Unit 36: Advanced Mechanical Principles

Unit Workbook 1

in a series of 4 for this unit

Learning Outcome 1

Characteristics of Materials
1.1 Behavioural Characteristics:

When choosing a material for a design, it is important to know how it will react under loading. For example, porcelain and clay will not make a good elastic band, and bridges are not made of polyester. Choosing the right material to ensure proper performance is vital in the design process.

1.1.1 Tensile Tests

The behavioural characteristics of materials is determined by a tensile test, in which a tensile force is applied to the material and the extension is noted, this process can be done until destruction.

![Fig.1.1: A tensile test of mild steel](image)

Fig. 1.1 shows a tensile test of mild steel; the graph can be broken down into ten important sections:

i) Between A and B, Hooke’s law applies, and Stress is directly proportional to strain.

ii) Point B is the limit of proportionality, and stress is no longer proportional to strain.

iii) Point C is the elastic limit, if the material is released at this point, it will return to its original length with negligible permanent extension.

iv) Point D is the yield point, after this point there is a huge extension for a small amount of load, the yield stress of the material is calculated as Eq.1.1.

\[
\sigma_y = \frac{\text{load where yield takes place}}{\text{original cross-sectional area}}
\]  

(Eq.1.1)

v) After point J the material strain hardens, and the slope becomes about 1/50th of that of A to B.

vi) Between points D and E, extension takes place over the whole gauge length of the specimen.

vii) Point E signals the ultimate tensile stress (UTS), the maximum load that the material can withstand calculated with Eq.1.2.

\[
UTS = \frac{\text{max load}}{\text{original cross-sectional area}}
\]  

(Eq.1.2)

viii) Point E to F is the material beginning to fail, as the cross-sectional area of the material greatly decreases and forms a “neck”.

ix) Point F is the point of failure, the material will fracture.

x) Point G to H is the permanent elongation of the material.
1.1.2 Tensile Tests for Different Materials

Fig.1.2 shows the tensile tests for four different materials. All show a different relationship between load and extension.

From Fig.1.2:

- Material A shows a brittle material, there is high load with little extension, but there is no plastic deformation present. There is just an immediate, catastrophic failure. Materials like this are typically ceramics.
- Material B is a strong material that is not ductile, there is a small amount of plastic deformation before failure, there is a lot of elastic strain energy in these materials, wires made of a material such as this can whiplash on failure and be very dangerous. These is typically brittle metals.
- Material C is a ductile material, similar to the mild steel used in Fig.1.1.
- Material D is a plastic material such as a polymer, there is little elastic deformation before a large amount of plastic deformation.
1.2 Elastic Constants

1.2.1 Young’s Modulus of Elasticity

Young’s modulus of elasticity \( E \) is the ratio of stress to strain of a material. The equation for Young’s modulus is shown by Eq. 1.3 below. The equation only applies to the elastic region of the material (between points A and B on Fig. 1.1), where \( \sigma \) is the stress and \( \epsilon \) is the strain of the material.

\[
E = \frac{\sigma}{\epsilon} \quad \text{(Eq. 1.3)}
\]

1.2.2 Modulus of Rigidity

The modulus of rigidity \( G \), is similar to Young’s modulus, but it concerns shear stress and shear strain. Again, this equation is only applicable to the elastic region of the material. The equation for the modulus of rigidity is seen in Eq. 1.4, where \( \tau \) is the shear stress and \( \epsilon \) is the shear strain of the material.

\[
G = \frac{\tau}{\epsilon} \quad \text{(Eq. 1.4)}
\]

1.2.3 Bulk Modulus

Bulk modulus \( K \) describes the compressibility of a solid or fluid. The bulk modulus describes the elastic properties of a material when compressed on all surfaces and is a ratio of the pressure applied to the material relative to the volumetric strain created. The equation for bulk modulus is given as Eq. 1.5, where \( p \) is the pressure, \( V_0 \) is the original volume, and \( V \) is the new volume of the material.

\[
K = \frac{p}{(V_0 - V)/V_0} \quad \text{(Eq. 1.5)}
\]

1.2.4 Poisson’s Ratio

Poisson’s ratio \( \nu \) is the ratio of lateral strain to axial strain. Or the change in strain in the direction of the load, to the change in strain normal to the load. Fig. 1.3 and Fig. 1.4 show the effect of Poisson’s ratio when pulled on the x-axis in 2-D and 3-D, respectively. As you can see, when a tensile stress elongates the x-direction, the y and z axes shrink. Poisson’s ratio is given as Eq. 1.6 below. This only applies to a stress on one axis, strain for complex loading will be discussed later in Section 1.3

\[
\nu = -\frac{\epsilon_{\text{Lateral}}}{\epsilon_{\text{Axial}}} \quad \text{(Eq. 1.6)}
\]
Fig. 1.4: 3-D deformation with a tensile force.