



# 4.1 Ideal Heat Engine Cycles

## 4.1.1 Second Law of Thermodynamics with Heat Engines

The second law of thermodynamics is a series of observations that concerns the way things flow as time progresses forward. Typical observations are "water flows from high to low", and "heat flows from hot to cold". In the context of heat engines, however, the second law can be summed up as: **"No heat engine can be 100% efficient"**.

## 4.1.2 Carnot Cycle

### Theory

The Carnot cycle is a theoretical heat engine design, that is meant to the ideal operating system of a heat engine. It consists of four closed processes:

**1-2:** Fig.4.1 shows the first stage of the Carnot cycle, and its affect on the T-S and P-V diagram. As an isentropic system  $\Delta Q = \Delta S = 0$ .



**2-3:** Fig.4.2 represents the second stage, the isothermal process means that there is a heat input, but the process also produces a work output.



**3-4:** Fig.4.3 shows the isentropic expansion of the system, as with stage 1-2,  $\Delta Q = \Delta S = 0$ .





Fig.4.3: Stages 1-2-3-4 of the Carnot cycle

#### 4-1: The final stage, isothermal compression, completes the Carnot cycle, illustrated by Fig.4.4





We know, from the previous learning objectives, that  $Q_{net} = W_{net}$ , we can calculate the thermal efficiency of the system as Eq.4.1:

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_{cold}\Delta s_{cold}}{T_{hot}\Delta s_{hot}}$$
(Eq.4.1)

Since  $\Delta s_{cold} = \Delta s_{hot}$ , then thermal efficiency can be reduced to Eq.4.2:

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$$
(Eq.4.2)

This gives the **Carnot efficiency**, the ideal efficiency of an engine that can not be attained in practical systems.

#### Example 1

What is the maximum possible efficiency of an engine where  $T_{cold} = 50K$  and  $T_{hot} = 320K$ ?

$$\eta_{th} = 1 - \frac{50}{320} = 0.844$$

#### Example 2

A claim that a new engine has been developed with a thermal efficiency of 75%. It draws in air at  $10^{\circ}C$  and its exhaust releases gas at  $680^{\circ}C$ . Comment on whether a system such as this is possible.



#### Answer: Remembering to convert the temperatures to K

$$\eta_{th} = 1 - \frac{273 + 10}{273 + 680} = 0.703 = 70.3\%$$

The maximum possible efficiency is 70.3%, the claim cannot possible be true.

#### 4.1.3 Otto Cycle

**Theory** The Otto cycle is built to represent a more realistic engine system. This process is more commonly associated with a spark ignition (SI) engine, which will be discussed in Section.4.2. The complete P-V and T-S diagrams of the cycle are shown in Fig.4.8 below, the defining feature of the Otto cycle is its constant pressure heat addition in the system. The point marked  $r_c$  on Fig.4.5 is known as the compression ratio of the system, it is the ratio of volumes between the top dead centre (TDC) of the piston, relative to the bottom dead centre (BDC) of the piston, further detailed by both Eq.4.3.

$$r_c = \frac{V_{BDC}}{V_{TDC}} \tag{Eq.4.3}$$

The Otto cycle can be broken down into four stages:

1-2: Isentropic compression, Fig.4.5 shows the first stage of the system on the P-V and T-S diagram.



**2-3:** Constant volume heat addition, Fig.4.6 shows the second stage in the cycle.





Fig.4.6: Stages 1-2-3 of the Otto cycle

**3-4:** Isentropic expansion back to  $V = V_1$ , Fig.4.7 shows the next stage of the Otto cycle.



**4-1:** Constant pressure heat rejection, the cycle is completed, and the working fluid returns to the original conditions at point 1.



The thermal efficiency of the Otto cycle,  $\eta_{th,0}$ , is given as Eq.4.4. The derivation of the Eq.4.4 is not required, but can be found in "Other Resources" on Moodle.

$$\eta_{th,OTTO} = 1 - \frac{1}{r_c^{\gamma - 1}}$$
(Eq.4.4)

## 4.1.4 Diesel Cycle

Theory

The Diesel cycle is used to represent realistic compression ignition (CI) engines, which will be discussed in more detail later in Section.4.2, we can break it down into four stages:

**1-2:** Isentropic compression, Fig.4.9 shows the effect on the P-V and T-S diagrams.



## 4.2 Heat Engine Equations

### 4.2.1 Engine Geometry

Purpose

The geometry of an engine can help determine certain performance characteristics of an engine, such as the volumetric efficiency, discussed in Section 4.2. Fig.4.19 shows the

geometry of a cylinder, along with the crankshaft, piston and connecting rod.



Fig.4.19: The geometry of a single cylinder and piston.

