Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 19: Electrical and Electronic Principles

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Circuits with Sinusoidal Voltages & Currents

Contents

INTRODUCTION

Analyse simple circuits with sinusoidal voltages and currents

Fundamental quantities of periodic waveforms:

Frequency, period, peak value, phase angle, waveforms, the importance of sinusoids.

Mathematical techniques:

Trigonometric representation of a sinusoid. Rotating phasors and the phasor diagram. Complex notation applied to represent magnitude and phase.

Reactive components:

Principles of the inductor and capacitor. Basic equations, emphasising understanding of rates of change (of voltage with capacitor, current with inductor). Current and voltage phase relationships with steady sinusoidal quantities, representation on phasor diagram.

Circuits with sinusoidal sources:

Current and voltage in series and parallel RL, RC and RLC circuits. Frequency response and resonance.

Mains voltage single-phase systems. Power, root-mean-square power quantities, power factor.

Ideal transformer and rectification:

The ideal transformer, half-wave and full-wave rectification. Use of smoothing capacitor, ripple voltage.

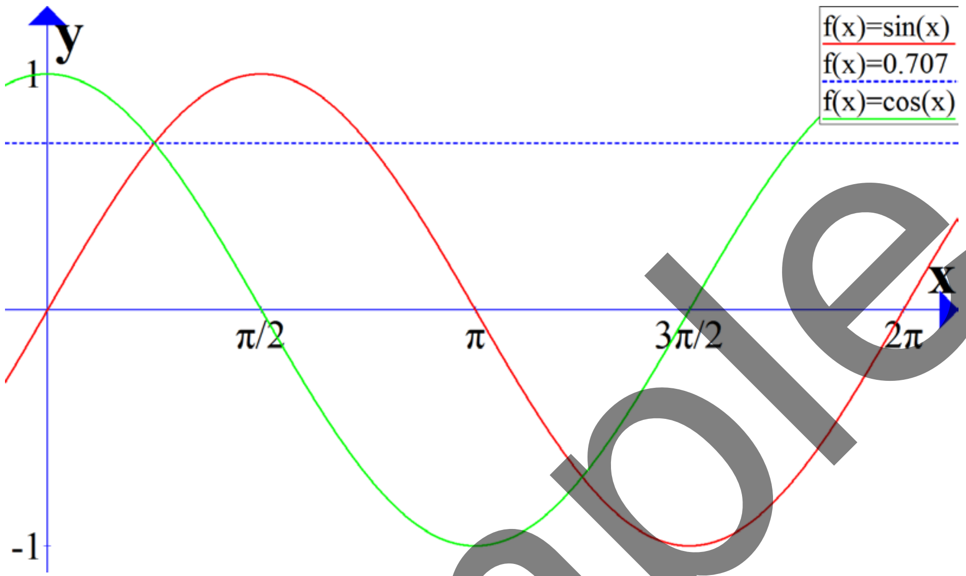
SIMULATOR DOWNLOADS

MicroCap

TINA-TI

Fundamental quantities of periodic waveforms:

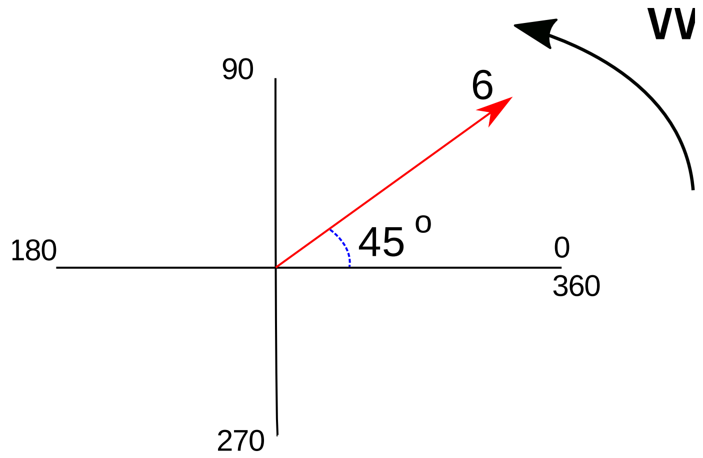
Sinusoidal voltages and currents complete a full period (cycle) in 2π radians (360 degrees). A basic sine wave is shown in red below. Also shown (green) is a cosine wave which leads the sine wave by π/2 radians (90 degrees). Both are sinusoidal in nature/shape, the difference is in the phase (i.e. cosine leads).



