Pearson BTEC Level 4 Higher Nationals in Engineering (RQF)

Unit 30: Operations and Plant Management

Unit Workbook 3

in a series of 4 for this unit

Learning Outcome 1

Static & Dynamic Fluid Systems

Contents

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

Purpose — Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

Theory — Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

Example — The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself. Many of the examples given resemble assignment questions which will come your way, so follow them through diligently.

Question — Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence. As an Online Learner it is important that the answers to questions are immediately available to you. Contact your Unit Tutor if you need help.

Challenge — You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

Video — Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

1.1 Pressure and Force

1.1.1 Defining Fluid

Before beginning the unit, it is important to know exactly what a fluid is. Fluids are commonly mistaken as a liquid, and while this is technically correct, fluids are defined as something that flows, which means that fluids actually encompass both liquids ***and*** gases. Fluids do not have a fixed shape and will yield easily to external pressure.

1.1.2 Pascal's Law

Pascal's law is an important consideration in fluid mechanics that was first coined by the French scientist Blaise Pascal. Pascal's Law states:

"A pressure change in one part of a fluid at rest in a closed container will transmit to every portion of the fluid and the walls of the container without any losses"

This is an important principle when considering hydraulic systems. Pressure P can be defined by Eq.1.1, where F is the force and A is the area.

$$\mathrm{P} = \frac{\mathrm{F}}{\mathrm{A}} \quad (1.1)$$

This explains why being stood on by a high heel is more painful than being stood on by a flat shoe. The force applied will still be the same, but the pressure on the concentrated heel will be much greater than a flat shoe.

1.1.3 Hydraulics

Pascal's principle is applied to hydraulic systems. A force applied to a given area can be used to exert a force elsewhere through fluid transmission.

Hydraulics operate with two classes of piston, the "Master" and the "Slave" piston. The master piston is the dictator in the system and controlled by the operator, while the slave piston is the one that will move as a result. One of the most common hydraulic systems in day-to-day life are the brakes in cars. Fig.1.1 shows a basic schematic of a hydraulic system in place, the master piston is shown as the one receiving the operator's "effort force" F_1, and the slave piston is applying the "load force" F_2.

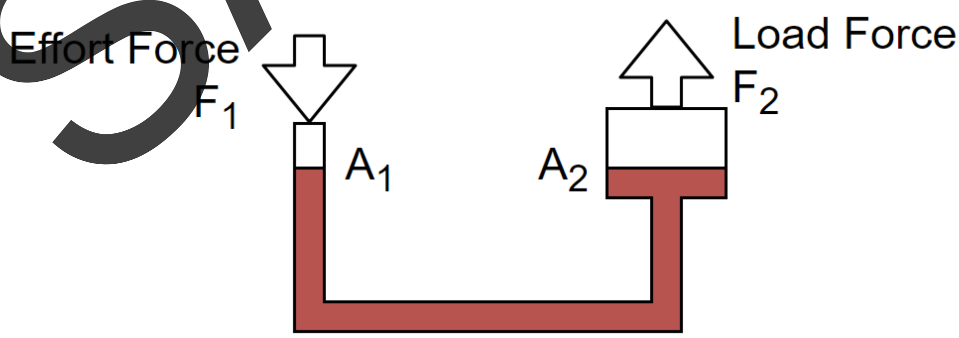


Figure 1.1: Hydraulic diagram

Since hydraulics are closed systems, the energy and work transmitted out of the system is zero, which means that the pressure on each piston is the same. The load force generated can be calculated using Eq.1.2.

The deepest recorded depth of a manned submersible is currently held by the Bathyscaphe Trieste, that reached 10,911 metres below sea level to view the Challenger Deep, the deepest known point in the ocean in the Mariana Trench, Atlantic Ocean. The deepest solo journey was by Director James Cameron, who followed the same journey and reached 10,898 metres in the more advanced Deepsea Challenger. The pressures at this depth are 1092 bar, or 110.6 MPa. The only living things at this depth are microorganisms.

1.2.2 Thrust on Immersed Surfaces

The thrust (or force) on a horizontal surface submerged in a fluid is calculated as:

$$\mathrm{F} = \mathrm{P_a A} = \rho \mathrm{ghA} \quad (1.5)$$

Where:

- $\mathrm{P_a}$ is the average pressure acting on the surface (Pa)
- A is the submerged area ($\mathrm{m^2}$)
- ρ is the fluid density
- h is the depth
- g is the acceleration due to gravity

As discussed, the change in pressure is linear, therefore the pressure profile can be described as Fig.1.4.

