Pearson BTEC Levels 4 and 5 Higher Nationals in Engineering (RQF)

Unit 2: Engineering Maths (core)

Unit Workbook 1

in a series of 4 for this unit

Learning Outcome 1

Mathematical Methods

Contents

Section 1: Mathematical Methods

INTRODUCTION

Identify the relevance of mathematical methods to a variety of conceptualised engineering examples.

Mathematical concepts:

Dimensional analysis.

Arithmetic and geometric progressions.

Functions:

Exponential, logarithmic, circular and hyperbolic functions

GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

Purpose — Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

Theory — Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

Example — The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself. Many of the examples given resemble assignment questions which will come your way, so follow them through diligently.

Question — Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence. As an Online Learner it is important that the answers to questions are immediately available to you.. Contact your Unit Tutor if you need help.

Challenge — You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

Video — Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

1 Mathematical Concepts

1.1 Dimensional analysis

Very often in engineering we encounter numbers with associated physical units, for example; 3 volts (3 V), 5 kilograms per metre (5 kg/m or 5 kg.m^{-1}), 12 cubic metres (12 m^3) etc. Dimensional Analysis is a neat way to analyse those physical units, enabling us to check the validity of an equation (an equation has just one unknown and an 'equals' sign) or a formula (a formula has two or more unknowns and an 'equals' sign). We can even use dimensional analysis to generate an equation or formula.

Dimensional analysis uses the basic dimensions of **Length (L)**, **Mass (M)** and **Time (T)** to formulate the dimensions of other quantities. For example, the area of a football field can be expressed in square metres, and square metres is 'a length times a length', of course. Let's take a look at some notation for this simple analysis…

$$[Area] = [length \times length] = L^2$$

The use of the square brackets [ ] here denotes 'dimensions of'.

Another example could be 'density'. Let's look at that one…

$$[Density] = \left[\frac{mass}{volume}\right] = \frac{M}{L^3} = ML^{-3}$$

Yet another could be velocity (speed)…

$$[Velocity] = \left[\frac{metres}{seconds}\right] = \left[\frac{length}{time}\right] = \frac{L}{T} = LT^{-1}$$

Let's take a look at some rules which we need to be aware of when undertaking dimensional analysis…

Rule 1:

Constants (i.e. numbers) must be ignored. For example, if we see 3 m as a length then the dimensions are just L. The 3 is ignored.

Rule 2:

Angles must be ignored. An angle cannot be represented with our basic dimensions of length, mass or time.

Rule 3:

If physical quantities are to be added or subtracted, they MUST have the same dimensions. We cannot add apples to oranges, nor can we add voltage to current. However, we can add voltage to voltage.

At this point it is worth noting the dimensions of some common quantities encountered in engineering…

[mass]	**M**	**M**
[length]	**L**	**L**
[time]	**T**	**T**
[area]	**[length × length]**	L^2
[volume]	**[area × length]**	L^3
[density]	**[mass/volume]**	ML^{-3}
[velocity]	**[length/time]**	LT^{-1}
[acceleration]	**[velocity/time]**	LT^{-2}
[force]	**[mass × acceleration]**	MLT^{-2}
[moment of force, torque]	$MLT^{-2} \times L$	ML^2T^{-2}
[impulse]	**[force × time]**	MLT^{-1}
[momentum]	**[mass × velocity]**	MLT^{-1}
[work]	**[force × distance]**	ML^2T^{-2}
[kinetic energy]	**[mass × (velocity)^2]**	ML^2T^{-2}
[power]	**[work/time]**	ML^2T^{-3}
[electrical charge]	**[length x (force)$^{0.5}$]**	$L^{1.5}M^{0.5}T^{-1}$
[current]	**[electrical charge/time]**	$L^{1.5}M^{0.5}T^{-2}$
[electric potential energy]	**[charge^2/distance]**	L^2MT^{-2}
[voltage]	**[electric potential energy/charge]**	$L^{0.5}M^{0.5}T^{-1}$
[capacitance]	**[charge/voltage]**	**L**
[inductance]	**[voltage x time/current]**	$L^{-1}T^2$
[electrical resistance]	**[voltage/current]**	$L^{-1}T$

Worked Example 1

Ohm's Law relates resistance (R), voltage (V) and current (I) as;

$$I = \frac{V}{R}$$

Use the dimensions of V and R to determine the dimensions of I.

ANSWER

$$[V] = L^{0.5}M^{0.5}T^{-1}$$

$$[R] = L^{-1}T$$

$$\therefore \quad [I] = \left[\frac{V}{R}\right] = \frac{L^{0.5}M^{0.5}T^{-1}}{L^{-1}T} = L^{0.5--1}M^{0.5}T^{-1-1}$$

$$\therefore \quad \boldsymbol{[I] = L^{1.5}M^{0.5}T^{-2}}$$

Worked Example 2

A guitar string has mass (m), length (l) and tension (F, i.e. a force). It is proposed that a formula for the period of vibration (t) of the string might be;

$$t = 2\pi\sqrt{\frac{F}{ml}}$$

Use dimensional analysis to determine whether this formula might be correct.

ANSWER

The first thing to note here is that 2π is just a number, so, according to Rule 1 it should be ignored. We can now use a ‘proportionality’ (∝) symbol instead of an equals sign to represent the proposition…

$$t \propto \sqrt{\frac{F}{ml}}$$

$$\left[\sqrt{\frac{F}{ml}}\right] = \left(\frac{MLT^{-2}}{ML}\right)^{0.5} = (T^{-2})^{0.5} = T^{-1}$$

This proposed formula has dimensions of T^{-1} which are clearly not the dimensions of time (the period of vibration of the guitar string, which has dimensions of T). Therefore, we have shown, using dimensional analysis, that the proposed answer is invalid.

Worked Example 3

Two positively charged particles, Q_1 and Q_2, are separated by a distance r. Each particle has a mass m. We are unsure whether Q, r and m are all required in an equation to represent the force (F) between the particles. Use dimensional analysis to develop a formula which represents the force between the particles.

ANSWER

We commence with an overview of the quantities which are possibly involved…

F = force **MLT^{-2}**

Q_1 = charge **$L^{1.5}M^{0.5}T^{-1}$**

Q_2 = charge **$L^{1.5}M^{0.5}T^{-1}$**

r = distance **L**

m_1 = mass **M**

m_2 = mass **M**

We may now suggest a general expression regarding the force, F…

$$F = function\{Q, r, m\}$$

Which simply means that force is possibly a function of (or governed by) charge, distance and mass.

To make progress towards a solution, we prefer to represent the problem as follows…

$$F \propto Q^A r^B m^C$$

and then we may say…

$$[F] = [Q^A r^B m^C]$$

Our job will be to find values for those unknowns A, B and C.

We now replace those quantities with their dimensions…

$$MLT^{-2} = (L^{1.5}M^{0.5}T^{-1})^A L^B M^C$$

Now let's bring together those dimensions and powers…

$$MLT^{-2} = L^{1.5A+B}M^{0.5A+C}T^{-A}$$

Now we look at each dimension in turn and equate those powers, left to right…

From L: 1 = 1.5A + B [1]

From M: 1 = 0.5A + C [2]

From T: -2 = -A [3]

From [3]: **A = 2** [4]

Sub [4] into [1]: 1 = 1.5(2) + B so **B = -2**

Sub [4] into [2]: 1 = 0.5(2) + C so **C = 0**

Almost there. Earlier we had…