We can scale the basic amplitude (strength) of these waves from 1 to any value we wish. For example, the UK mains has a peak value (from 0 all the way to the top of the crest) of 325.2 V. This is not a value we normally associate with the UK mains, we are used to seeing 230 V. The 230 V figure is derived from the root-mean-square (RMS) value of the sinusoid and is the equivalent voltage that a DC source might have to supply to deliver the same power. This RMS value comes out to be 0.707 times the peak value, so 0.707 x 325.2 gives around 230. This 0.707 level of the 1V basic sinusoid is shown as the blue dotted line on the figure.

We may also scale the frequency of the wave (i.e. how many cycles should there be in one second). The UK mains has a frequency of 50 Hz, of course, so that means there are 1/50 = 20 ms in a full cycle. If we want a 1 MHz sinewave then it will have a period of 1 μs (i.e. 1/1,000,000).

We may also change the phase to any value required. Each of these parameters is very useful to know when we come to realise just how useful sine waves and cosine waves are as basic building blocks for other types of signal.



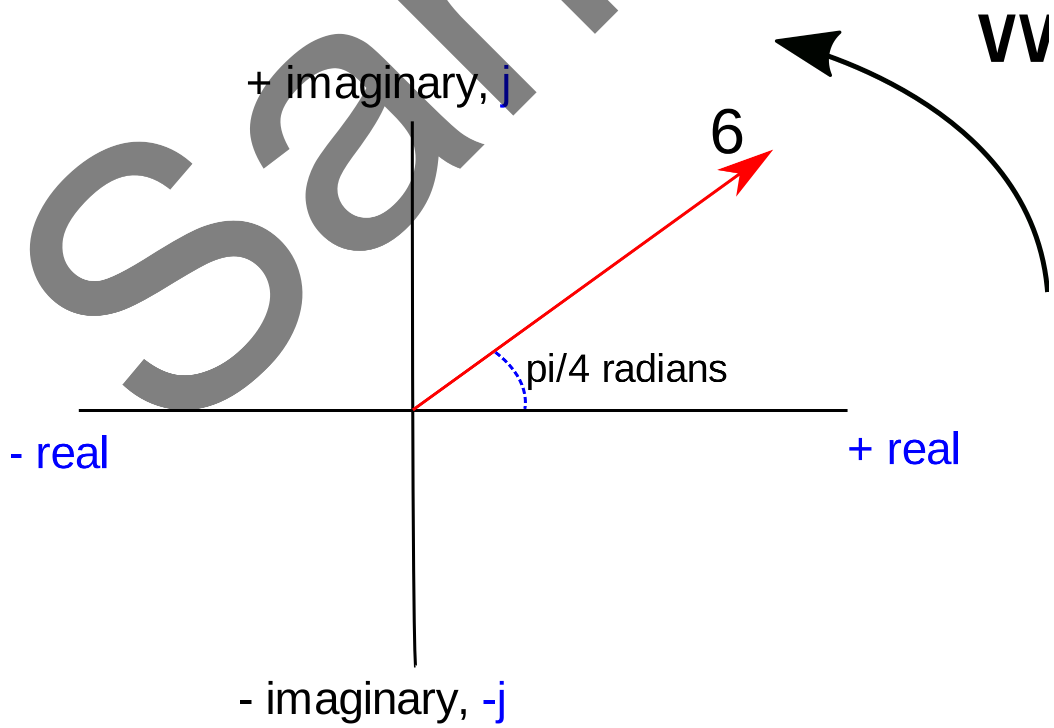
The phasor featured above represents a sine wave with a peak value of 6 (perhaps volts) and an angle of 45 degrees. Here we adopt the convention that the peak value of the sinusoid is used as the phasor length.

