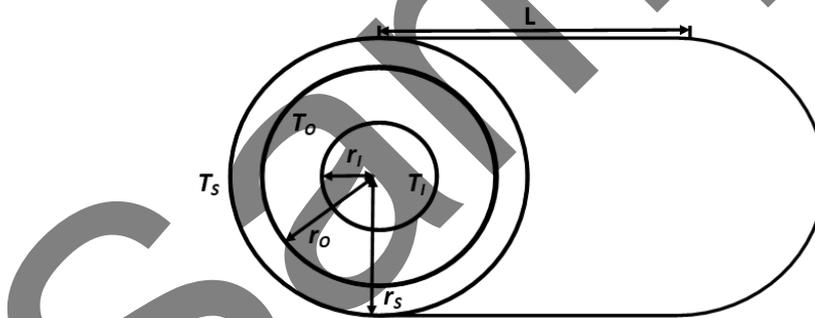

Fig.3.5: A diagram of a lagged pipe

The equation for the rate of heat loss is now demonstrated with Eq.3.17, where k_s is the thermal conductivity of the pipe.

$$\dot{Q} = \frac{T_S - T_I}{[(ln(r_O/r_I)/2\pi kL) + (ln(r_S/r_O)/2\pi k_S L)]} \qquad (Eq.3.17)$$

The equation for the temperature profile will differ when including the insulation. The temperature on the outside of the pipe must be calculated using Eq.3.16 first, before using Eq.3.19.

$$T = T_O - \dot{Q}[ln(r_S/r_O)/2\pi k_S L] \qquad (Eq.3.19)$$

Example 3

The pipe in Example 2 has now been covered with $20mm$ thick magnesia insulation ($k_s = 0.055Wm^{-1}K^{-1}$).
Calculate the temperature at the outer surface.

Answer:

a) Rate of heat loss:

$$\dot{Q} = \frac{T_S - T_I}{[(ln(r_O/r_I)/2\pi kL) + (ln(r_S/r_O)/2\pi k_S L)]}$$

$$= \frac{55}{[(ln(12.5/7.5)/2\pi \cdot 401 \cdot 1) + (ln(32.5/12.5)/2\pi \cdot 0.055 \cdot 1)]} = 19.8 \, W$$

b) The temperature profile across the pipe is

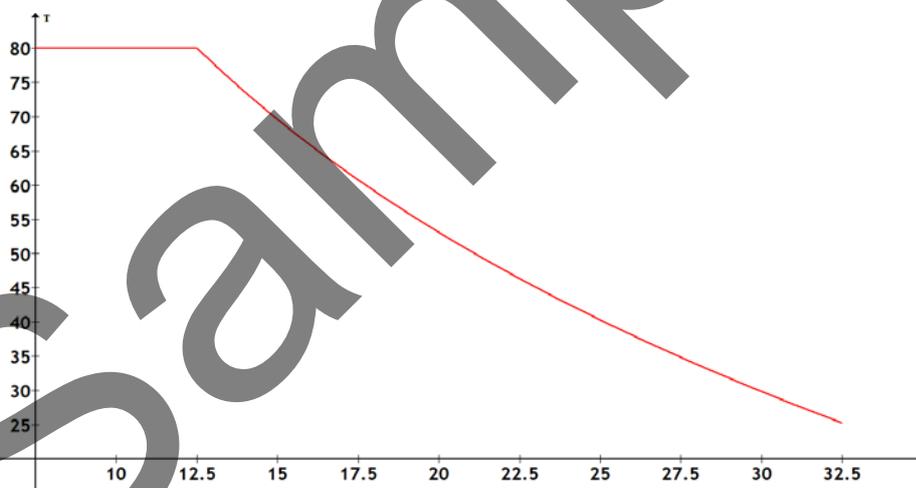$$T = T_I - \frac{\dot{Q}ln(r/r_I)}{2\pi kL} \qquad r_I \leq r \leq r_O$$