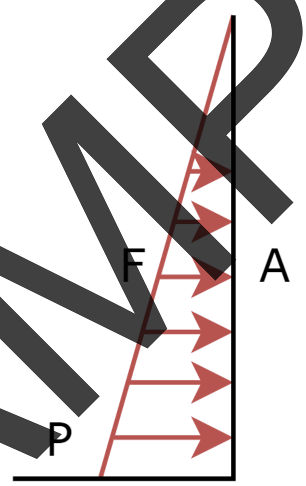


Figure 1.4: The effect of varying pressure on the force

Thus, the average force on a vertical surface $\mathrm{F_a}$ can be considered as Eq.1.6:

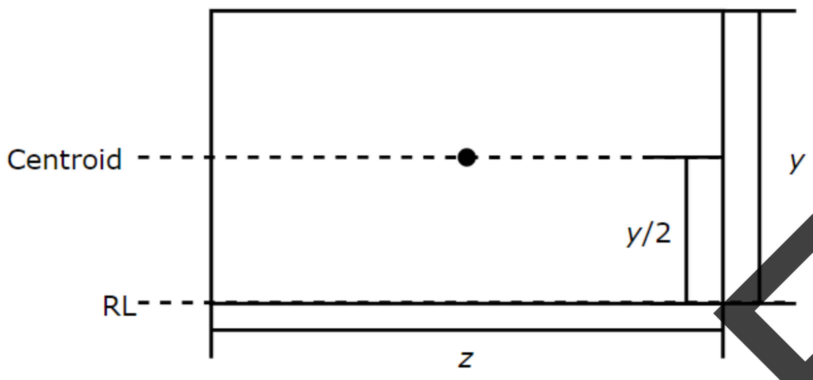
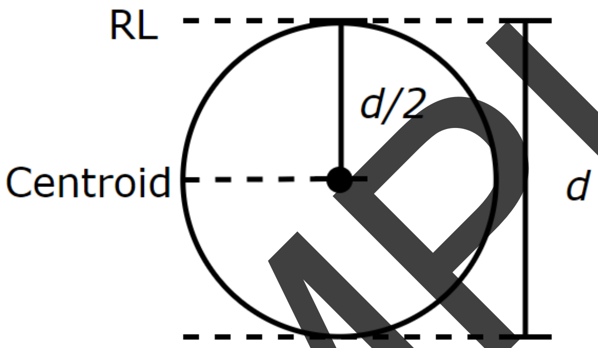
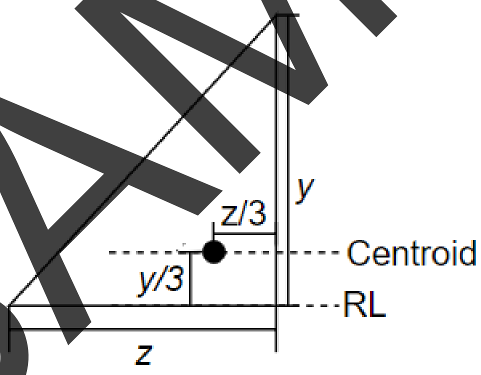
$$\mathrm{F_a} = \frac{\mathrm{P_t} + \mathrm{P_b}}{2}\mathrm{A} = \rho \mathrm{gA}\frac{\mathrm{h_t} + \mathrm{h_b}}{2} \quad (1.6)$$

Where:

- P_t and P_b are the pressures at the top and bottom of the surface, respectively.
- h_t and h_b are the height of the top and bottom of the surface, respectively.

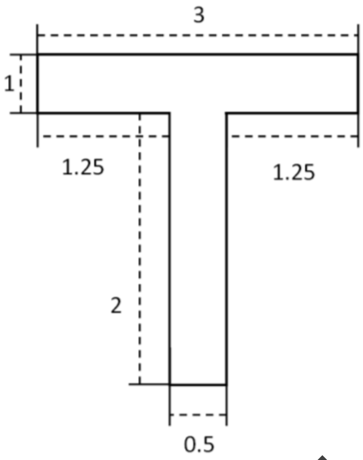
Table 1.2 gives the second moments of area for certain shapes:

Table 1.2: General second moments of area for different shapes

<table>
<tr><th>Shape</th><th>Diagram</th><th>Equation</th></tr>
<tr><td>Rectangle</td><td>
</td><td>$$I = \frac{zy^3}{12} \quad (1.13)$$</td></tr>
<tr><td>Circle</td><td>
</td><td>$$I = \frac{\pi d^4}{64} \quad (1.14)$$</td></tr>
<tr><td>Right-Angled Triangle</td><td>
</td><td>$$I = \frac{zy^3}{36} \quad (1.15)$$</td></tr>
</table>

Example 5

Calculate the second moment of area for the T-beam below



The beam can be broken down into a 3×1 rectangle at the top (which we call element 1), and a 2×0.5 rectangle at the bottom (which we call element 2). The beam has a vertical line of symmetry, meaning the horizontal coordinate of the centroid ($\bar{x}$) is 1.5 (i.e. half the width of the diagram), so we only need to find the vertical coordinate, $\bar{y}$.

Let the area of the element 1 be a_1 and that of element 2 be a_2.

$$\therefore \quad a_1 = 1 \times 3 = 3\, m^2$$

$$\therefore \quad a_2 = 0.5 \times 2 = 1\, m^2$$

$$\therefore \quad \sum a = a_1 + a_2 = 3 + 1 = 4\, m^2$$

Let y_1 be the vertical distance of the centroid in element 1 to the base (bottom edge of the 'T').

