Pearson BTEC Levels 4 and 5 Higher Nationals in Engineering (RQF)

Unit 3: Engineering Science (core)

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Mechanical Engineering Systems

Mechanics can be split into two distinct systems, static or dynamic. Static means that the system is in equilibrium and in a steady state. Dynamic systems are time dependent and will change depending on the timeframe that the system is observed.

2.1 Statics

Statics are most commonly associated with beams and support systems, by determining the reaction forces of the support beams we can look into more advanced details of the system, such as (but not limited to) the deflection of the beam from its original, unloaded position, or the rate at which the deflection changes, or the bending moment on the system.

2.1.1 Free Body Diagrams

When analysing mechanics problems, its always best to start off with a free body diagram (FBD), this helps clear the picture of what is happening in the system. A FBD is a drawing of all the forces that are acting on the system. Including all moments as well. Fig.2.1 shows a free body diagram of a beam.

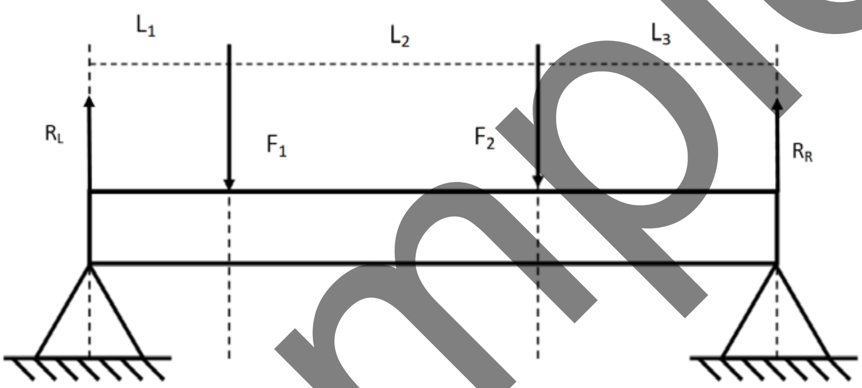


Fig.2.1: FBD of a beam supported at two ends with two vertical forces acting on it

2.1.2 Finding Static Equilibrium

The most important part about statics is that all forces are balanced: horizontal, vertical and moments are all equal (see Eq.2.1, 2.2, 2.3). If it is not, then the system is not stable, and in real applications this can be catastrophic.

$$\sum F_x = 0 \qquad \text{(Eq.2.1)}$$

$$\sum F_y = 0 \qquad \text{(Eq.2.2)}$$

$$\sum M = 0 \qquad \text{(Eq.2.3)}$$

So, in the case of the FBD in Fig.2.1:

- There are no horizontal forces, so we know that this is balanced.
- There are four vertical forces, we generally take up as positive and down as negative and our equations is therefore:

$$R_L - F_1 - F_2 + R_R = 0$$

- You can take moments about any point on the beam, however we have two unknowns, and this is the best way to eliminate one. By analysing the moments about one of the supports then (by remembering $M = Fd$, then when looking at the support ($d = 0 \therefore M = 0$). For convention, clockwise moments are negative, and counter-clockwise moments are positive. Looking at the moments about R_L:

$$R_L(0) - F_1(L_1) - F_2(L_1 + L_2) + R_R(L_1 + L_2 + L_3)$$

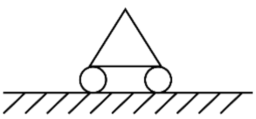
$$\therefore F_1(L_1) + F_2(L_1 + L_2) = R_R(L_1 + L_2 + L_3)$$

  And if we took moments about R_R:

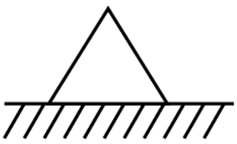
$$F_1(L_3 + L_2) + F_2(L_3) = R_L(L_1 + L_2 + L_3)$$

  We will then have R_L or R_R, and we can then calculate the remaining reaction force from Eq.2.1 or Eq.2.2.

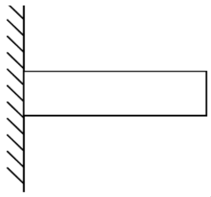
Different beam supports offer different reaction forces, such as suppressing vertical forces, horizontal forces, and moments, shown below. The application of a reaction force will add to the bending moment equation, and it's important to know what reactions the beam will apply.



