Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 19: Electrical and Electronic Principles

Unit Workbook 2

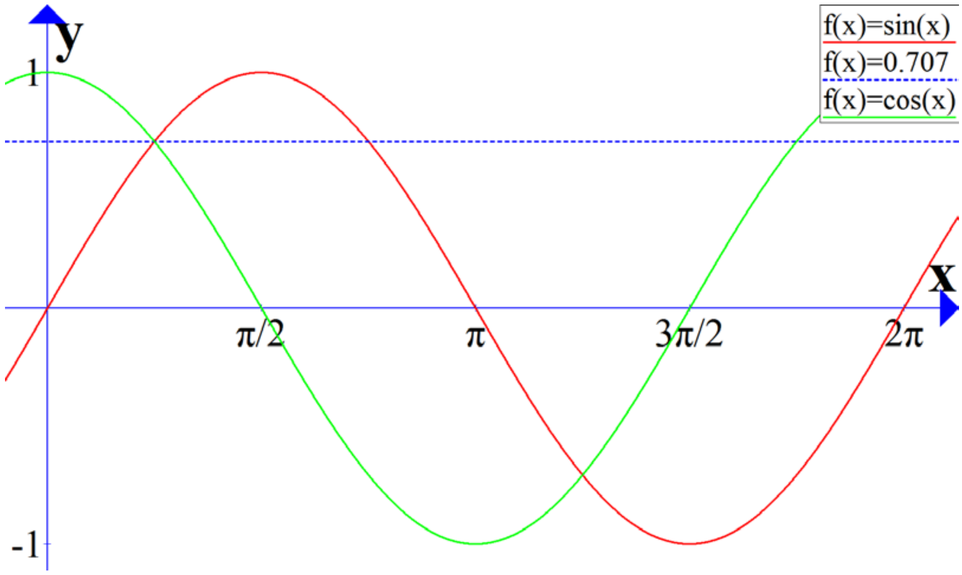
in a series of 4 for this unit

Learning Outcome 2

Circuits with Sinusoidal Voltages & Currents

Fundamental quantities of periodic waveforms:

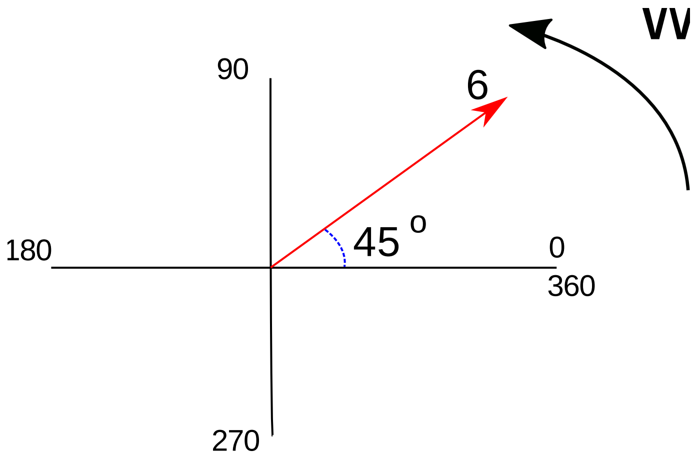
Sinusoidal voltages and currents complete a full period (cycle) in 2π radians (360 degrees). A basic sine wave is shown in red below. Also shown (green) is a cosine wave which leads the sine wave by π/2 radians (90 degrees). Both are sinusoidal in nature/shape, the difference is in the phase (i.e. cosine leads).



We can scale the basic amplitude (strength) of these waves from 1 to any value we wish. For example, the UK mains has a peak value (from 0 all the way to the top of the crest) of 325.2 V. This is not a value we normally associate with the UK mains, we are used to seeing 230 V. The 230 V figure is derived from the root-mean-square (RMS) value of the sinusoid and is the equivalent voltage that a DC source might have to supply to deliver the same power. This RMS value comes out to be 0.707 times the peak value, so 0.707 x 325.2 gives around 230. This 0.707 level of the 1V basic sinusoid is shown as the blue dotted line on the figure.

We may also scale the frequency of the wave (i.e. how many cycles should there be in one second). The UK mains has a frequency of 50 Hz, of course, so that means there are 1/50 = 20 ms in a full cycle. If we want a 1 MHz sinewave then it will have a period of 1 μs (i.e. 1/1,000,000).

We may also change the phase to any value required. Each of these parameters is very useful to know when we come to realise just how useful sine waves and cosine waves are as basic building blocks for other types of signal.



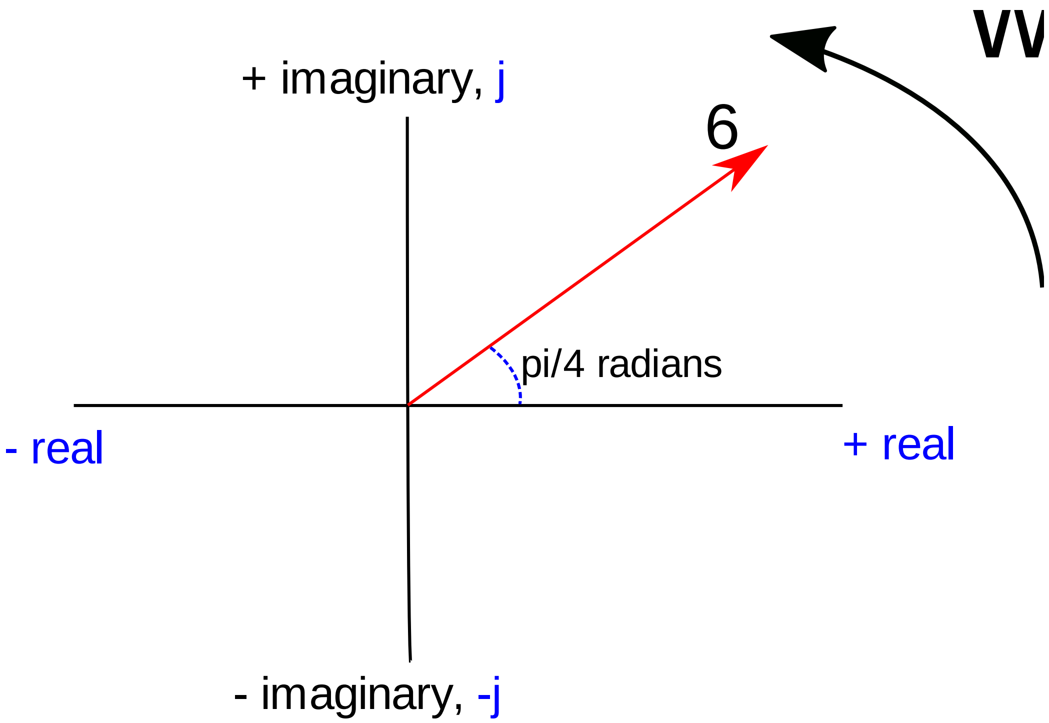
The phasor featured above represents a sine wave with a peak value of 6 (perhaps volts) and an angle of 45 degrees. Here we adopt the convention that the peak value of the sinusoid is used as the phasor length.

Trigonometrically, this can be represented as…

$$6\sin(2\pi ft + 45^o)$$

The sign ω in the diagram represents 'angular velocity', which is the same as 2πf, and indicates that the phasor rotates anticlockwise.

Rather than have to place angles on the phasor diagram (i.e. 0, 90, 180, 270, 360 degrees, as above) it is more conventional to use Complex Notation to represent the magnitude and phase of a signal/wave. Consider the diagram below…



This diagram is said to be in the 'complex plane'. Don't be put off by that terminology though. All we have done is to label the horizontal axis as real and the vertical axis as imaginary.