Let y_2 be the vertical distance of the centroid in element 2 to the base (bottom edge of the 'T').

$$\therefore \quad y_1 = 3 - 0.5 = 2.5\, m$$

$$\therefore \quad y_2 = 2 - 1 = 1\, m$$

Now let's determine the ay products…

$$a_1 y_1 = 3 \times 2.5 = 7.5\, m^3$$

$$a_2 y_2 = 1 \times 1 = 1\, m^3$$

$$\therefore \quad \sum ay = 7.5 + 1 = 8.5\, m^3$$

Now we determine the ay^2 products…

$$a_1 {y_1}^2 = (a_1 y_1)(y_1) = (7.5)(2.5) = 18.75\, m^4$$

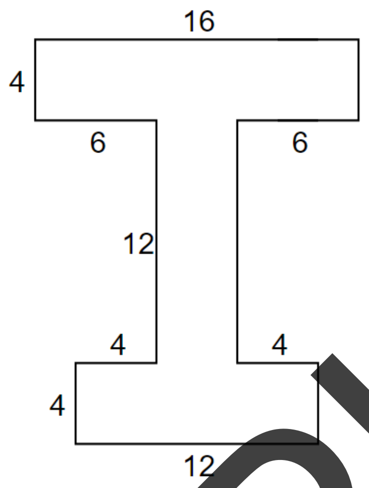
$$a_2 {y_2}^2 = (a_2 y_2)(y_2) = (1)(1) = 1\, m^4$$

$$\therefore \quad \sum ay^2 = 18.75 + 1 = 19.75\, m^4$$

$$C_P = \frac{I}{S} \quad (1.18)$$

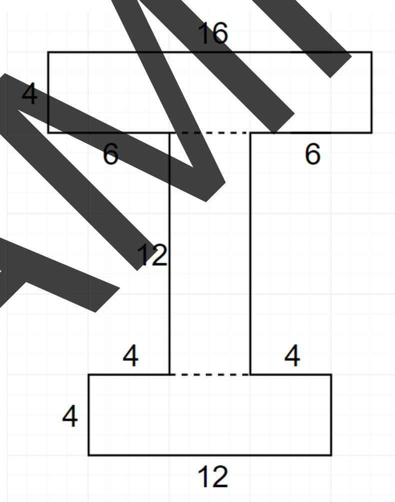
Example 6

Find the centre of pressure for the beam below.



Answer:

The shape can be broken down into three rectangles, shown below:



S is found using Eq.1.16, which will then be used to find the centroid.

$$S = A\bar{y} = \sum \bar{y}_c A_c$$

edexcel Approved Centre

2.1 Viscosity

Viscosity is a fluid's resistance to deformation under shear stresses.

Viscosity is an important property of any fluid, as it also helps determine its behaviour and motion against solid boundaries (such as pipes, gears, sliding contacts etc.). The viscosity is determined by the inter-molecular friction that is seen when one layer slides over the other. Or to put it simply, ***viscosity is how runny the fluid is***. The higher the viscosity, the thicker, and less runny, the fluid is.

It is very important to note that viscosity is temperature dependent. When considering a shortlist of fluids to a given application, it is vital that the temperature of the system is also considered.

2.1.1 Dynamic Viscosity

Dynamic viscosity is the fluid's resistance to flow when an external force is applied. Dynamic viscosity can be thought of as the tangential force per unit area required to move one plane (layer) of fluid with respect to another. The velocity between layers of a laminar fluid moving in straight parallel lines for a Newtonian fluid can be seen in Fig.2.1.

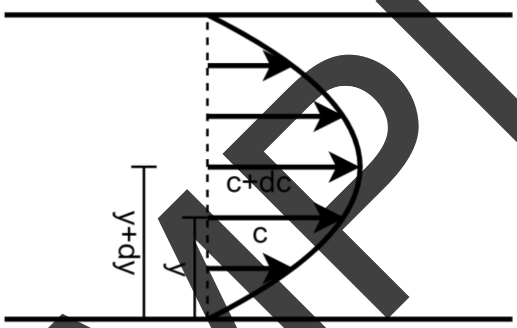


Figure 2.1: Velocity between layers of a laminar fluid

The shear stress τ can be defined by Eq.2.1, where μ is the dynamic viscosity, c is the velocity of the fluid, y is the height from the surface. dc/dy is also known as the "shear rate".

$$\tau = \mu \frac{dc}{dy} \quad (2.1)$$

The SI units for dynamic viscosity is $\mathrm{Pa \cdot s}$, the values used are typically very low (e.g., the dynamic viscosity of water at $20°\mathrm{C}$ is $0.0010005\ \mathrm{Pa \cdot s}$. More commonly the units that are used are the Poise, P, or centipoise, cP, where $10\ \mathrm{P} = 1\ \mathrm{Pa \cdot s}$, therefore the dynamic viscosity of water at $20°\mathrm{C}$ is $0.010005\ \mathrm{P}$ or $1.0005\ \mathrm{cP}$.

2.1.2 Kinematic Viscosity

Kinematic viscosity is the fluid's resistive flow under its own weight (no external forces are applied, just gravity). The substance with the highest kinematic viscosity is tar pitch, which, despite appearing to be a solid and even shatters when it is hit with a hammer, is actually an incredibly viscous liquid, and will drip roughly once every ten years. An experiment widely recognised as the longest running in the University of Queensland, Australia, is analysing the drip of tar pitch and began in 1927. Since the drip occurs around once every ten years, it has never actually been seen; the last time it did drip, the webcam failed and missed it.