Meaning the temperature on the outside wall is:

$$T_O = T_I - \frac{\dot{Q}ln(12.5/7.5)}{2\pi \cdot 401 \cdot 1} = 79.995°C$$

And the temperature profile across the insulation is

$$T = T_O - \frac{\dot{Q}ln(r_O/r_I)}{2\pi k_S L} \qquad r_o \leq r \leq r_s$$

Giving an overall temperature profile of

3.2.3 Improving the Accuracy of Pipe Calculations

Theory

We can improve the accuracy of the system by including convective heat transfer with the fluid in the pipe and the air shown by Eq.3.20.

$$\dot{Q} = \frac{T_I - T_S}{\left[\frac{1}{2\pi h_f r_I L} + \frac{ln(r_O/r_I)}{2\pi kL} + \frac{ln(r_S/r_O)}{2\pi k_S L} + \frac{1}{2\pi h_a r_S L}\right]}$$
(Eq.3.20)

Where h_f and h_a are the convective heat transfers of the fluid in the pipe and the air surrounding the pipe, respectively.
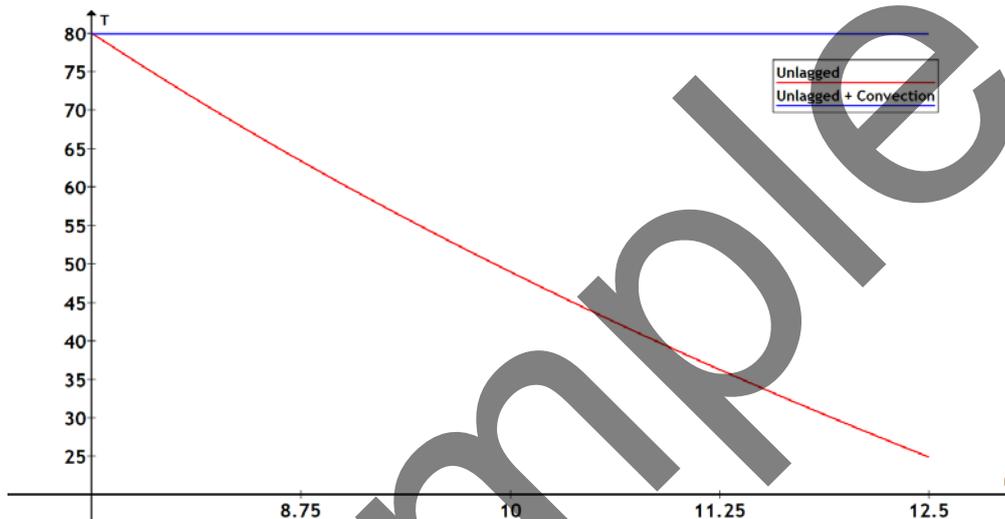
Example 4

We will consider the impact of convection in Example 2, where h_f and h_a are $3000Wm^{-2}K^{-1}$ and $5Wm^{-1}K^{-1}$, respectively.

$$\dot{Q} = \frac{55}{\left[\dfrac{1}{2\pi \cdot (12.5 \cdot 10^{-3}) \cdot 3000 \cdot 1} + \dfrac{ln(12.5/7.5)}{2\pi \cdot 401 \cdot 1} + \dfrac{1}{2\pi \cdot (12.5 \cdot 10^{-3}) \cdot 5 \cdot 1}\right]} = 21.56W$$

The temperature at the inside surface of the pipe is:

$$T_I = T_f - \frac{\dot{Q}}{2\pi \cdot 7.5 \cdot 10^{-3} \cdot 3000 \cdot 1} = 79.84$$

We can compare this to Example 2:

Which is as you would expect (if you touch a bare pipe it feels hot, it doesn't feel the same temperature as the room).