Rolling support: Provides a vertical reaction only



Pinned support: Provides a vertical and horizontal reaction

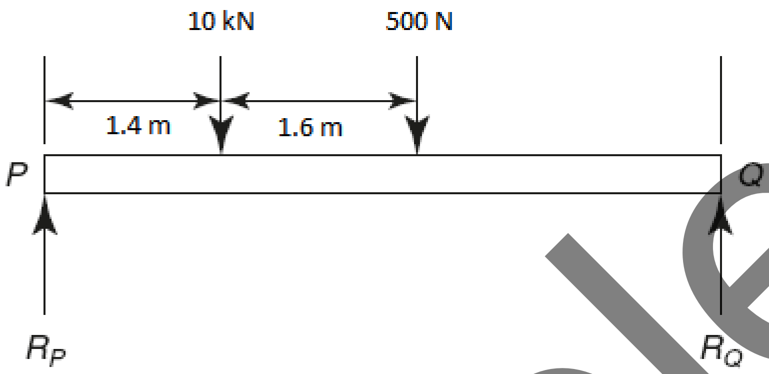


Beam is pinned directly to the surface: Provides a vertical, horizontal and moment reaction

Typically, in statics equations, we know what the forces we are looking to apply are, and so we are looking at the reaction forces that the supports will generate to fulfil the three equations above.

Worked Example 1

A beam PQ is $6.0\ \mathrm{m}$ long and is supported at its ends in a horizontal position as shown in the figure below. The mass is assumed to act as a point load of $500\ \mathrm{N}$ at its centre, as shown. A point load of $10\ \mathrm{kN}$ acts on the beam in the position shown.



When the beam is in equilibrium, determine the reactions of the supports.

ANSWER

Taking moments about P:

$$10{,}000(1.4) + 500(3) = R_Q(6)$$

$$\therefore \quad 14{,}000 + 1500 = 6R_Q$$

$$\therefore \quad R_Q = \frac{15{,}500}{6} = \mathbf{2.58\ \mathit{kN}}$$

Taking moments about Q:

$$500(3) + 10{,}000(4.6) = R_P(6)$$

$$\therefore \quad 1500 + 46{,}000 = 6R_P$$

$$\therefore \quad R_P = \frac{47{,}500}{6} = \mathbf{7.92\ \mathit{kN}}$$

CHECK: For equilibrium, upward forces must equal downward forces…

$$2.58\ kN + 7.92\ kN = 10.5\ kN = 10\ kN + 500\ N = 10.5\ kN$$

Therefore, the calculation is correct

2.2 Dynamics

2.2.1 Newton's Three Laws

Newton is renowned for many laws and theories in maths and physics, he is credited as the father of calculus and is responsible for the following three laws:

- An object will remain at rest or in uniform motion in a straight line unless there is an external forced acted upon it. If there is no force acting on the object, then the object will remain at a constant velocity.
- If there is a force F applied to the system, the force must be equal to the change in momentum per change in time (conservation of momentum is important considering dynamics) With this in mind we get Eq.2.4, where m is the mass of the object, v is the velocity, t is time, and a is acceleration.

$$F = mv/t = ma \qquad \text{(Eq.2.4)}$$

- For every action, there is an equal and opposite reaction, if you push down on a table, you hand does not go through it as it produces an equal and opposite reaction back onto your hand.

2.2.2 D'Alembert's Principle

This principle is used as a way to convert a dynamic equation into a static equation. It is easier to calculate bodies in equilibrium than if it is in motion, so we add an imaginary inertial force. With this we then use the equations used in statics to calculate the imaginary inertial force, which will be the equal and opposite force applied to the dynamic system.

2.2.3 Angular Velocity

Angular velocity is when an object performs a circular motion around a certain point. There is always a force acting towards the centre of the circle (known as a centripetal force). If this force is removed, then the object will fly off in a straight line at a tangent to the point where the force was removed.

The tangential velocity of the object v is calculated using Eq.2.5.

$$\mathrm{v} = \omega \mathrm{r} \qquad \text{(Eq.2.5)}$$

We also calculate the centripetal acceleration using Eq.2.6:

$$\mathrm{a} = \omega^2 \mathrm{r} = \frac{\mathrm{v}^2}{\mathrm{r}} \qquad \text{(Eq.2.6)}$$

The centripetal force F can be calculated using Eq.2.7, where m is the mass of the object, r is the radius of the circle and ω is the angular velocity.

$$\mathrm{F} = \mathrm{m}\omega^2 \mathrm{r} \qquad \text{(Eq.2.7)}$$

Which you will notice bears an uncanny resemblance the force in a straight line (Eq.2.4)

Worked Example 2

A conical pendulum rotates at a horizontal angular velocity of 7 rad/s. If the length of the string is 2.4 m and the pendulum mass is 0.5 kg, determine the tension in the string.