Kinematic viscosity, v, can be calculated using Eq.2.2, where ρ is the density of the fluid

$$v = \frac{\mu}{\rho} \quad (2.2)$$

To calculate the VI, there are two pieces of information which must be known from the start; the Kinematic Viscosity (KV) at 40°C and at 100°C. The KV at 40°C is noted as U and the KV at 100°C is noted as Y in the following method.

If the value of KV at 100°C is between 2 cSt and 70 cSt then the table opposite is used to look up the corresponding values of 'L' and 'H', these letters merely signify some constants of viscosity at certain predefined temperatures. If the KV at 100°C is above the value of 70 cSt then the table is not used to find 'L' and 'H', but rather a separate formula is used, as follows:

$$\mathrm{L} = 0.8353\mathrm{Y}^2 + 14.67\mathrm{Y} - 216 \quad (2.3)$$

$$\mathrm{H} = 0.1684\mathrm{Y}^2 + 11.85\mathrm{Y} - 97 \quad (2.4)$$

The viscosity index in either case is then calculated using Eq.2.5:

$$\mathrm{VI} = 100\frac{\mathrm{L} - \mathrm{U}}{\mathrm{L} - \mathrm{H}} \quad (2.5)$$

2.1.4.2 $VI > 100$

In the rare case that lubricants have a viscosity index greater than 100, then the following steps need to be taken.

1. Determine the kinematic viscosity of the sample at $40^{\circ}\mathrm{C}$ and $100^{\circ}\mathrm{C}$
2. Determine the value of H
   i. If $2\ \mathrm{mm}^2/s < \mathrm{Y} < 70\ \mathrm{mm}^2/s$, the ATSM standards can be used; these values are shown in Table 2.1.
   ii. If $70\ \mathrm{mm}^2/s < \mathrm{Y}$, then H is calculated using Eq.2.4:
3. The viscosity index is therefore given as Eq.2.6:

$$\mathrm{VI} = \frac{10^{\mathrm{N}} - 1}{0.00715} + 100 \quad (2.6)$$

Where:

$$\mathrm{N} = \frac{\log_{10}\mathrm{H} - \log_{10}\mathrm{U}}{\log_{10}\mathrm{Y}} \quad (2.7)$$

A lab has bought a new capillary viscometer to test the viscosity of an unknown liquid, with the only known property being the density $\rho = 558\ \mathrm{kg/m^3}$. Unfortunately, the manufacturer of the viscometer has not provided the capillary constant. The lab technicians decide to find the capillary constant by measuring the time taken for water at $20°\mathrm{C}$, which has a kinematic viscosity of $1.0023\ \mathrm{cSt}$, which took $4.36\ \mathrm{s}$. The technicians then performed the test on the unknown liquid and it took $6.24\ \mathrm{s}$. Calculate:

a) The capillary constant $\mathrm{K_c}$
b) The dynamic viscosity of the unknown liquid

Answers:

a) Using the values obtained with water

$$k_c = \frac{v}{t} = \frac{1.0023 \times 10^{-6}\ [m^2/s]}{4.36\ [s]} = 2.30 \times 10^{-7}\ [m^2/s^2]$$

b) The kinematic viscosity of the unknown liquid is:

$$v = K_c \cdot t = 2.30 \times 10^{-7} \cdot 6.24 = 1.43 \times 10^{-6}\ m^2/s = 1.43\ cSt$$

The dynamic viscosity is therefore:

$$\mu = v \cdot \rho = 1.43 \times 10^{-6} \cdot 558 = 7.98 \times 10^{-4} = 0.798\ cP$$

One problem with capillary viscometers is that they require a transparent or translucent liquid to operate effectively, since they require a visual cue for the start and stop mark. The visual cue can also be a problem in itself, timing it requires a judgement through the human eye, and reflexes to stop the timer, both of which cause inaccuracies and errors (although they are not substantial). Electrical systems can be implemented to avoid this, albeit at an increased cost.

2.2.2 Falling Sphere Viscometer

The falling sphere viscometer is exactly as it sounds, timing how long it takes for a sphere to fall through the fluid, and using an appropriate calculation to find the viscosity. The timer begins once the bottom of the sphere is aligned with the ring mark, and stops when the top of the sphere aligns with it. A diagram of the falling sphere viscometer can be seen in Fig.2.5.

The dynamic viscosity of the sample fluid is calculated using Eq.2.9, where t is the travelling time of the ball; ρ_1 is the density of the ball; ρ_2 is the density of the fluid; $\mathrm{K_b}$ is the ball constant and F is the working angle constant (at $80°$, then $\mathrm{F} = 1$).

$$\mu = t(\rho_1 - \rho_2)K_b \cdot F \quad (2.9)$$

The kinematic viscosity is therefore calculated using Eq.2.10:

$$v = \frac{\mu}{\rho_2} \quad (2.10)$$

The falling sphere viscometer experiences similar problems to the capillary viscometer, as it relies on visual cues, and it cannot use opaque fluids.