The equation to calculate y as a function of theta is not required for this unit, but is in "Other Resources" for further information.

Theory

The displaced volume,  $V_d$  is the volume the piston covers between moving from TDC to BDC, and is calculated using Eq.4.7, where S is the stroke length of the piston, and B is the bore of the cylinder.

diameter of the cylinder.

 $V_d = \frac{\pi B^2}{4} S = \frac{\pi B^2}{4} (2r)$ 

(Eq.4.7)

The volume ratio of the cylinder is calculated using Eq.4.8, where  $V_c$  is the clearance volume of the cylinder.

(Eq.4.8)

The size of a cylinder can be broken down into three classes.

- Over square: B > S
- Square: B = S
- Under square: B < S

Over square engines have a higher possible rotational speed and lower the crank stress due to a lower piston acceleration. But this increases the heat loss in the system and lowers the thermal efficiency of the system.

## 4.2.2 Engine Power, Torque and Work

Theory

The real heat engine cycles do not match the ideal curves in Fig 4.8, Fig.4.12 and Fig.4.18 entirely due to:



- Variation of specific heats with temperature
- Residual gases from previous cycles
- Combustion is a gradual process, not instantaneous
- Heat losses through the cylinder wall
- Frictional losses and leakages

A more realistic P-V diagram of a heat engine cycle will look more like Fig.4.20. The green area represents the work done by the combustion process, the red area represents work lost through pumping.



The total indicated work of the cycle,  $\dot{W}_i$ , is calculated using Eq.4.9, where  $\dot{W}_{i,g}$  is the indicated work of the gas per cycle and  $\dot{W}_p$  is the pumping work

$$\dot{W}_i = \int P \, dV = \dot{W}_{i,g} - \dot{W}_p \tag{Eq.4.9}$$

The indicated power per cycle,  $\dot{P}_i$ , is calculated using Eq.4.10, where n is the number of revolutions per second, and  $n_r$  is the number of revolutions per power stroke. In a four-stroke engine, there are two revolutions per power stroke ( $n_r = 2$ ), whereas in a two-stroke engine, there is one revolution per power stroke ( $n_r = 1$ ).

$$\dot{P}_i = W_i \cdot \frac{n}{n_r} \tag{Eq.4.10}$$

The indicated power, however is not the power that you see advertised on a car, we are shown the brake power. The brake power per cycle,  $\dot{P}_b$ , is the remaining energy outputted to the crankshaft and is simply represented as Eq.4.11, where  $\dot{P}_f$  is the frictional power loss per cycle.

$$\dot{P}_b = \dot{P}_i - \dot{P}_f \tag{Eq.4.11}$$

Or it can be represented by the brake work per cycle seen in Eq.4.12.

$$\dot{P}_b = \dot{W}_b \cdot \frac{n}{n_r} \tag{Eq.4.12}$$

We relate engine torque to the brake power through Eq.4.13, where N is the rotational velocity  $(rad \cdot s^{-1})$ .

$$P_b = 2\pi NT \tag{Eq.4.13}$$



The pressure of the combustion process on the cylinder wall is calculated using the mean effective pressure. Eq.4.14 shows the indicated mean effective pressure,  $\bar{p}_i$ , and Eq.4.15 shows the brake mean effective pressure,  $\bar{p}_b$ .

$$\bar{p}_i = \frac{\dot{w}_i}{v_d}$$
(Eq.4.14)  
$$\bar{p}_b = \frac{\dot{w}_b}{v_d}$$
(Eq.4.15)

#### 4.2.3 Engine Chemistry

The most common fuel burned in todays heat engines is a Hydrocarbon chain, a long chain of Carbon atoms with Hydrogen atoms branched around it. They can also have additional constituent parts, which will alter the chemistry of the hydrocarbon, such as turning it into alcohols, fats and acids. Fig.4.21 shows the Hydrocarbon chain, Octane, one of the alkanes found in petrol.



In the ideal air cycles, we model the combustion phase as a heat addition, this occurs when the chemical bonds between the atoms break down and release energy. The net heat release of fuel is classified as its calorific value (CV), the energy released during complete combustion, the case of Octane is shown in Eq.4.16.

$$C_8 H_{18} + 13\frac{1}{2}O_2 \to 8CO_2 + 9H_2O$$
 (Eq.4.16)

The ratio of complete combustion of the fuel with is known as the "stoichiometric" ratio, Typically, the stoichiometric ratio is 14.7 for most petroleum based fuels, meaning an engine needs 14.7 parts of air to one part of fuel. This does not provide the highest work output, but running as close to stoichiometric is the best in terms of pollutants, the combustion products are:

- Stoichiometric
  - Products:  $CO_2$ ,  $H_2O$ ,  $N_2$
- Fuel-rich (More than Stoichiometric)
  - Major Products:  $CO_2$ ,  $H_2O$ ,  $O_2$ ,  $N_2$
  - Minor Products:  $NO_x$ , CO, soot
- Fuel-lean (Less than Stoichiometric)
  - Major Products:  $CO_2$ ,  $H_2O$ ,  $O_2$ ,  $N_2$
  - Minor Products: *CO NO<sub>x</sub>*, unburnt fuel



The volumetric efficiency is essentially how well the engine "breathes", how much air the engine can take in relative to its displacement volume. For a naturally aspirated engine,  $\eta_V$  can have a value up to 100%, a turbocharged or supercharged engine, that uses forced induction, can have a value greater than 100%. The equation for  $\eta_V$  is given as Eq.4.19, where  $\rho_a$  is the density of air.

$$\eta_V = \frac{\text{airflow into intake}}{\text{volume displacement rate}} = \frac{\dot{m}_a \rho_a}{V_d \cdot \frac{n}{n_r}}$$
(Eq.4.19)

The thermal efficiency can be calculated by Eq.4.20.

$$\eta_{th} = \frac{\dot{P}_i}{\dot{m}_f CV}$$
(Eq.4.20)

Mechanical efficiency, the ratio of brake power to indicated power, indicating frictional losses is shown by Eq.4.21.

$$\eta_{mech} = \frac{\dot{P}_b}{\dot{P}_i} \tag{Eq.4.21}$$

The overall efficiency of the system is given by Eq.4.22.

$$\eta_O = \frac{\dot{P}_b}{\dot{m}_f \cdot CV} = \frac{1}{sfc \cdot CV} = \eta_{th} \cdot \eta_{mech}$$
(Eq.4.22)

Example 3

A two-stroke engine is running at 5000 rpm. The displaced volume in the engine is  $1200cm^3$ , with a volumetric efficiency of 130%. The indicated and mechanical efficiency is 45% and 85%, respectively. The air-fuel ratio is 15 and the calorific value of the fuel used is 42MJ/kg. Calculate:

- a) The overall efficiency
- b) The specific fuel consumption
- c) The air mass flow rate
- d) The fuel mass flow rate
- e) The brake power per cycle
- f) The brake work per cycle
- g) The brake means effective pressure.

## Answers:

- a) Overall efficiency:
- b) Specific fuel consumption:

 $\eta_o = \eta_i \eta_{mech} = 0.45 \cdot 0.85 = 0.383$ 

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$$sfc = \frac{1}{\eta_0 \cdot CV} = \frac{1}{0.44 \cdot 42} = 0.0622kg/MJ$$

c) Air mass flow rate:

$$\dot{m}_a = \eta_{VOL} \cdot \rho_a \cdot V_d \cdot \frac{n}{n_R} = 1.3 \cdot 1.18 \cdot 1200 \cdot 10^{-6} \cdot \frac{5000/60}{1} = 0.153 kg/s$$

d) Fuel mass flow rate:

$$\dot{m}_f = \frac{\dot{m}_a}{\lambda} = \frac{0.153}{15} = 0.0102 kg/s$$

e) Brake power per cycle:

$$\dot{P}_b = \frac{\dot{m}_f}{sfc} = \frac{0.0102}{0.0622 \cdot 10^{-6}} = 164kW$$

f) Brake work per cycle:

$$\dot{W}_b = \frac{P_b \cdot n_R}{n} = \frac{164 \cdot 10^3 (5000/60)}{1} = 1971 J$$

g) Brake mean effective pressure

$$\bar{p}_b = \frac{\dot{W}_b}{V_d} = \frac{1971}{1200 \cdot 10^{-6}} = 1.64 MPa$$

## 4.4 Real Engines

#### 4.4.1 Two-stroke vs. Four-stroke

**Theory** A two-stroke engine is one that has a complete combustion cycle: Intake-Compression-Combustion-Exhaust, in one full rotation of the crankshaft, whereas a four-stroke engine completes its cycle with two full rotations of the crankshaft. A better visual representation can be seen in "Other Resources". There are several performance differences between two-stroke and four-stroke, the details of which are better explained in Table.4.1.

Table.4.1: A comparison between two-stroke and four-stroke engines

Four-Stroke Engine	Two-Stroke Engine
Power to weight ratio is lower	Power to weight ratio is higher
Complex design and construction increasing weight, initial costs and maintenance costs	Simple design and construction lowers weight, initial cost and maintenance costs
Lesser cooling and lubrication requirements	Greater cooling and lubrication requirements
Fuel efficiency is comparatively high	Fuel efficiency is comparatively low
Comparatively more torque at low rpm	Comparatively more torque at high rpm
Longer life expectancy	Shorter life expectancy