Complex notation can be in one of the forms…

$$a + jb$$

or

$$a - jb$$

It is far more convenient to use complex notation when representing signals, or even when representing the sum or difference of signals, than it is to draw phasor diagrams. Here is a simple example…

Suppose we have a voltage (v_1) and a voltage (v_2), defined below, and wish to add these together.

$$v_1 = 10 + \mathrm{j}12$$

$$v_2 = 15 + \mathrm{j}8$$

All we need to do is to add the real components and then add the imaginary (j) components, thus…

$$v_T = v_1 + v_2 = (10 + 15) + j(12 + 8)$$

$$\therefore \quad v_T = 25 + j20 \qquad [volts]$$

Using Pythagoras we may deduce the magnitude of the phasor which represents v_T as…

$$|v_T| = \sqrt{(25)^2 + (20)^2} = \sqrt{1025} = 32.02\ volts$$

We may also use simple trigonometry to deduce the phase angle…

$$\emptyset = tan^{-1}\left(\frac{20}{25}\right) = 38.7^o$$

The very long way to solve this problem with phasor diagrams would have been to draw $v_1 = 10 + \mathrm{j}12$ then draw $v_2 = 15 + \mathrm{j}8$. Then you would have needed to add their horizontal components to find the horizontal resultant of adding the horizontals, then add their vertical components to find the vertical resultant of adding the verticals. The resultant phasor, drawn with extreme care, would have a length of 32.02 volts and a phase angle of 38.7 degrees. Engineers prefer to use the short and accurate method; complex notation, as above.

Reactive components:

The capacitor (possibly two parallel plates which do not touch) is a device designed to store electrical charge. The larger the value of a capacitor the greater its capacity to store additional electrical charge. The current through a capacitor is proportional to the rate of change of voltage across its terminals, and its capacitance value. Put mathematically…

$$i_c = C\frac{dv}{dt} \quad [Amps]$$

That equation can be processed with a little transposition and Calculus to yield another equation which represents the voltage across a capacitor…

$$v_c = \frac{1}{C}\int i\,dt \quad [Volts]$$

It is important to remember that a capacitor cannot be fully-charged in zero time, nor can it be fully-discharged in zero time. A good analogy is to try to fill a bathtub in zero time, or empty it in zero time – can't be done.

RC Circuits

A capacitor presents an obstacle to the flow of current, just as a resistor does. The 'resistance' of a capacitor is frequency dependent and is termed its '***reactance***' and given the symbol X_c. There is a formula you need to remember for capacitive reactance…

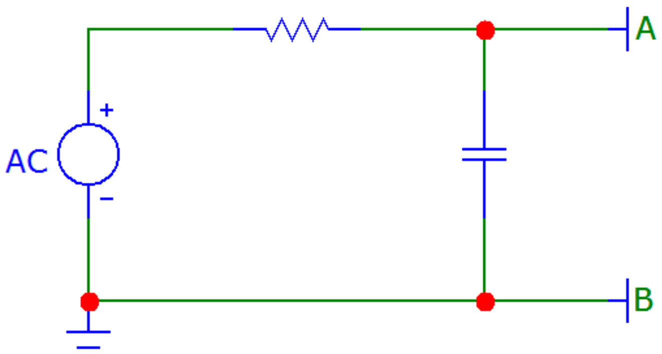
$$X_c = \frac{1}{2\pi f C} \quad [\Omega]$$

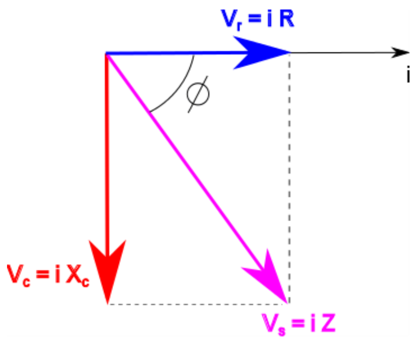
Here, f represents the frequency and C the capacitor value. The square brackets indicate that the units for this quantity are in Ohms.

If a resistor and capacitor are connected in series then we need to find the resultant obstacle to current flow. This resultant obstacle is known as '***impedance***' and is given the symbol Z. The formula for the impedance of a series RC circuit is…

$$Z = \sqrt{R^2 + {X_c}^2} \quad [\Omega]$$

This comes about because the current through a capacitor leads the voltage across it by 90 degrees. Consider the series RC circuit and its phasor diagram below…



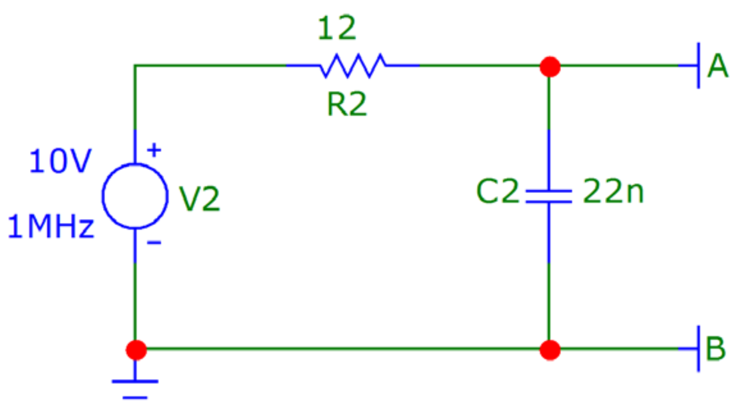


The reason why the current through a capacitor leads the voltage across it is because the current is at a peak the moment when charging commences – the peak voltage occurs across the capacitor sometime later. Each phasor is multiplied by the current i, so we could easily remove this i to give an impedance diagram containing just R, X_C and Z. This is a right-angled triangle situation which invokes Pythagoras' Theorem. So, the square of Z equals the square of R plus the square of X_C.

Let's now put all of this new knowledge into practice by way of a worked example.

Worked Example 1

Consider the RC circuit below.



Calculate the:

a) Reactance of the capacitor

Calculate the:

a) Reactance of the coil
b) Impedance of the circuit
c) Supply Current

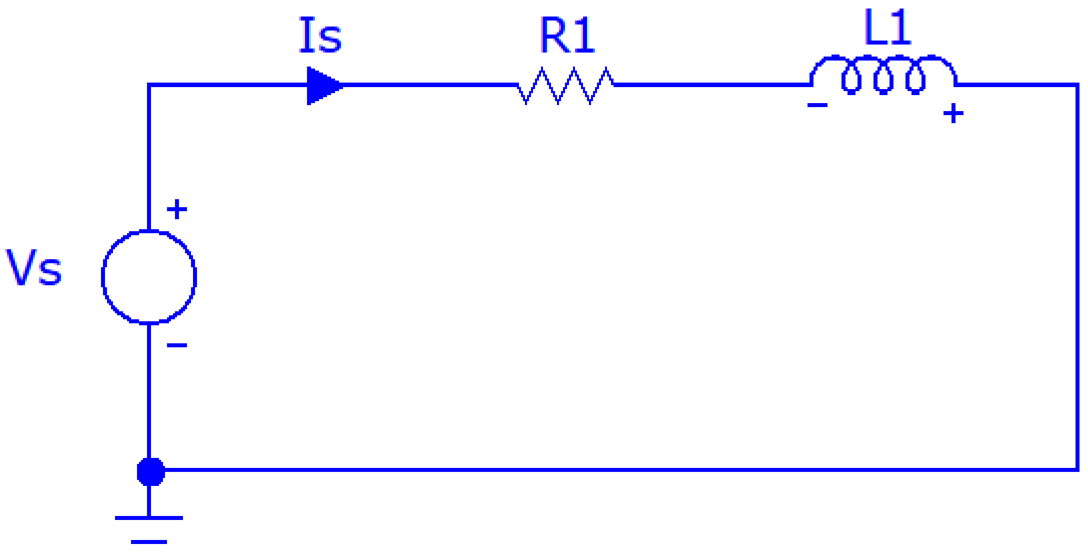
a) $X_L = 2\pi f L = 2\pi \times 1 \times 10^6 \times 1.5 \times 10^{-6} = 9.42\ \Omega$

b) $Z = \sqrt{R^2 + {X_L}^2} = \sqrt{12^2 + 9.42^2} = \sqrt{144 + 88.7} = \sqrt{232.7} = 15.3\ \Omega$

c) $i = \frac{V_s}{Z} = \frac{10}{15.3} = 654\ mA$

Circuits with sinusoidal sources:

Series RL Circuit



Relevant formulae:

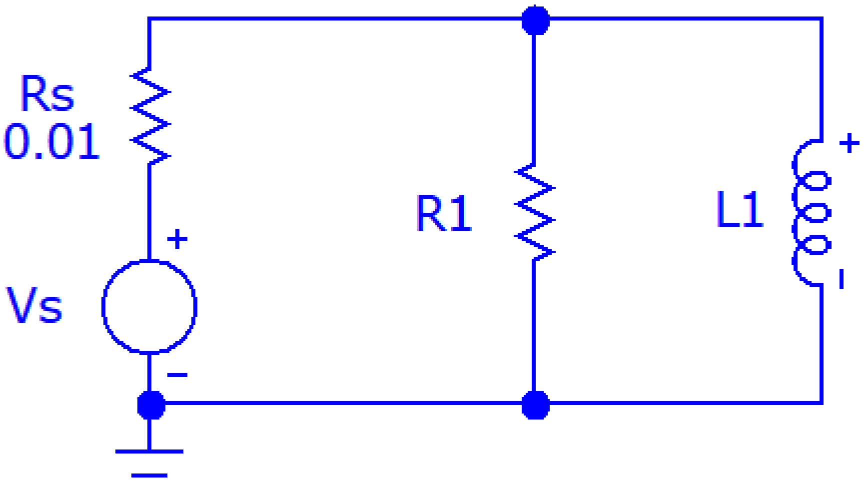
$Inductive\ Reactance, X_L = 2\pi f L$

$Impedance\ magnitude, Z = \sqrt{R^2 + {X_L}^2}$

$Source\ current\ magnitude, i_s = \frac{V_s}{Z}$

$Resistor\ voltage\ magnitude, v_R = iR$

Parallel RL Circuit



Relevant formulae:

$$Inductive\ Reactance, X_L = 2\pi f L$$

$$Impedance\ magnitude, Z = \frac{RX_L}{\sqrt{R^2 + X_L{}^2}}$$

$$Source\ current\ magnitude, i_s = \frac{V_s}{Z}$$

$$Resistor\ current\ magnitude, i_R = \frac{v_s}{R}$$

$$Inductor\ current\ magnitude, i_L = \frac{v_s}{X_L}$$

Worked Example 4

A parallel RL circuit has; V_s = 10 V rms; f_s = 1 kHz; R = 100 Ω; L = 10 mH. Determine…

a) X_L
b) Z
c) i_s
d) i_R
e) i_L

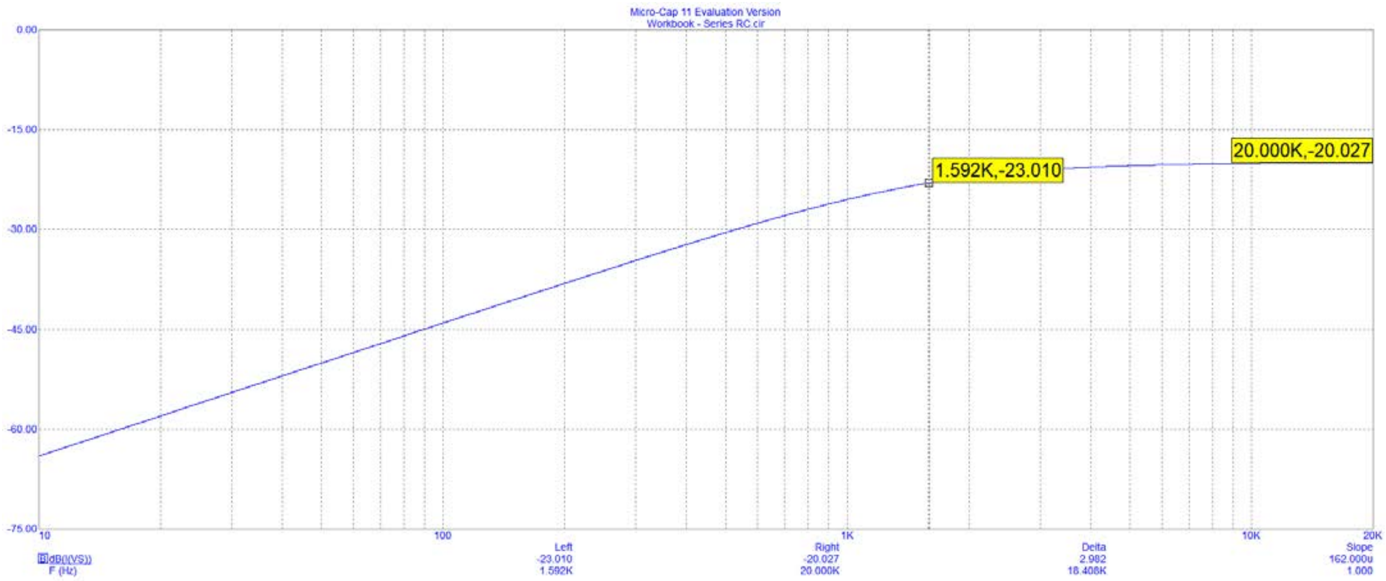
Worked Example 5

A series RC circuit has; V_s = 10 V rms; f_s = 1 kHz; R = 100 Ω; C = 1 μF. Determine…

a) X_C
b) Z
c) i_s
d) v_R
e) v_L
f) f_c

ANSWERS

a) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1000)(1\times10^{-6})} = 159\ \Omega$
b) $Z = \sqrt{R^2 + {X_C}^2} = \sqrt{100^2 + 159^2} = 187.8\ \Omega$
c) $i_s = \frac{V_s}{Z} = \frac{10}{187.8} = 53.2\ mA$
d) $v_R = iR = 0.0532 \times 100 = 5.32\ V$
e) $v_C = iX_C = 0.0532 \times 159 = 8.46\ V$
f) $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(100)(1\times10^{-6})} = 1.59\ kHz$



ANSWERS

a) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1000)(1\times10^{-6})} = 159\ \Omega$

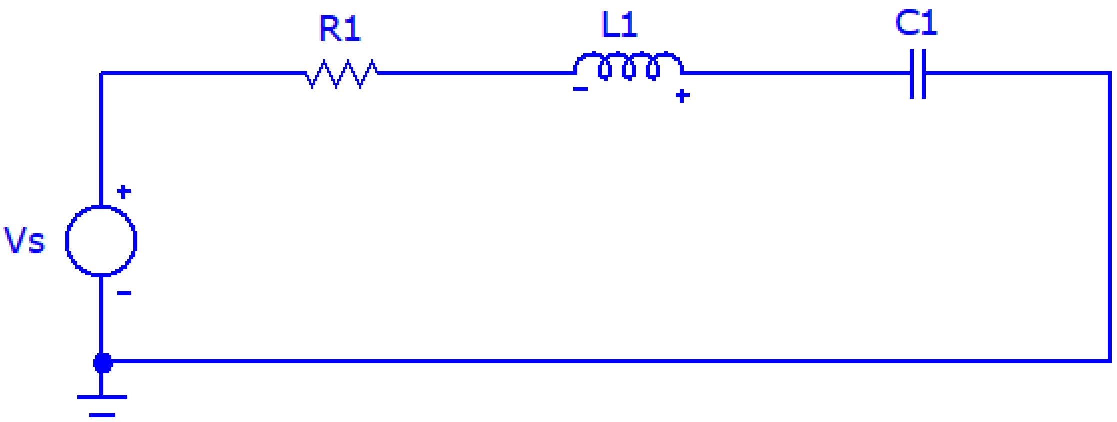
b) $Z = \frac{RX_C}{\sqrt{R^2+X_C{}^2}} = \frac{100(159)}{\sqrt{100^2+159^2}} = 84.6\ \Omega$