ANSWER

$$\omega = 7\,rad \cdot s^{-1} \quad ; \quad m = 0.5\,kg \quad ; \quad L = 2.4\,m$$

$$T = m\omega^2 L = 0.5 \cdot 7^2 \cdot 2.4 = \mathbf{58.8\ N}$$

Worked Example 3

A locomotive travels around a curve of 700 m radius. If the horizontal thrust on the outer rail is 1/30th of the locomotive's weight, determine the speed of the locomotive (in km/h).

NOTE: The surface that the rails are on may be assumed to be horizontal and the horizontal force on the inner rail may be assumed to be zero. Take g as 9.81 m/s^2.

ANSWER

Centripetal force on outer rail $= \frac{mg}{30}$

$$\therefore \quad \frac{mv^2}{r} = \frac{mg}{30}$$

$$\therefore \quad v^2 = \frac{gr}{30} = \frac{9.81 \times 700}{30} = 228.9\,m^2.s^{-2}$$

$$\therefore \quad v = \sqrt{228.9} = 15.13\,m.s^{-1}$$

Convert this velocity to kilometres per hour...

$$\frac{m}{s} = \frac{km}{1000} \div \frac{hour}{3600} = \frac{km}{1000} \times \frac{3600}{hour} = 3.6\,km/hour$$

$$\therefore \quad v = 3.6 \times 15.13 = \mathbf{54.47\ km/hour}$$

2.3 Fluid Mechanics

Fluids are considered to be anything that flows, this can mean both liquids and gases, a common slip up is to get confused thinking that fluid means liquid.

2.3.1 Archimedes' Principle

Archimedes is a famous Greek inventor and mathematician who was asked by Hieron, the king of Syracuse, to prove that the crown given to him by a goldsmith was not made of gold. This proved to be a difficult task, as while it is easy to calculate the volume of a uniform object, a crown is not uniform and almost impossible to calculate its volume with measurements.

This was until Archimedes filled his bathtub up to the brim and noticed that when he got in that the water was displaced by his body. He then realised that the weight of the water displaced was equal to the weight of his own body. He then realised he could determine the volume of the crown, and knowing the mass, he could determine the density of the material and prove that it was not gold. Caught up in the excitement he allegedly ran out into the streets of Syracuse naked screaming "Eureka!".

There are two ways to calculate the volume of the object, we can use the "Eureka jug", water is filled up to the spout, and when the object is submersed then the water will be forced out of the spout, which we can then calculate the volume of the water displaced, Fig.2.2 shows a Eureka jug.

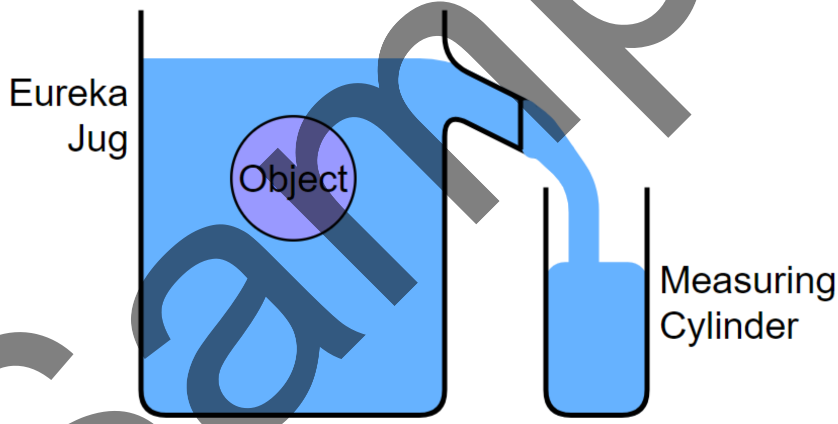


Fig.2.2: Eureka jug

Or we can use the change in weight that the object has when it is completely submerged. The body will weigh more in air than it will in water due to the buoyancy forces. We can calculate the weight of the water displaced W_W with Eq.2.5, where W_{AIR} is the weight of the object in air, and W_{SUB} is the weight when the object is completely submerged in water.

$$W_{AIR} - W_{SUB} = W_W \qquad \text{(Eq.2.5)}$$

Archimedes' principle can be expressed as Eq.2.6, where V is the volume of the body, ρ is the density of the liquid, g is the acceleration due to gravity on Earth.

$$W_w = V\rho g \qquad \text{(Eq.2.6)}$$

If, like Archimedes, we want to find out the density of the material that the object is made of, ρ_O, then we simply use Eq.2.7, where m_O is the mass of the object.

$$\rho_O = \frac{m_O}{V} \qquad \text{(Eq.2.7)}$$

Worked Example 4

A body weighs 3.1N in air and 2.2N when completely immersed in water of density 1020kg/m^3. Calculate the volume of the body. Assume gravitational acceleration is 9.81m/s^2.