- Searle Principle

Viscometers also measure torque using two different systems.

- Servo systems
- Spring systems

2.2.3.1 Couette Principle

The Couette principle relies on a bob to be suspended in a container filled with the test fluid. In this case, the driving force is acting on the container itself, meaning that the bob is the stationary frame of reference in the system (shown in Fig.2.6). This design avoids any problems with turbulent flow, but it is rarely used in commercial applications as it can be difficult to ensure that the container is well insulated and sealed in the rotating cup.

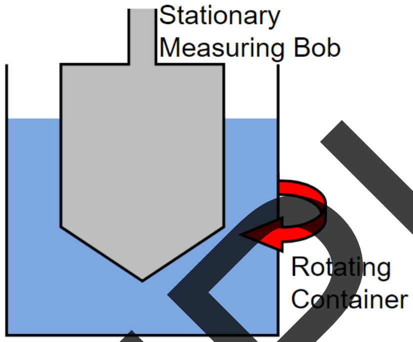


Figure 2.6: Couette principle rotational viscometer

2.2.3.2 The Searle Principle

The Searle principle holds the container stationary, and instead spins the measuring bob (as can be seen in Fig.2.7). In this case, the viscosity is proportional to the motor torque that is required for turning the bob against the resistive viscous forces of the fluid. These are much more common viscometers; however, the measuring bob must be kept at a low enough velocity to ensure that the flow in the container does not become turbulent.

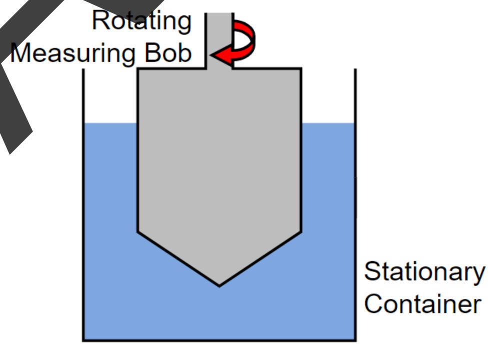


Figure 2.7: Searle principle rotational viscometer

2.2.3.3 Servo Devices

Servo systems use a servo motor to drive the main shaft, which will turn the measuring bob. Tachometers or high resolution digital encoders measure the rotational speed. The current drawn by the motor is proportional to the torque caused by the viscosity of the test fluid, meaning viscosity can be calculated using the rotational speed of the servo and current demand.

c) The dynamic viscosity

Answers:

a) The shear rate is given as:

$$\frac{dc}{dy} = \frac{2\omega R_c^2}{(R_c^2 - R_b^2)}$$

We need to find ω, R_c, R_b in the appropriate dimensions

$$\omega = 2500 \text{ rpm} \equiv \left(\frac{2500}{60}\right)(2\pi) = 261.8 \text{ rad/s}$$

$$R_c = 0.5d_c = 7.5\text{cm} = 0.075\text{m}$$

$$R_b = 0.5d_b = 5\text{cm} = 0.05\text{m}$$

With this information:

$$\frac{dc}{dy} = \frac{2\omega R_c^2}{(R_c^2 - R_b^2)} = \frac{2(261.8)(0.075)^2}{(0.075^2 - 0.05^2)} = \mathbf{942.5\ s^{-1}}$$

b) Shear stress is given as:

$$\tau = \frac{T}{2\pi R_b^2 h}$$

h in the appropriate dimension is:

$$h = 20\text{cm} = 0.2\text{m}$$

and therefore τ is:

$$\tau = \frac{T}{2\pi R_b^2 h} = \frac{0.05}{2\pi(0.05)^2(0.2)} = \mathbf{15.9\ Pa}$$

c) The dynamic viscosity is therefore:

$$\mu = \tau \div \frac{dc}{dy} = 15.9/942.5 = \mathbf{0.0169\ Pa \cdot s}$$
$$= \mathbf{0.169\ P}$$
$$= \mathbf{16.9\ cP}$$

2.2.4 Orifice Viscometers

Orifice viscometers are used in the oil industry because of their simplicity and ease of use. The system consists of a reservoir, an orifice and a receiver. The method is simple; the sample fluid is poured into the reservoir, which is temperature controlled in a water bath. Once the sample fluid has reached the desired temperature (that of the water bath), a valve at the base of the reservoir is opened and the time taken for a specific amount of sample fluid to flow out of the orifice is measured. While the industry has several types of orifice viscometers, this workbook will only look at the Saybolt and Redwood viscometers.

Other orifice viscometers include:

2.3 Newtonian and Non-Newtonian

Fluid behaviour can be split into two different classes, either Newtonian or Non-Newtonian. These are useful descriptors that will quickly help to predict their behaviour.

2.3.1 Newtonian Fluids

To understand what makes a fluid "Newtonian", it is important to look at Eq.2.1.

$$\tau = \mu \frac{dc}{dy}$$

Let's say temperature was kept constant. Fig.2.11 shows what would happen to the shear stress τ as the shear rate dc/dy increases. The first thing observed is that the graph is a straight line. Thus, the definition of a Newtonian fluid is:

A fluid that has a shear stress that is linearly proportional to the strain rate.

