

Pearson BTEC Levels 4 and 5 Higher Nationals in Engineering (RQF)

## Unit 2: Engineering Maths (core)

# Unit Workbook 3

in a series of 4 for this unit

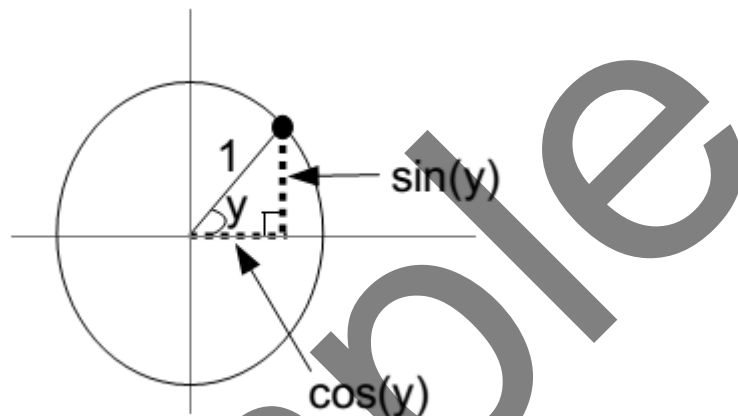
Learning Outcome 3

# Analytical and Computational Methods

## 3.1 Sinusoidal Waves

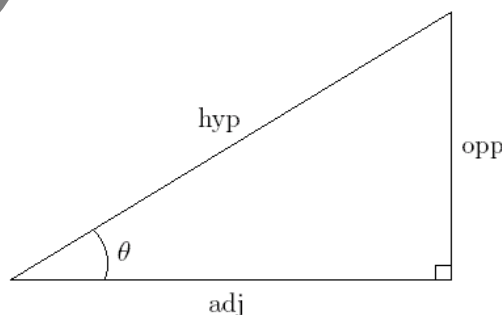
### 3.1.1 Trigonometric and Hyperbolic Identities

So far you have been used to thinking of trigonometry as the analysis of right-angled triangles. The graph below uses a unit circle to define the sine and cosine functions. You will notice that the triangle is contained within the circle.

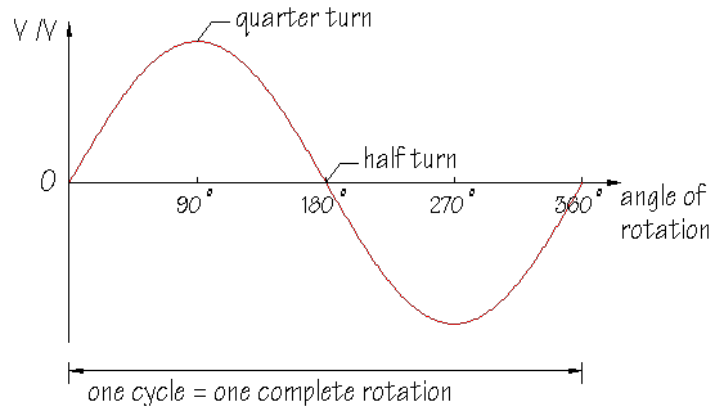


If you imagined this circle to be an electrical generator, spinning anti-clockwise with the dot starting at 3 o'clock, then measurement of the height of the dot versus time would trace out a very familiar sine wave. The mains electricity supply has a sinusoidal nature and this is produced by a circular generator at the power station. For this reason we can say that **trigonometric functions such as sin, cos and tan are CIRCULAR functions.**

A quick reminder about the common circular functions...



