Pearson BTEC Higher Nationals in Electrical and Electronic Engineering (RQF)

## Unit 2: Engineering Maths (core)

# Unit Workbook 4

in a series of 4 for this unit

Learning Outcome 4

# Calculus

# Contents

# GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

**Purpose**

Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

**Theory**

Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

**Example**

The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself.

**Question**

Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence.

**Challenge**

You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

**Video**

Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

# 3.1 Calculus

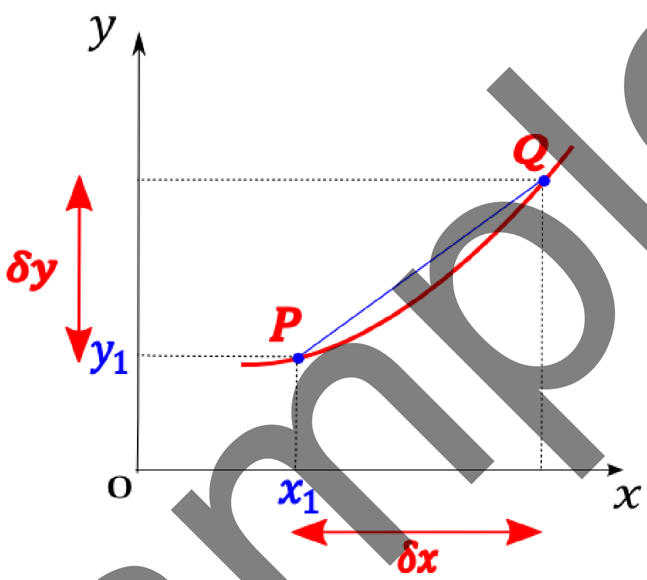
There are a couple of great webpages for checking your answers to Calculus problems…

Check Differentiation answers

Check Integration answers

## 3.1.1 The Concept of the Limit, Continuity and the Derivative

Calculus deals with functions which continually vary and is based upon the concept of a limit and continuity. Let's refer to a diagram to understand the concept of a limit…




Here we have indicated a point P on part of a function (the red curve) with Cartesian co-ordinates $(x_1, y_1)$. We require another point Q on the function to be a small increment away from P and will designate a small increment with the symbol $\delta$ (delta).

What we then have is…

$\boldsymbol{P = (x_1, y_1)}$

$\boldsymbol{Q = (x_1 + \delta x,\ \ y_1 + \delta y)}$

Look now at the chord PQ (drawn as the straight line in blue). If we can determine the slope of this chord and then make it infinitesimally short we will end up with a tangent to the function. It is the slope of this tangent which forms the basis of Differential Calculus (normally called *differentiation*).

By inspection, we see that the slope of the chord is given by…

$$\boldsymbol{slope\ of\ chord\ PQ = \frac{\delta y}{\delta x}}$$

If we deliberately make $\delta x$ approach zero (i.e. make it as short as possible) then we shall reach a limit, which may expressed mathematically as…

$$\frac{dy}{dx} = \delta x \xrightarrow{limit} 0 \ \frac{\delta y}{\delta x}$$

The term $dy/dx$ is written in Leibnitz notation and indicates the slope of the chord when the chord only touches the function at one single point. This is achieved by *continual* reduction of $\delta x$.

What we can now say is that we are able to find the slope of any function by adopting this process. Hence, given a function $f(x)$ we are able to differentiate that function, meaning find its slope at *all* points. We can write…

The slope at any point of a function $f(x)$ is given by $\frac{d(f(x))}{dx}$

***This process is called finding the derivative of a function.***

## 3.1.2 Derivatives of Standard Functions

What we don't want to be doing is to spend too much time drawing graphs of functions just to work out the derivative. Fortunately there are standard ways to determine the derivative of functions and some of the frequent ones which engineers meet are given in the table below.

| Function | Derivative |
|---|---|
| $Ax^n$ | $nAx^{n-1}$ |
| $\mathrm{A}\sin(x)$ | $\mathrm{A}\cos(x)$ |
| $\mathrm{A}\cos(x)$ | $-A\sin(x)$ |
| $A\,e^{kx}$ | $kAe^{kx}$ |
| $A\,log_e(x)$ | $A/_x$ |
| $\mathrm{A}\sinh(x)$ | $\mathrm{A}\cosh(x)$ |
| $\mathrm{A}\cosh(x)$ | $\mathrm{A}\sinh(x)$ |

Let's look at some examples of using these standard derivatives…

**Worked Example 1**

**Differentiate the function $y = 3x^4$ with respect to $x$.**

We see that this function is similar to that in row 1 of our table. We can see that $A = 3$ and $n = 4$. We may then write…

$$\frac{dy}{dx} = nAx^{n-1} = (4)(3)x^{4-1} = 12x^3$$

**Worked Example 2**

**Differentiate the function $y = 6x^{-5}$ with respect to $x$.**

We see that this function is similar to that in row 1 of our table. We can see that $A = 6$ and $n = -5$. We may then write…

$$\frac{dy}{dx} = nAx^{n-1} = (-5)(6)x^{-5-1} = -30x^{-6}$$

**Worked Example 3**

**Differentiate the function $v = 12\, sin(t)$ with respect to $t$.**

Don't worry that $y$ and $x$ have disappeared here. We can use any letters we like. The letter $v$ means voltage and $t$ is time. All we want to do here then is to find ${}^{dv}/_{dt}$.

We see that the function is similar to that in row 2 of our table. We may write…

$$\frac{dv}{dt} = A\cos(t) = 12\cos(t)$$

**Worked Example 4**

**Differentiate the function $i = 5\, cos(t)$ with respect to $t$.**

The letter $i$ means current and $t$ is time. All we want to do here then is to find ${}^{di}/_{dt}$.

We see that the function is similar to that in row 3 of our table. We may write…

$$\frac{di}{dt} = -A\sin(t) = -5\sin(t)$$