c) $i_s = \frac{V_s}{Z} = \frac{10}{84.6} = 118.2\ mA$

d) $i_R = \frac{v_s}{R} = \frac{10}{100} = 100\ mA$

e) $i_C = \frac{v_s}{X_C} = \frac{10}{159} = 62.9\ mA$

Series RLC Circuit



Relevant formulae:

$$Resonant\ frequency, f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$Inductive\ Reactance, X_L = 2\pi f L$$

$$Capacitive\ Reactance, X_C = \frac{1}{2\pi f C}$$

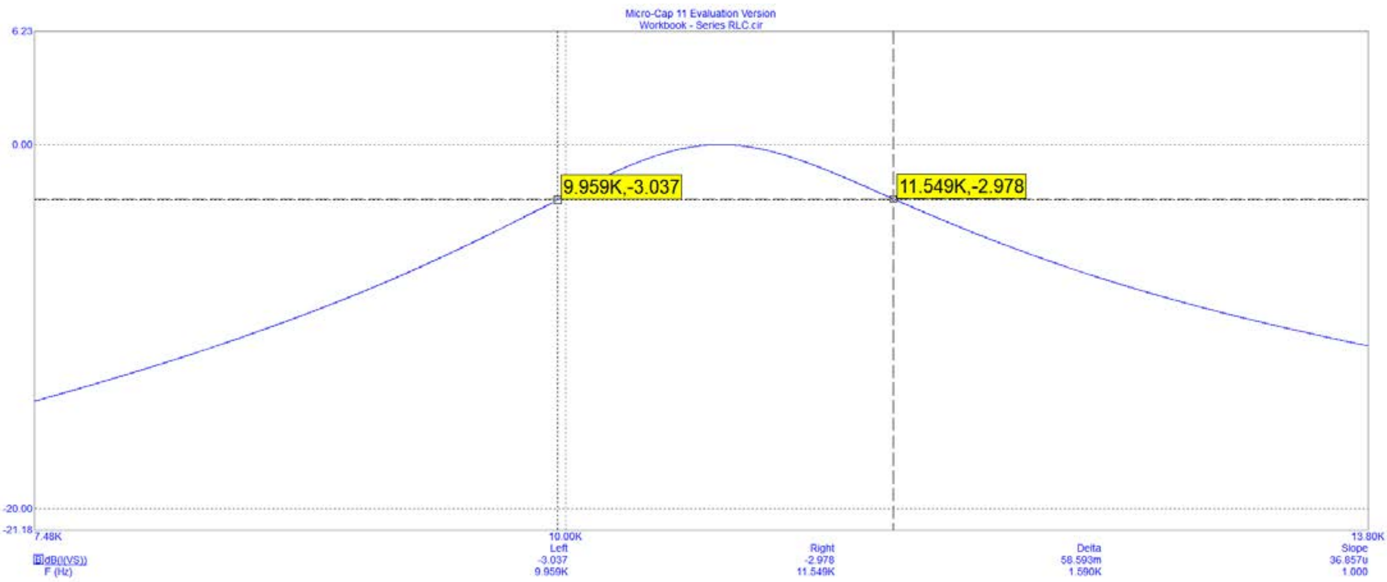
$$Impedance\ magnitude, Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Source\ current\ magnitude, i_s = \frac{V_s}{Z}$$

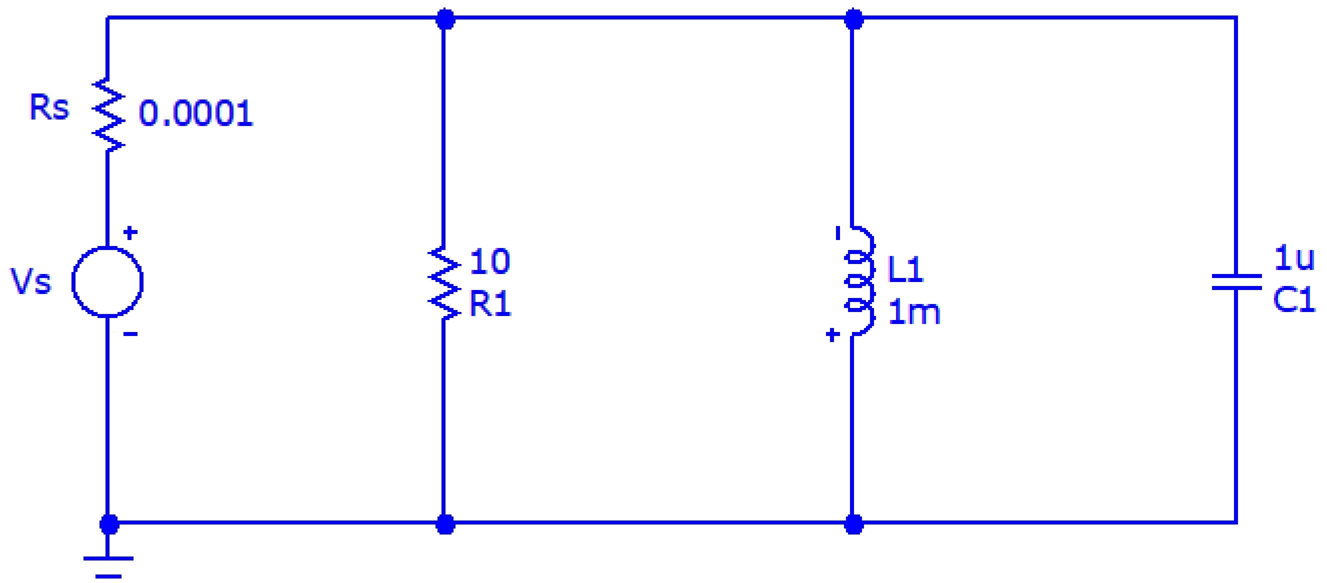
$$Resistor\ voltage\ magnitude, v_R = iR$$

$$Inductor\ voltage\ magnitude, v_L = iX_L$$

$$Capacitor\ voltage\ magnitude, v_C = iX_C$$



Parallel RLC Circuit



Relevant formulae:

$$Resonant\ frequency, f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$Inductive\ Reactance, X_L = 2\pi f L$$

$$Capacitive\ Reactance, X_C = \frac{1}{2\pi f C}$$

c) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(5030)(1\times10^{-6})} = 31.6\ \Omega$

d) $Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{31.6} - \frac{1}{31.6}\right)^2}} = 10\ \Omega$