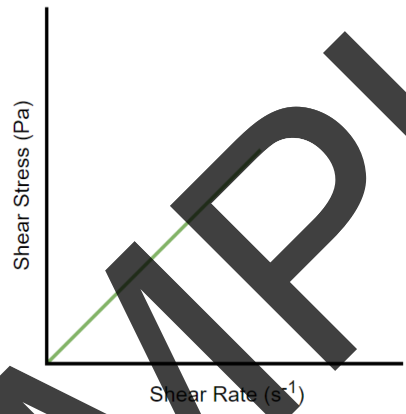


Figure 2.11: Shear Stress against shear strain

In essence, viscosity remains constant as shear strain and shear rate change. It's important to note that while no fluid is "perfectly Newtonian", fluids that are close enough to be considered Newtonian are the likes of:

- Water
- Mineral oils
- Air

2.3.2 Non-Newtonian

The definition of Non-Newtonian fluids can be self-explanatory:

Shear stress is not linearly proportional to the strain rate.

Non-Newtonian itself can be broken down into several more distinct categories.

2.3.2.1 Dilatant Fluids

A Dilatant fluid behaves as a fluid until it is met with a certain shear rate, after that, the fluid beings to solidify; in essence, it will turn solid when agitated. The presence of shear forces produces an almost exponential increase in viscosity, giving the impression that the substance is solid. The two URLs below show the effect of a dilatant fluid that has been agitated by a certain force. The first link is a corn-starch and water mixture placed atop of a speaker. The vibrations on the speaker create a force which begins to solidify the mixture, and makes it almost look alive. The second link is from Brainiac, showing a full swimming pool of custard,

$$\mathrm{Re} = \frac{\rho \mathrm{uL}}{\mu} = \frac{uL}{v} \quad (3.1)$$

Where:

- ρ is the density of the fluid $(\mathrm{kg/m^3})$
- u is the velocity of the fluid $(\mathrm{m/s})$
- L is the length that you are measuring over (m)
- μ is the dynamic viscosity $(\mathrm{Pa \cdot s})$
- v is the kinematic viscosity $(\mathrm{m^2/s})$

The corresponding value for the Reynold's number defines the flow as:

- $\mathrm{Re} < 2000$: Flow is laminar
- $\mathrm{Re} = 2000$: Known as the critical Reynold's number, flow is no longer laminar and will start to transition towards turbulent flow
- $2000 < \mathrm{Re} < 4000$: Flow is considered transitional, or unstable, it is not laminar, but it is not fully turbulent yet either.
- $4000 < \mathrm{Re}$: Flow is turbulent

Example 1

Give the flow characteristic of:

a) Honey $(1450\ \mathrm{kg/m^3},\ 14.095\ \mathrm{Pa \cdot s})$ flowing through a $3\ \mathrm{m}$ length of pipe at $0.3\ \mathrm{m/s}$.
b) Castor oil $(961\ \mathrm{kg/m^3},\ 950\ \mathrm{cP})$ flowing through $1\ \mathrm{m}$ length of pipe at $20\ \mathrm{m/s}$
c) Water $(1000\ kg/m^3,\ 1\ cP)$ flowing through a $1\ m$ length of pipe at $0.3\ m/s$.

Answers:

a) Re is given as:

$$\mathrm{Re} = \frac{\rho \mathrm{uL}}{\mu} = \frac{1450(0.3)(3)}{14.095} = \mathbf{92.6}$$

The flow is laminar

b) $950\ \mathrm{cP} = 9.5\ \mathrm{Pa \cdot s}$, so Re is:

$$\mathrm{Re} = \frac{961(20)(1)}{9.5} = \mathbf{2023}$$

The flow is transitional

c) $1\ \mathrm{cP} = 0.01\ \mathrm{Pa \cdot s}$, so Re is:

$$\mathrm{Re} = \frac{1000(0.3)(1)}{0.01} = \mathbf{30,000}$$

The flow is turbulent

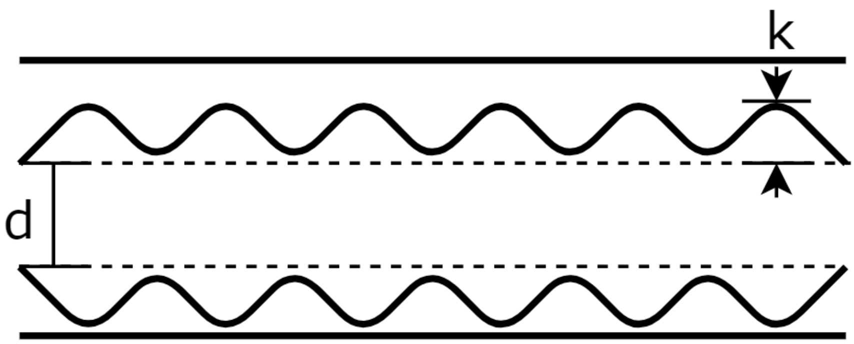


Figure 3.5: The measurement of the diameter and height of irregularities in a pipe.

4.2.2 Moody Diagrams

A Moody diagram is a diagram to measure the coefficient of friction in pipes (noted in this workbook as f, although some sources will use λ). Calculating the coefficient of friction (sometimes referred to as the friction factor) depends on both the flow in the pipe and its relative roughness.