Example 5

Building from Example 3, we will now consider the convection of the fluid inside the pipe and the surrounding air.

$$\dot{Q} = \frac{55}{\left[\dfrac{1}{2\pi \cdot (7.5 \cdot 10^{-3}) \cdot 3000 \cdot 1} + \dfrac{ln(12.5/7.5)}{2\pi \cdot 401 \cdot 1} + \dfrac{ln(32.5/12.5)}{2\pi \cdot 0.055 \cdot 1} + \dfrac{1}{2\pi \cdot (32.5 \cdot 10^{-3}) \cdot 5 \cdot 1}\right]} = 14.67W$$

The temperature at the inside wall of the pipe is

$$T_I = T_f - \frac{\dot{Q}}{2\pi \cdot (7.5 \cdot 10^{-3}) \cdot 3000 \cdot 1} = 79.90°C$$

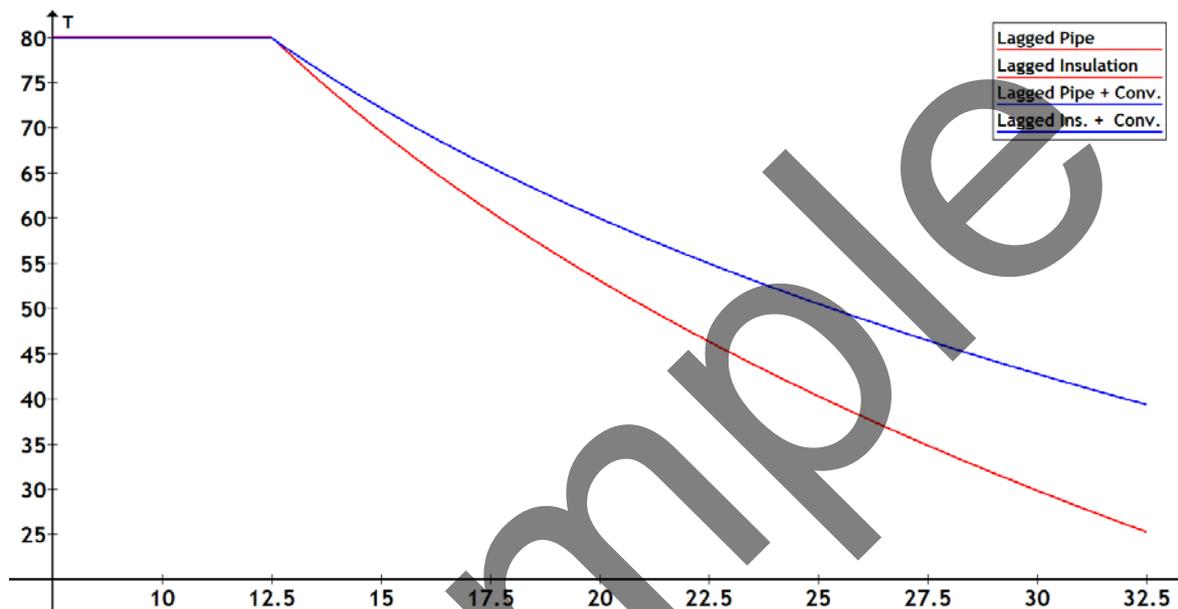It's clear that there is not a substantial impact on the temperature of the pipe when considering the convection at the pipe.

The temperature at the outside wall of the pipe:

$$T_O = T_I - \frac{\dot{Q}ln(r_O/r_I)}{2\pi kL} = 79.90°C$$

The temperature of the surface of the insulation is

$$T_S = T_O - \frac{\dot{Q}ln(r_S/r_O)}{2\pi k_sL} = 39.34°C$$

And the temperature profile, compared to Example 3.

Again, it creates a more realistic temperature profile (an insulated pipe will feel warm, not room temperature)

3.3 Heat Exchangers

Heat exchangers are an effective way to transfer heat between a barrier, with several different methods of accomplishing this task.