e) $i_s = \frac{V_S}{Z} = \frac{10}{10} = 1\ A$

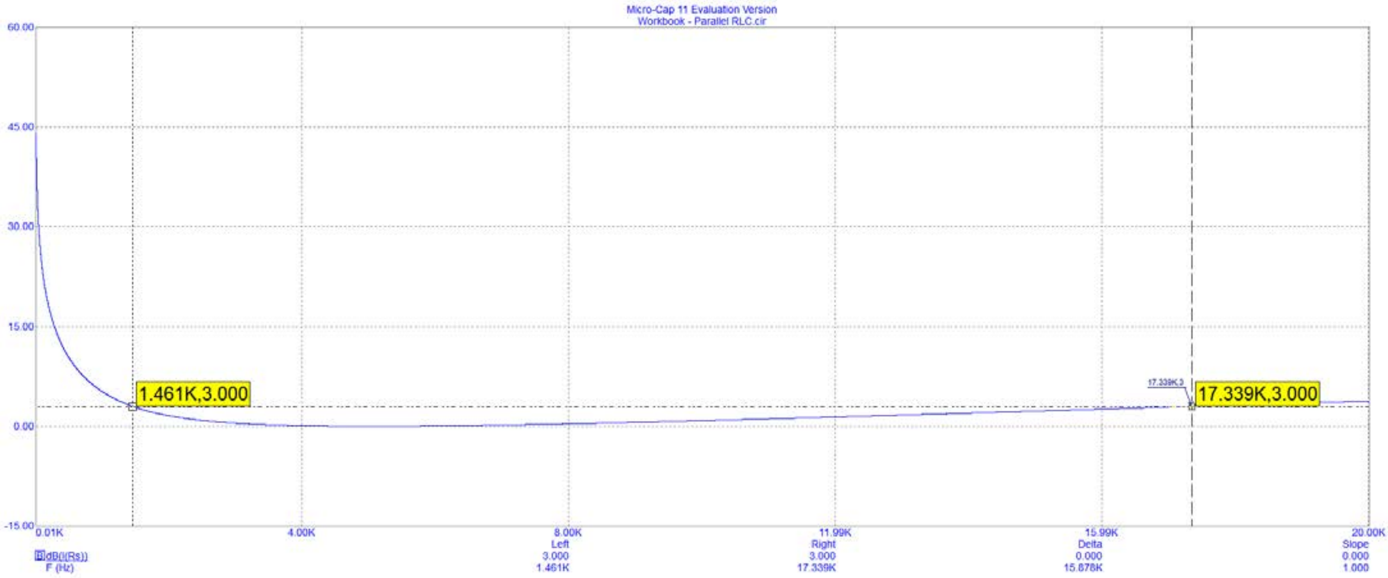
f) $i_R = \frac{v_S}{R} = \frac{10}{10} = 1\ A$

g) $i_L = \frac{v_S}{X_L} = \frac{10}{31.6} = 0.32\ A$

h) $i_C = \frac{v_S}{X_C} = \frac{10}{31.6} = 0.32\ A$

i) $Q = R\sqrt{\frac{C}{L}} = 10\sqrt{\frac{1\times10^{-6}}{0.001}} = 0.32$

j) $BW = \frac{f_o}{Q} = \frac{5030}{0.32} = 15.7\ kHz\ (asymmetric)$



Phase angles and j notation

When we have circuits containing just resistors then life is so easy in terms of circuit analysis. Most useful circuits also contain capacitors and inductors (usually coils and windings). The introduction of capacitors and inductors into circuits causes 'phase angles' in our calculations. The study of these phase angles is made much easier by introducing complex numbers.

From your level 3 studies you will have come across Inductive Reactance (X_L) and Capacitive Reactance (X_C). These terms are used to quantify the amount of 'opposition' caused by capacitors and inductors to changes in current or voltage. The term 'reactance' is brought about because capacitors cannot be charged or discharged in zero time, and inductors cannot be energised or de-energised in zero time. A good analogy for capacitors is, as mentioned earlier, the amount of water in a bathtub. It is impossible to fill a bathtub in zero time, and it's also impossible to empty a bathtub in zero time. The amount of reactance from capacitors and inductors is a function of their manufactured properties and the frequency of operation. Let's review the equations for these reactances…

$$X_L = 2\pi f L \quad [\Omega]$$

$$X_C = \frac{1}{2\pi f C} \quad [\Omega]$$

where;

$X_L = inductive\ reactance\ (measured\ in\ Ohms, \Omega)$

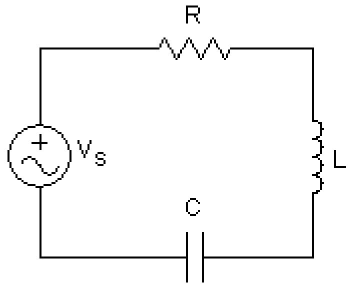
$X_C = capacitive\ reactance\ (measured\ in\ Ohms, \Omega)$

$f = frequency\ (measured\ in\ Hertz, Hz)$

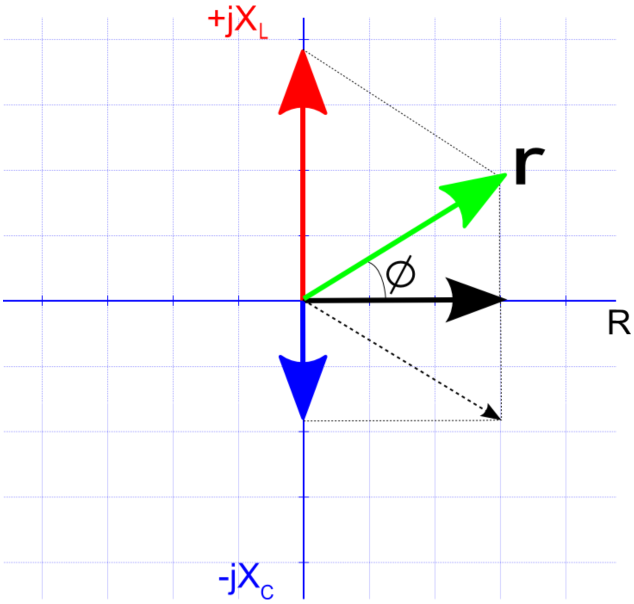
$L = inductance\ (measured\ in\ Henries,$ H)

$C = capacitance\ (measured\ in\ Farads, F)$

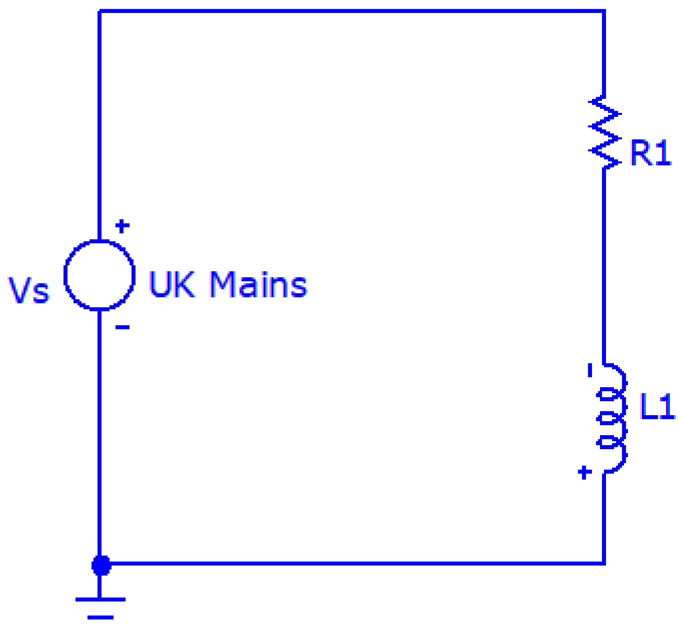
Consider the RLC circuit below…



We can draw a phasor diagram for this circuit, as follows…



Mains voltage single-phase systems



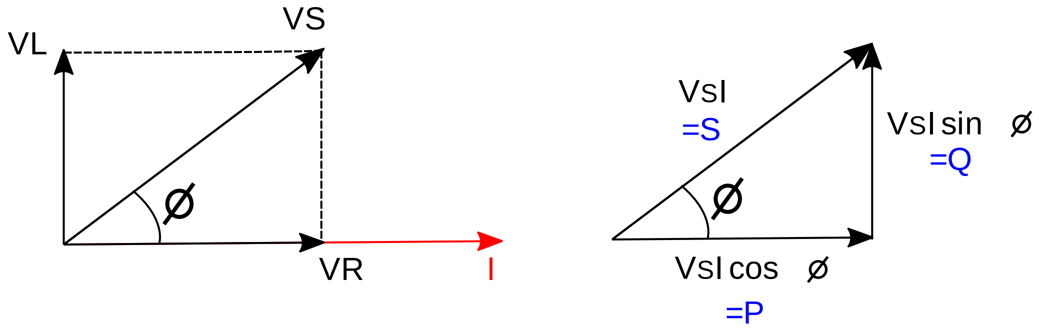
In the circuit above we have a single-phase UK mains source. This source has a frequency of 50 Hz and a peak voltage value of 325.27 V. Usually it is not the peak voltage value which is stated for a source, but rather its root-mean-square value (its 'rms' value).