The equations used for calculating f are:

Laminar: $$\mathrm{f} = \frac{16}{\mathrm{Re}} \qquad (3.3)$$

Turbulent Smooth pipes: $$\mathrm{f} = 0.079Re^{-0.25} \qquad (3.4)$$

Turbulent Rough pipes: $$\frac{1}{\sqrt{f}} = -3.6\, log_{10}\left[\frac{6.9}{Re} + \left(\frac{k}{3.71d}\right)^{1.11}\right] \qquad (3.5)$$

These equations are quite long to calculate, with the exception of the laminar equation (Eq.3.3). So, alternatively, the Moody chart is available for reference. The Moody chart is a graph that has already plotted the values for the friction factor across a range of Reynold's numbers, and relative roughness, to give a quick (and fairly accurate) estimate. Fig.3.6 shows a Moody chart, and the lines show the variation of friction factor at a given relative roughness, but a varying Reynold's number. Most Moody diagrams will also include an absolute roughness value for some materials. The absolute roughness value is a typical estimate for ϵ for certain materials.

3.2.3 Bernoulli's Equation

Bernoulli's equation is a conservation of energy equation used in fluid mechanics. This can draw similarities to the thermodynamic Steady Flow Energy Equation (SFEE).

$$Q - W = \left(U_2 + \frac{1}{2}mc_2^2 + mgz_2\right) - \left(U_1 + \frac{1}{2}mc_1^2 + mgz_1\right)$$

Where:

- Q is the heat transferred through the system
- W is the work done in the system
- U_i is the internal energy of the system at point i
- $\frac{1}{2}mc_i^2$ is the kinetic energy at point i
- mgz_i is the potential energy at point i

The same principle can be applied to flow through a pipe, since no heat is transferred in the flow of the pipe. Bernoulli's equation is given as Eq.3.7:

$$\mathrm{P_1} + \frac{1}{2}\rho \mathrm{u_1^2} + \rho \mathrm{gh_1} = \mathrm{P_2} + \frac{1}{2}\rho \mathrm{u_2^2} + \rho \mathrm{gh_2} \quad (3.7)$$

Where:

- P is the pressure at a given point (Pa)
- ρ is the density (kg/m^3)
- u is the velocity at a given point (m/s)
- g is acceleration due to gravity (m/s^2)
- h is the height at a given point (m)

When using Bernoulli's equation, it's always important to know the equation for volumetric flow rate $\dot{\mathrm{Q}}$, which is Eq.3.8, where A is the area of the pipe.

$$\dot{\mathrm{Q}} = \mathrm{u} \cdot \mathrm{A} \quad (3.8)$$

Where:

- u is the velocity of the fluid (m/s)
- A is the cross-sectional area (m^2)

Example 4

Fig.3.7 shows water $\rho = 1000\ \mathrm{kg/m^3}$ travelling through a converging-diverging pipe. The figure also gives the known values for each point in the pipe. Calculate

a) The volumetric flow rate
b) The velocity at point 2
c) The pressure at point 2

3.3 Drag

3.3.1 Drag Around a Sphere

Consider Fig.3.8, which shows fluid streams passing around a sphere. From the Figure, it can be seen that the fluid will move around the sphere and slowly move back to its original state, there is also a generation of turbulent eddy currents that get caught behind the sphere.

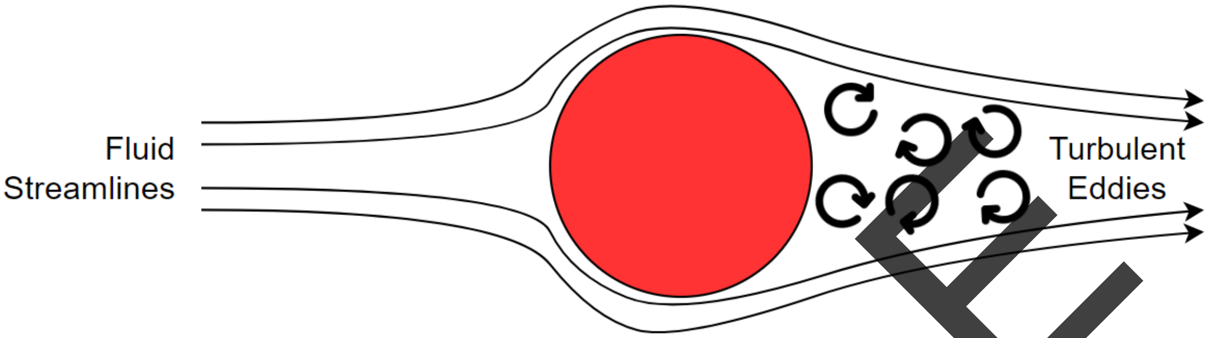


Figure 3.8: Streamlines against a sphere

There are two types of drag that can exist in a system; viscous drag and pressure drag.