ANSWER

Apparent loss of weight: $3.1\ N - 2.2\ N = 0.9\ N$

So, the weight of the displaced water is 0.9 N. Rearranging Eq

$$W_w = V\rho g$$

$$\therefore \quad V = \frac{W_w}{\rho g} = \frac{0.9}{(1020)(9.81)} = \mathbf{9 \times 10^{-5}\ m^3}$$

- This volume may also be expressed in cubic centimetres: $9 \times 10^{-5} \times 10^6\ cm^3 = \mathbf{90\ cm^3}$
- This volume may also be expressed in cubic millimetres: $9 \times 10^{-5} \times 10^9\ mm^3 = \mathbf{90,000\ mm^3}$

Worked Example 5

A rectangular watertight box is 570mm long, 420mm wide and 215mm deep. It weighs 235N. If it floats with its sides and ends vertical in water of density 1040kg/m^3, what depth of the box will be submerged?

ANSWER

$$235\ N = V\rho g = V \times 1040\ kg.m^{-3} \times 9.81\ m.s^{-2}$$

$$\therefore \quad V = \frac{235\ N}{1040\ kg.m^{-3} \times 9.81\ m.s^{-2}} = 0.023\ m^3$$

Also, the submerged volume of the box, V, is given by the equation below, where L, b and d are the length, breadth and submerged depth of the box, respectively.

$$V = Lbd$$

$$\therefore \quad d = \frac{V}{Lb} = \frac{0.023}{(0.57)(0.42)} = 0.096\ m = \mathbf{96\ mm}$$

2.3.2 Continuity of Mass Flow

Fluids can be considered to be either compressible or incompressible. Every fluid is compressible, however most liquids usually have a minute compressibility, and so we assume they are incompressible to make calculations simpler. This is important to consider when we look at making sure our equations are correct. If the equation points towards the idea that some of the volume of an incompressible fluid has disappeared, then we can quickly state that there is a mistake somewhere. If we split a litre bottle into two 500ml bottles then we should not somehow have 505ml and 500ml in the two bottles. This helps us build the continuity equation shown in Eq.2.8.

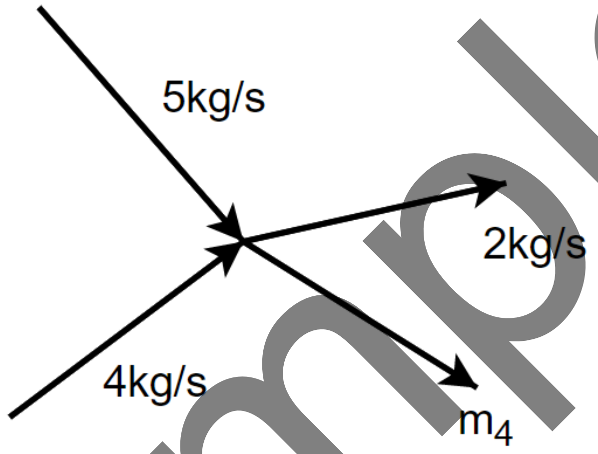
The same principle applies to mass flow, which is noted as $\dot{m}$, the dot above the m is used to note that this is a flow rate.

$$\therefore \sum \dot{m}_{in} = \sum \dot{m}_{out} \qquad \text{(Eq.2.8)}$$

Or, essentially: $\sum \dot{m} = 0$

Worked Example 6

Consider the schematic for the pipe arrangement below. Calculate m_4.



ANSWER

Using Eq.2.8 we get:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\therefore 5 + 4 = 2 + \dot{m}_4$$

$$\therefore \dot{m}_4 = 5 + 4 - 2 = \mathbf{7kg/s}$$

2.3.3 Continuity of Volume Flow

From the mass flow rate, we can also consider the volume flow rate of a system, it is not always a case that the flow is split into different pipes, engineers can also change the area of the pipe to alter the behaviour of the fluid.

The principles are still the same, and we consider Eq.2.8. However, we must also consider the following relationships discussed in Eq.2.9, 2.10, 2.11 and 2.12. where $\dot{V}$ is the volumetric flow rate (m^3/s), ρ is density (kg/m^3), v is the specific volume (m^3/kg) u is the fluid velocity (m/s^2) and A is the area of the pipe (m^2).