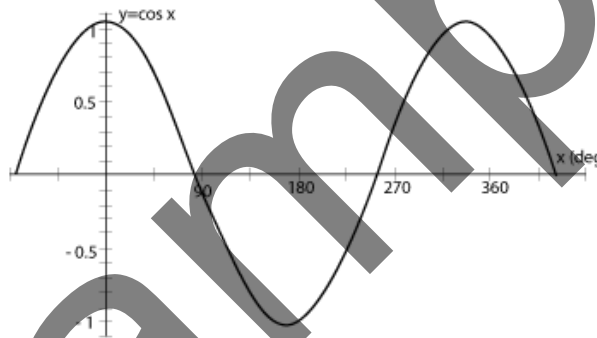
$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}; \quad \cos(\theta) = \frac{\text{adj}}{\text{hyp}}; \quad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$



If we say  $\sin(90^\circ) = 1$  then what this means, in relation to the sine wave above, is that the sine function reaches an amplitude of 1 at  $90^\circ$ . This is reading the function from the bottom up and then left. What if we were to read the function from the left first of all and then across and to the bottom? This could be expressed mathematically as...

$$\sin^{-1}(1) = 90$$

By the same token we could look at a cosine wave...



Here we can say...

$$\cos(0) = 1$$

and we can inverse this as...

$$\cos^{-1}(1) = 0$$

These inverse trigonometric functions are very useful to us because they allow the release of variables which are 'trapped' inside circular functions. When  $\sin^{-1}$  directly meets  $\sin$  then the two disappear (handy). The same goes for  $\cos^{-1}$  directly meeting  $\cos$ .

**Worked Example 1**

**An instantaneous signal voltage ( $v_s$ ) is described by the equation...**

$$v_s = 10 \sin(2\pi ft - \pi/8)$$

Make  $f$  the subject of this formula.

We commence our solution by tidying up a bit and then observing that our target variable  $f$  is 'trapped' inside the sine function. Taking  $\sin^{-1}$  of both sides should free up our  $f$  variable...

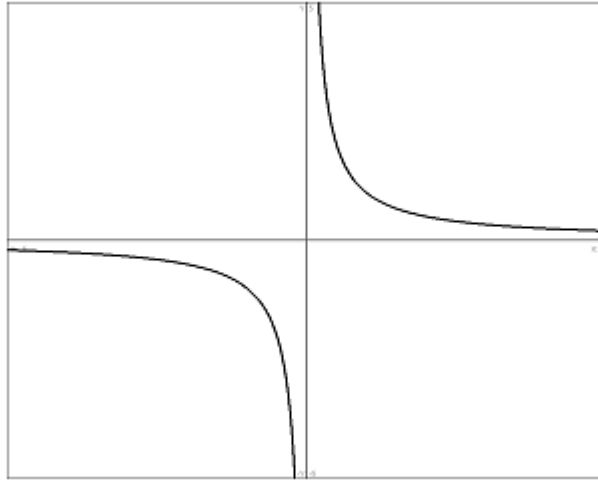
$$\begin{aligned}\frac{v_s}{10} &= \sin(2\pi ft - \pi/8) \\ \therefore \sin^{-1}\left(\frac{v_s}{10}\right) &= \sin^{-1}(\sin(2\pi ft - \pi/8)) \\ \therefore \sin^{-1}\left(\frac{v_s}{10}\right) &= 2\pi ft - \pi/8 \\ \therefore \sin^{-1}\left(\frac{v_s}{10}\right) + \frac{\pi}{8} &= 2\pi ft \\ \therefore f &= \frac{\sin^{-1}\left(\frac{v_s}{10}\right) + \frac{\pi}{8}}{2\pi t}\end{aligned}$$

**NOTE: Here the angle involves  $\pi$ . Whenever  $\pi$  is involved in the angle you MUST have your calculator in radians mode (RAD) otherwise you will be in degrees mode and find the wrong answer.**

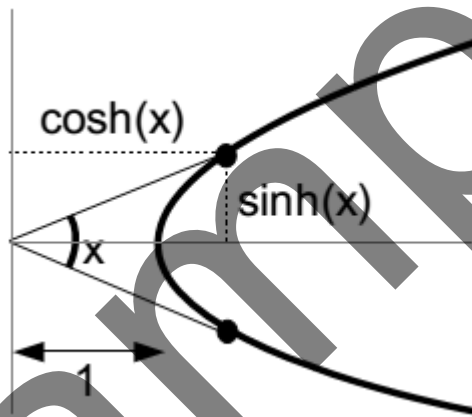
There are many facets of science, engineering and nature where simple trigonometry is an inadequate tool for analysis. Let's move on to look at HYPERBOLIC functions. Hyperbolae arise in many situations, such as...