The rms value is that voltage value of the waveform which would produce an equivalent heating effect from a DC source. This voltage happens to be the peak value divided by $\sqrt{2}$.

$$v_{rms} = \frac{v_{pk}}{\sqrt{2}}$$

$$\therefore \quad v_{rms} = \frac{v_{pk}}{\sqrt{2}} = \frac{325.27}{\sqrt{2}} = 230\ V$$

Consider the phasor diagram for this series circuit…

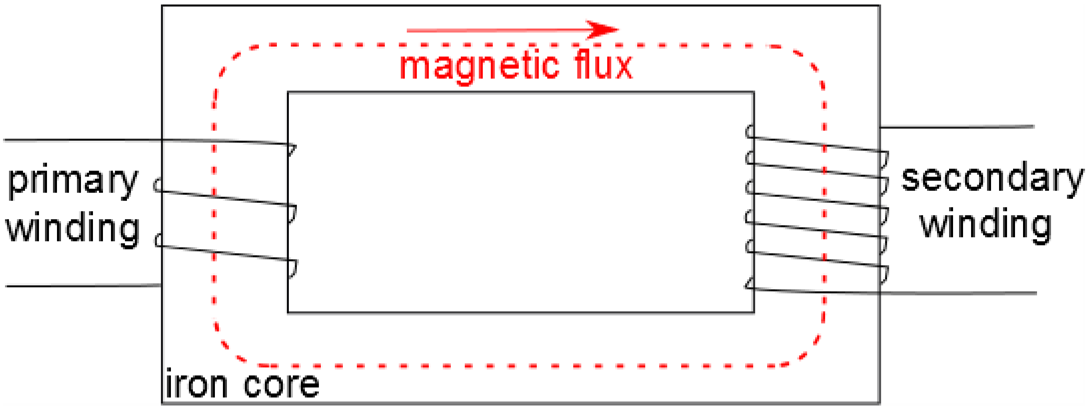


The phasor diagram, on the left, shows that the voltage across the resistor is in phase with the current. However, the voltage across the inductor leads the current by 90 degrees. The resultant of the resistor and

Ideal transformer and rectification:

Transformer Principles

A ***transformer*** can transfer energy by means of electromagnetic induction. A typical transformer will have a primary winding and a secondary winding, both around a common iron core, as shown below.

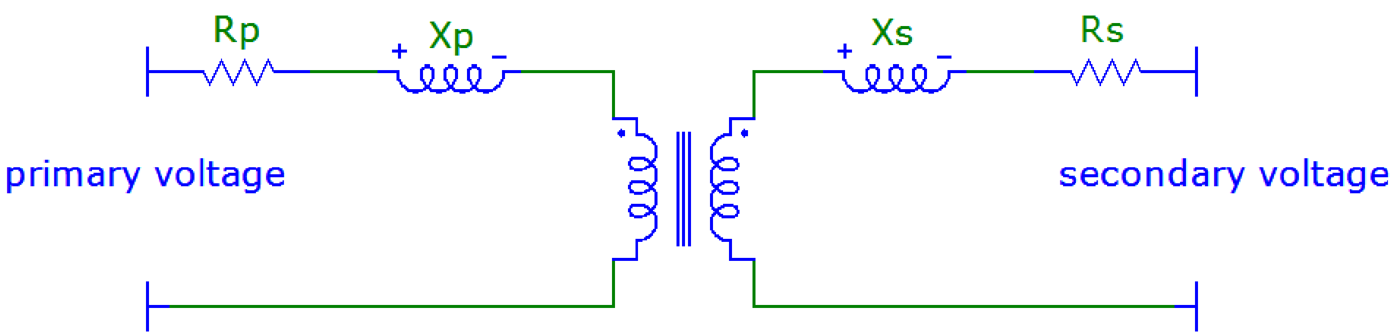


When an AC current is applied to the primary winding this generates magnetic flux within the iron core, which links and cuts the windings on the secondary. This concept of magnetic flux linkage produces a voltage on the secondary.

Video 1 **This short video introduces you to basic transformer principles**

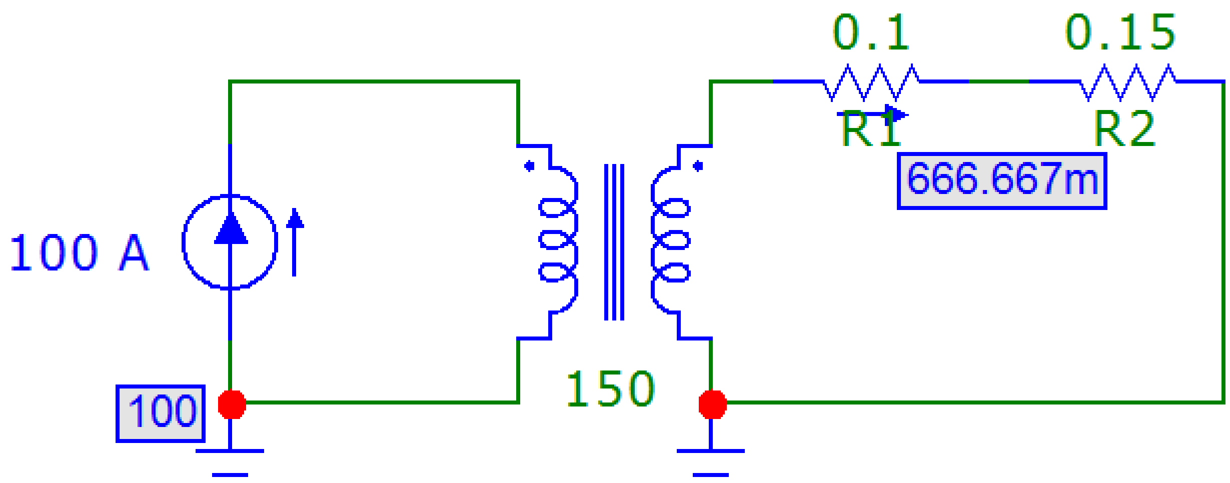
Video 2 ***Check out types of transformer windings in this short video***

A simplified equivalent circuit for a transformer is shown below.



Here the resistance and reactance of the primary and secondary windings are accounted for.

These results are confirmed with the following MicroCap simulation…



Worked Example 16

An audio amplifier output stage has an equivalent output resistance of 648Ω. It is desired to match this output stage to a loudspeaker of resistance 8Ω. Calculate the optimum turns ratio of the transformer.