3.3.2 Viscous Drag

Viscous Drag, or friction drag, occurs on the contact point of the surface of the object in the streamline. This type of drag is so-called because it is caused by the viscosity of the fluid itself and involves the transition from laminar drag to turbulent drag. It therefore stands that if the Reynold's number is higher, then there is less friction drag. Fig.3.9 shows the development of the "boundary layer" on a flat plate, which is the layer of fluid affected by viscous drag.

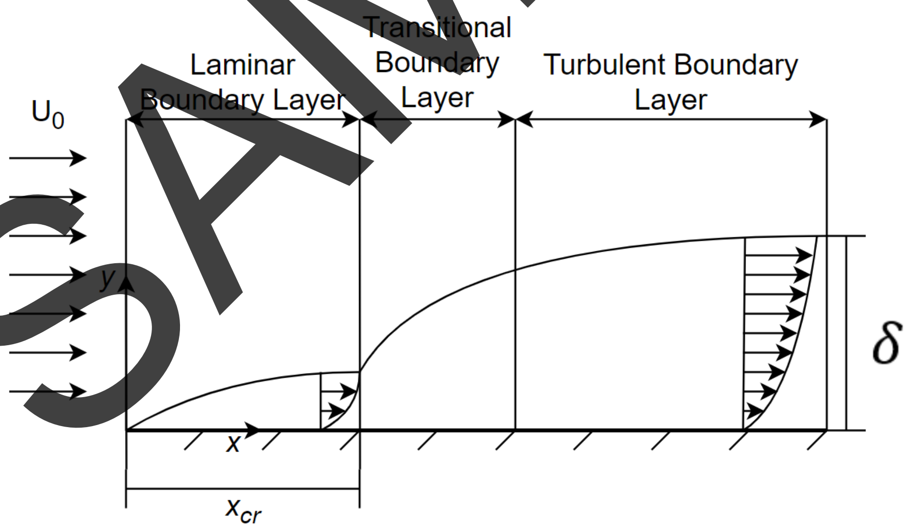


Figure 3.9: The development of a boundary layer along a flat plate.

The boundary layer is the stream that is travelling slower than the free stream speed, U_0. The further along the flat plate travelled shows an increase in the boundary layer thickness δ. There is also a given distance along the plate x_{cr} when the laminar boundary layer's Reynold's number moves higher than 2000, the fluid in the boundary layer transitional, before moving to turbulent.

3.4 Aerodynamics

Aerodynamics is how a body behaves when moving through a fluid (or if a fluid is moving through the body, depending on your frame of reference). By understanding the effects of aerodynamics, it can be applied to improve a number of systems.

3.4.1 Using the Drag Equation

The drag equation discussed in Section 3.3 has only discussed a body perpendicular to the fluid flow. But what if an "angle of attack" (commonly noted in mathematics as α) was introduced to the system. The angle of attack is altering the position of the body in order to direct the force, rather than the force just acting in the same direction as the fluid.

Consider the aerofoil in the Fig.3.12. In this scenario, the centreline of the aerofoil (otherwise known as the chord), is not in line with the flight path; as α increases, and the angle of attack becomes greater, there will be a greater pressure drag acting on the aerofoil, and the flow will begin to apply a force that will push the aerofoil upwards, giving "lift".

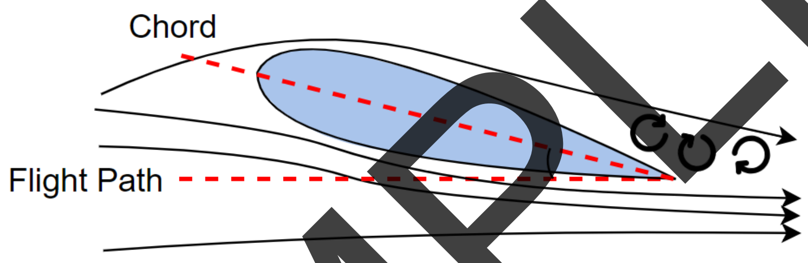


Figure 3.12: An aerofoil with an angle of attack against the flight path.

3.4.2 Coefficient of Lift

The coefficient of lift $\mathrm{C_L}$ is a term used to describe the lifting force $\mathrm{F_L}$ that acts on an aerofoil, as shown by Eq.3.10, and is essentially similar to Eq.3.9, since the lift relies on the drag force.

$$\mathrm{C_L} = \mathrm{F_L} \times \frac{1}{2}(\rho \mathrm{u}^2)\mathrm{DL} \quad (3.10)$$

The highest value for $\mathrm{C_L}$ occurs at the critical angle of attack α_{cr}. Once the angle passes this, $\mathrm{C_L}$ begins to drop, and C_d increases substantially, thus the system begins to "stall".

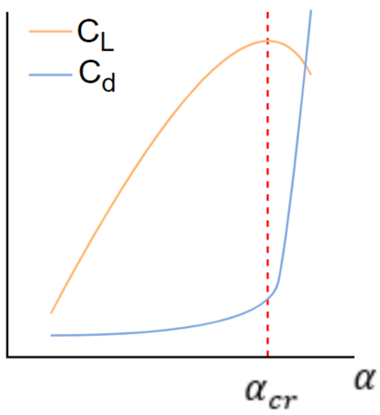


Figure 3.13: C_d and C_L relative to α

edexcel Approved Centre