3.3.1 Recuperative Heat Exchangers

Theory Recuperators are one of the simplest heat exchanger designs, the fluids are flowing next to each other, with a conductive layer separating the two. There are four types of flow with recuperators, shown below in Fig.3.6. The flows we will be analysing, however, are parallel-flow and counter-flow.
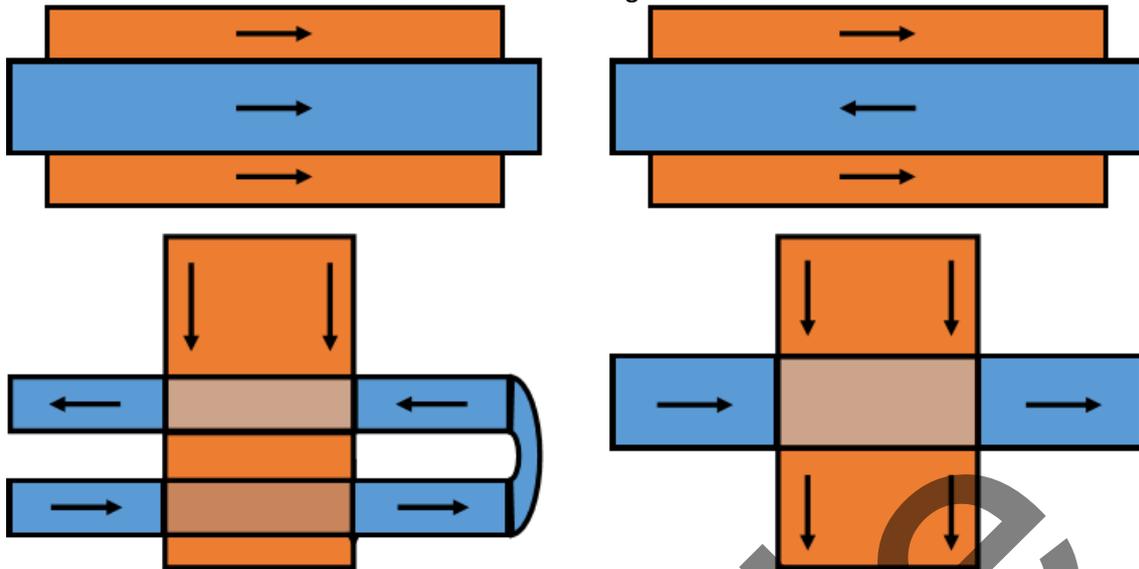
Fig.3.6: (Clockwise from top-left) Parallel-flow, Counter-flow, Cross-flow, Multi-pass cross-flow

When considering recuperative heat exchangers, we need to consider the heat transfer that is occurring. Convection will be working on both sides of the separator, and there is also conduction occurring through the separator. Since the temperatures ($T_h(x)$ and $T_c(x)$) varies across the pipe, then the rate of heat transfer varies across the system, shown by Eq.3.21. We assume that heat exchangers are isolated from their surroundings, meaning that the only heat transfer occurs between the two systems. **A common assumption of recuperative heat exchangers is that the thickness of the separator is negligible.**

We can find the rate of heat transfer for a parallel-flow heat exchanger by analysing each individual pipe using Eq.3.21.

$$\dot{Q} = -\dot{m}_h C_{p,h}\left(T_{2,h} - T_{1,h}\right) = \dot{m}_c C_{p,c}\left(T_{2,c} - T_{1,c}\right) \qquad \text{(Eq.3.21)}$$

This gives us the overall heat transfer required to get the temperature that we want, but we don't know how much is transferred with regards to distance, or radius. We need to get the overall heat transfer coefficient, which will be Eq.3.22.

$$U = \frac{1}{R} = \frac{1}{R_{conv,h} + R_{cond} + R_{conv,c}} \qquad \text{(Eq.3.22)}$$

The equation to develop the design parameters of the heat exchanger is called the **Log Mean Temperature Difference (LMTD)**, shown by Eq.3.23.

$$\dot{Q} = UA_s \frac{\Delta T_{in} - \Delta T_{out}}{ln(\Delta T_{in}/\Delta T_{out})} \qquad \text{(Eq.3.23)}$$

Typically, when we look at the design parameters for a heat exchanger, it will be the surface area, so adjusting the diameter and length of the system.

Heat exchange for counter-flow heat exchanger is given as Eq.3.24.