Trigonometrically, this can be represented as…

$$6\sin(2\pi f t + 45^{o})$$

The sign ω in the diagram represents 'angular velocity', which is the same as 2πf, and indicates that the phasor rotates anticlockwise.

Rather than have to place angles on the phasor diagram (i.e. 0, 90, 180, 270, 360 degrees, as above) it is more conventional to use Complex Notation to represent the magnitude and phase of a signal/wave. Consider the diagram below…



This diagram is said to be in the 'complex plane'. Don't be put off by that terminology though. All we have done is to label the horizontal axis as real and the vertical axis as imaginary.

Complex notation can be in one of the forms…

$$a + jb$$

or

$$a - jb$$

It is far more convenient to use complex notation when representing signals, or even when representing the sum or difference of signals, than it is to draw phasor diagrams. Here is a simple example…

Suppose we have a voltage (v_1) and a voltage (v_2), defined below, and wish to add these together.

$$v_1 = 10 + \mathrm{j}12$$

$$v_2 = 15 + \mathrm{j}8$$

All we need to do is to add the real components and then add the imaginary (j) components, thus…

$$v_T = v_1 + v_2 = (10 + 15) + j(12 + 8)$$

$$\therefore \quad v_T = 25 + j20 \qquad [volts]$$

Using Pythagoras we may deduce the magnitude of the phasor which represents v_T as…

$$|v_T| = \sqrt{(25)^2 + (20)^2} = \sqrt{1025} = 32.02\, volts$$

We may also use simple trigonometry to deduce the phase angle…

$$\emptyset = tan^{-1}\left(\frac{20}{25}\right) = 38.7^o$$

The very long way to solve this problem with phasor diagrams would have been to draw $v_1 = 10 + \mathrm{j}12$ then draw $v_2 = 15 + \mathrm{j}8$. Then you would have needed to add their horizontal components to find the horizontal resultant of adding the horizontals, then add their vertical components to find the vertical resultant of adding the verticals. The resultant phasor, drawn with extreme care, would have a length of 32.02 volts and a phase angle of 38.7 degrees. Engineers prefer to use the short and accurate method; complex notation, as above.

Reactive components:

The capacitor (possibly two parallel plates which do not touch) is a device designed to store electrical charge. The larger the value of a capacitor the greater its capacity to store additional electrical charge. The current through a capacitor is proportional to the rate of change of voltage across its terminals, and its capacitance value. Put mathematically…

$$i_c = C\frac{dv}{dt} \quad [Amps]$$

That equation can be processed with a little transposition and Calculus to yield another equation which represents the voltage across a capacitor…

$$v_c = \frac{1}{C}\int i\,dt \quad [Volts]$$

It is important to remember that a capacitor cannot be fully-charged in zero time, nor can it be fully-discharged in zero time. A good analogy is to try to fill a bathtub in zero time, or empty it in zero time – can't be done.

RC Circuits

A capacitor presents an obstacle to the flow of current, just as a resistor does. The 'resistance' of a capacitor is frequency dependent and is termed its '***reactance***' and given the symbol X_c. There is a formula you need to remember for capacitive reactance…

$$X_c = \frac{1}{2\pi f C} \quad [\Omega]$$

Here, f represents the frequency and C the capacitor value. The square brackets indicate that the units for this quantity are in Ohms.

If a resistor and capacitor are connected in series then we need to find the resultant obstacle to current flow. This resultant obstacle is known as '***impedance***' and is given the symbol Z. The formula for the impedance of a series RC circuit is…

$$Z = \sqrt{R^2 + {X_c}^2} \quad [\Omega]$$

This comes about because the current through a capacitor leads the voltage across it by 90 degrees. Consider the series RC circuit and its phasor diagram below…