Let's start our analysis here with the basic ideal transformer expression…

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

From this we can write…

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} \quad \therefore \quad V_p = V_s \times \frac{N_p}{N_s}$$

… and also…

$$\frac{N_s}{N_p} = \frac{I_p}{I_s} \quad \therefore \quad I_p = I_s \times \frac{N_s}{N_p}$$

We may now combine these expressions to give…

$$R_p = \frac{V_p}{I_p} = \frac{V_s \times \frac{N_p}{N_s}}{I_s \times \frac{N_s}{N_p}} = \frac{V_s}{I_s} \times \frac{{N_p}^2}{{N_s}^2} = R_s\left(\frac{{N_p}^2}{{N_s}^2}\right)$$

So our key expression is…

$$\frac{\boldsymbol{R_p}}{\boldsymbol{R_s}} = \frac{\boldsymbol{N_p}^2}{\boldsymbol{N_s}^2}$$

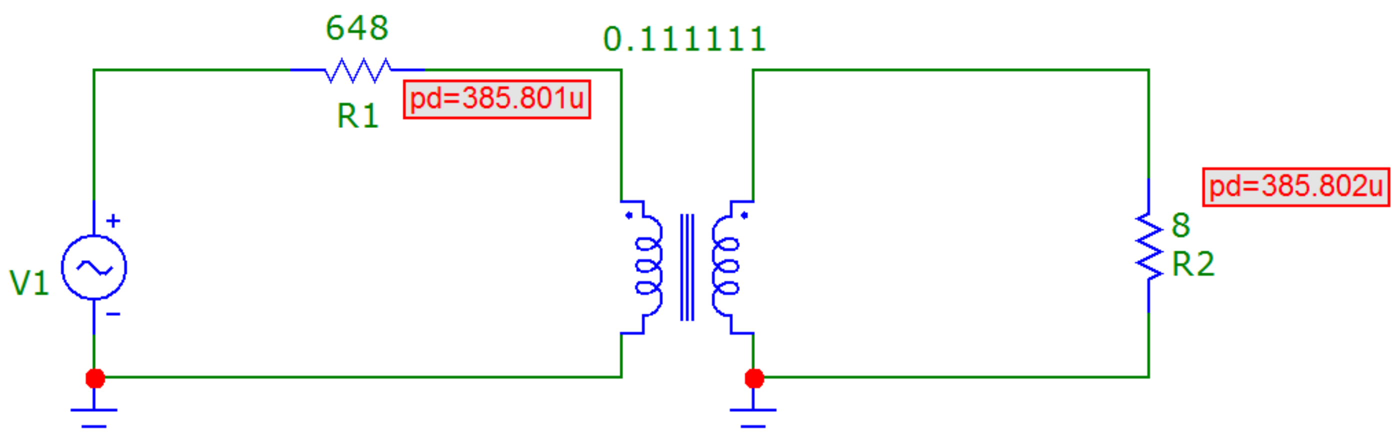
The optimum turns ratio of the transformer is the one whereby we match the output resistance of the amplifier to the resistance of the loudspeaker. Since there is no direct electrical connection between the amplifier output and the loudspeaker it is the job of the designer to find the optimum turns ratio of the transformer. Let's put the numbers in…

$$\frac{\boldsymbol{R_p}}{\boldsymbol{R_s}} = \frac{\boldsymbol{N_p}^2}{\boldsymbol{N_s}^2} = \frac{\mathbf{648}}{\mathbf{8}} = \mathbf{81}$$

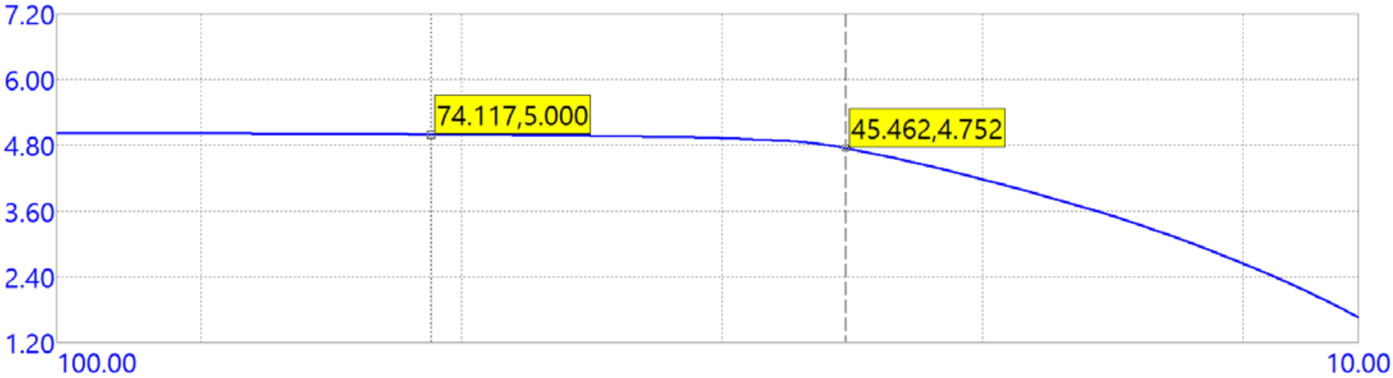
So, we need…

$$\frac{N_p{}^2}{N_s{}^2} = 81 \quad \therefore \quad \frac{N_p}{N_s} = 9$$

We have our answer. The optimal transformer to match the amplifier to the loudspeaker in this example has nine times the number of turns on the primary as it does on the secondary (i.e. a turns ratio of 9:1). These results are confirmed in the MicroCap simulation below…



Here, R1 represents the amplifier and R2 the speaker. Notice that since we have matched the amplifier and speaker with an optimal turns ratio transformer then we are invoking the maximum power transfer theorem (see workbook 1). This means that maximum power from the amplifier is developed in the 8 Ohm load. The simulation clearly shows that 385μW is developed in the amplifier and the load. The figure in green represents a turns ratio of 1:9, which is 0.111111. Try the simulation yourself, but ensure that you use an ***ideal*** transformer.



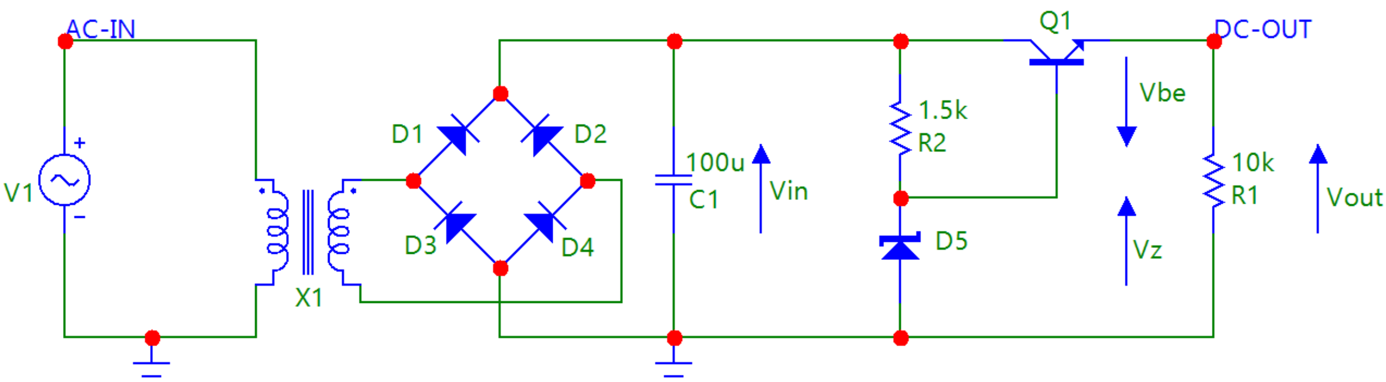
	Left	Right	Delta	Slope
V(OUTPUT) (V)	5.000	4.752	-248.546m	8.674m
R(RL)	74.117	45.462	-28.655	1.000

In the curve above, the load resistor's value (RL) is swept from 100 Ω on the left of the horizontal axis, down to 10Ω on the right. The effect of this sweep alerts us to the point where the circuit ceases to be a stable supply for the load – where RL is around the same value as R1. The yellow marker on the left indicates that the circuit is happily performing its intended function here (to deliver a nice steady 5V to the load) with a load resistance value of around 74Ω. The yellow marker on the right indicates that the output voltage has dropped to around 4.75V, when RL drops to around 45Ω, and has reached a dangerously low value to be able to reliably drive 5V logic circuits.