- The path of a spacecraft in open orbit around a planet and exceeding the planet's escape velocity
- The trace of a sundial shadow
- The shape of a hanging cable, chain or rope
- The scattering trajectory of a subatomic particle acted upon by a repulsive force
- The current at any point along a transmission line exhibiting leakage
- The shape of an object produced by cutting a vertical section through a cone

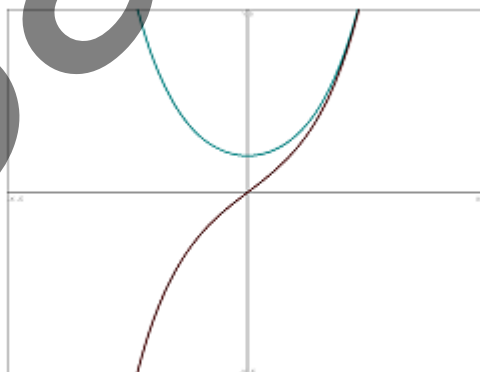
A simple hyperbola is produced by plotting the graph of  $y = 1/x$ ...



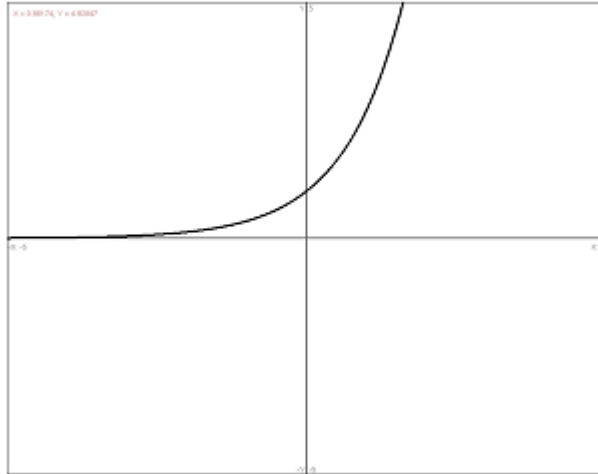
The graphic below helps us to analyse the hyperbola. From this we may define the hyperbolic sine and hyperbolic cosine functions (pronounced 'shine' and 'cosh' respectively). Notice that the apex of the hyperbola is one unit away from the vertical axis (much akin to our unit circle).



Let's take a look at the combined graphs of  $\sinh(x)$  and  $\cosh(x)$ ...



The black curve represents  $\sinh(x)$  and the curve resembling a hanging cable represents  $\cosh(x)$ . An apparently strange thing happens if we add these two graphs together...



We end up with a graph of  $e^x$ , as above. Let's write down mathematically what we have just noticed graphically...

$$e^x = \cosh(x) + \sinh(x)$$

Now, rather than add the  $\cosh(x)$  and the  $\sinh(x)$  we can also subtract them. This will give us a graph of  $e^{-x}$ . Now we may also write...

$$e^{-x} = \cosh(x) - \sinh(x)$$

If we add these last two results we get...

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

and subtracting the second from the first gives...

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

We may also try  $\cosh^2(x) - \sinh^2(x)$  and realise that this comes out to be 1...

$$\cosh^2(x) - \sinh^2(x) = 1$$

We can also divide  $\sinh(x)$  by  $\cosh(x)$  ...

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

These last four results are highlighted in blue because they are extremely important when you need to take your maths knowledge through to level 5 and beyond. Those topics will involve Complex Numbers, covered in Unit 35: Further Analytical Methods for Engineers.

**Video**

These videos will further boost your understanding of hyperbolic functions.

The table below lists the trigonometric and hyperbolic identities which you might meet most often.