If simple zener circuits such as this are to be used to provide high currents to a load then both R1 and the zener diode itself must be rated at considerable power dissipations, implying physical bulk, increased cost and wasted energy in the form of dissipated heat.

Series Transistor Regulated Power Supply

Consider the circuit given below. Rather than mimic a non-ideal AC supply, as done for the zener circuit above, we can design a full AC supply from the 230V mains. This circuit has a step-down transformer, bridge rectifier (consisting of those 4 diodes) and a reservoir capacitor. That will provide some raw DC

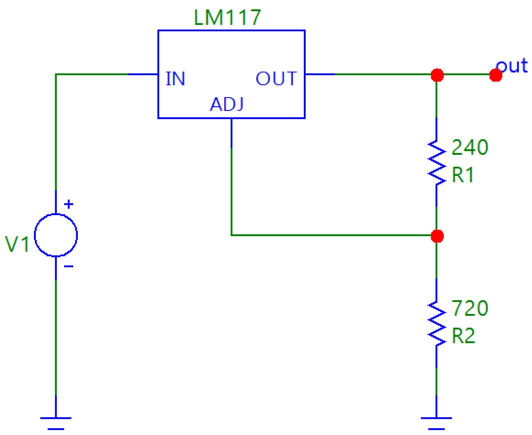


to the resistor-zener series combination. This particular zener can be quite low-wattage because it is the transistor which is really providing the regulation. Let's see how the regulation works. We have…

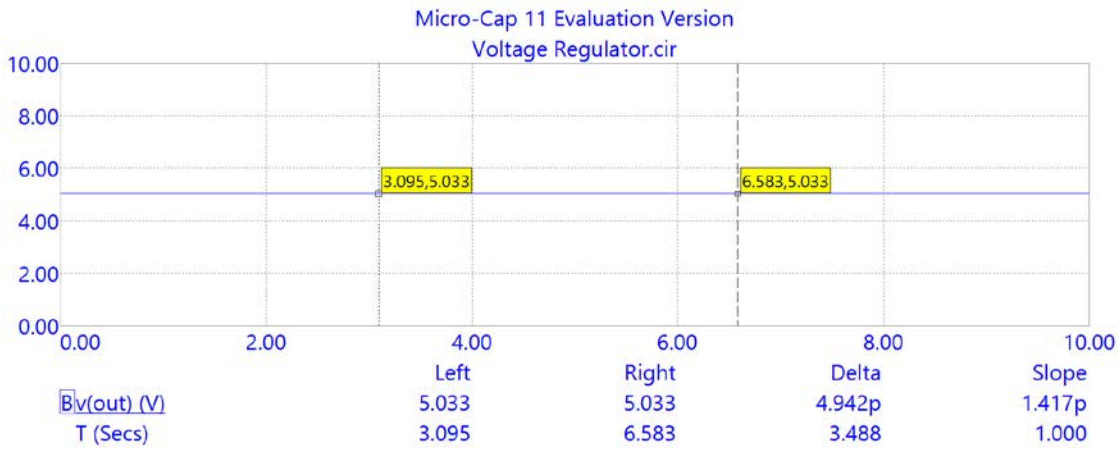
IC Regulated Power Supply

Many varieties of voltage regulator are packaged into single modules. The simplest of these provides a fixed output voltage from a variable input voltage (the input voltage will have minimum and maximum values). These are usually 3-pin types, such as the LM7805.

A more flexible 3-pin type will allow us to select the output voltage by configuring two external resistors. A good example is the LM117. A useful implementation of the LM117 is to provide a regulated 5 volt DC supply for a microprocessor system, as shown below.



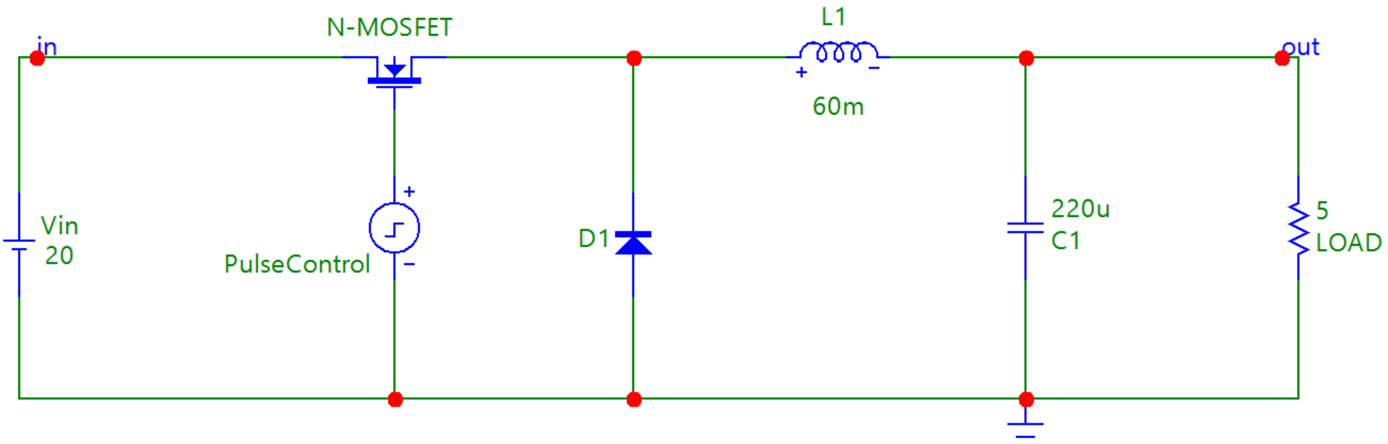
This chip can provide output voltages between 1.25V and 37V. The output voltage is selected by configuring the two resistor values. The voltage across R1 is kept constant at 1.25V and the current through R2 can be considered to be the same as the current through R1. Correct operation of the LM117 requires that R1 has a value of 240Ω. A little bit of basic circuit analysis then yields a 720Ω resistance for R2. A Transient Analysis for this circuit is as follows;



From this plot it is clear that a quite steady 5V is supplied by the regulator. Read the datasheet for more detailed information on this very useful device.

Switched Mode Power Supply

If we have a DC source and wish to regulate its voltage to a lower level, with high energy efficiency, then a Buck Converter is the ideal circuit. A typical Buck Converter is shown below.



The term 'Switched Mode' refers to an electronic switch, in this case the N-MOSFET shown. When a positive pulse is presented to the gate (control) pin of the transistor then it is turned fully on. That will mean that the voltage at the input (Vin) is presented to the junction formed by the diode/inductor. Current will gradually increase through the inductor, charging the capacitor and supplying the load with an increased voltage.

When the pulse is removed from the transistor gate then the transistor can be considered to be open circuit. That will mean that OV is now sitting at the junction of the diode/inductor. The inductor generates a back-emf to oppose this sudden change in potential, resulting in a current flowing in a clockwise fashion through the inductor/capacitor/diode combination. This means that less current flows through the load, with a consequent reduction in load voltage.

If we construct the pulse to the transistor gate in a way that it has a low periodic time (in the example circuit above it is 5µs) then the increase/decrease in load voltage happens so quickly that the output voltage can be considered constant (i.e. regulated).

If the control pulse has equal on and off times then it has a duty cycle of 50% and the output voltage will be 50% of the input (Vin), resulting in an output voltage of 10V. Should the pulse duty cycle be 25% (i.e ON for 1.25µs and OFF for 3.75µs) then the output voltage will be 25% of the input (i.e. 5V). The Transient Analysis shown below indicates an output voltage of close to 5V, due to a pulse control duty cycle of 25%.