Pearson BTEC Levels 5 Higher Nationals in Engineering (RQF)

Unit 36: Advanced Mechanical Principles

Unit Workbook 4

in a series of 4 for this unit

Learning Outcome 4

Dynamic Rotating Systems

The follower, during its travel, may have one of the following motions shown in Fig.4.2. Uniform Velocity shows a constant speed, and exhibits a linear displacement. Uniform acceleration-retardation is a constant acceleration and deceleration, which displaces in a cubic fashion; and simple harmonic motion, which is sinusoidal.

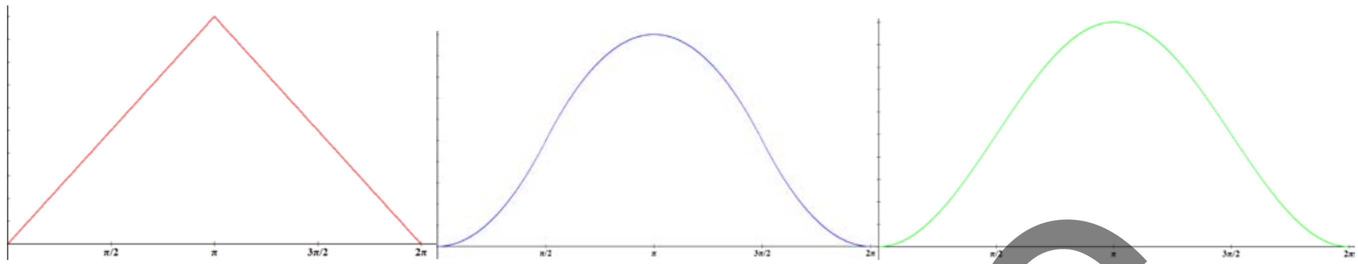


Fig.4.2: Displacement maps for uniform velocity (right), uniform acceleration-retardation (centre), and simple harmonic motion (right)

4.1.3 Developing Displacement Maps

A displacement map is typically built from several different graphs combined.

i. Uniform velocity – We know that the graph is linear, and the general equation for linear displacement is shown with Eq.4.1

$$y = mx + c \qquad \text{(Eq.4.1)}$$

The boundary conditions for this we know is that $y = 0$ at $x = 0, 2\pi$ and $y = y_{max}$ at a specified value for x. Which will give everything that is necessary to find the equations for each part.

ii. Uniform acceleration-retardation – the equation for constant acceleration, and its relationship with displacement is shown in Eq.4.2.

$$\frac{d^2y}{dx^2} = c \qquad \text{(Eq.4.2)}$$

Integrating twice will give a cubic equation, but it will also show linear velocity. We also know that $\frac{dy}{dx} = 0$ when y is at its maximum and minimum.

iii. Simple harmonic motion – The acceleration of the graph is shown by Eq.4.3.

$$\frac{d^2y}{dx^2} = cCos(x) \qquad \text{(Eq.4.3)}$$

Again, integrating twice will give the displacement equation. Using boundary conditions to figure out the necessary constants.

Example

A Rise-Return-Rise cam is to be designed with the following specifications:

Peak displacement of 80 mm occurs when the cam angle (x) is π radians, before returning to its original position $(y = 0)$ at a cam angle of 0 and 2π. Plot the displacement maps for:

a) Uniform velocity
b) Uniform acceleration-retardation
c) Simple harmonic motion

Answer:

For any questions regarding this, a simple hand-drawn graph with important points detailed (peak displacement, any mid-points or axis intersects) is perfectly acceptable. However, if you do wish to do generate a computer-generated graph, more detailed equations must be generated.

a) This needs to be broken down into two parts, to look at the rise and return. With uniform velocity we have the general equation $y = mx + c$
For the rise
$y(0) = 0$ and $y(\pi) = 80$ gives $c = 0$ and $m = 80/\pi$
For the return
$y(\pi) = 80$ and $y(2\pi) = 0$ gives $c = 160$ and $m = -80/\pi$; which gives

b) Uniform acceleration-retardation needs to be broken down into 3 sections
   i. Initial acceleration between 0 and π/2
   ii. Deceleration between π/2 and 3π/2
   iii. Acceleration between 3π/2 and 2π

We know that $\frac{d^2y}{dx^2} = c$, which integrates to become the velocity $\frac{dy}{dx} = cx + d$, and integrates again to become displacement $y = \frac{cx^2}{2} + dx + e$

   i. The boundary conditions we can use are $y(0) = 0$; $y(\pi/2) = 40$; $\frac{dy}{d(0)} = 0$;

$$y(0) = \frac{c(0)^2}{2} + d(0) + e = 0 \therefore e = 0$$

$$y\left(\frac{\pi}{2}\right) = \frac{c\left(\frac{\pi}{2}\right)^2}{2} + d\frac{\pi}{2} = 40 = \frac{\pi^2}{8}c + \frac{\pi}{2}d = 40$$

$$\frac{dy}{d(0)} = cx + d = 0 \therefore d = 0$$

And so, $\frac{\pi^2}{8}c = 40$ gives $c = \frac{8}{\pi^2}40 = \frac{320}{\pi^2}$

The displacement graph is $y = \frac{160}{\pi^2}x^2$

ii. A rise-return-rise graph that peaks at π means that $\frac{d^2y}{dx^2} = \pm c$, depending on whether the system is accelerating or decelerating. $\frac{dy}{dx} = -cx + f$, and integrates again to become displacement $y = \frac{-cx^2}{2} + fx + g$

The boundary conditions in this case are

$\frac{dy}{d(\pi)} = 0$ and $y(\pi) = 80$

$$\frac{dy}{d(\pi)} = -c(\pi) + f = 0 \therefore f = c\pi = \frac{320}{\pi}$$

$$y(\pi) = \frac{-320(\pi)^2}{2\pi^2} + \frac{320\pi}{\pi} + g = 80 \therefore g = 80 + 160 - 320 = -80$$

The displacement graph is therefore $y = \frac{-160(x)^2}{\pi^2} + \frac{320x}{\pi} - 80$

iii. The final part of the graph will have the equations $\frac{d^2y}{dx^2} = c$, $\frac{dy}{dx} = cx + h$, and $y = \frac{cx^2}{2} + hx + i$

The boundary conditions are

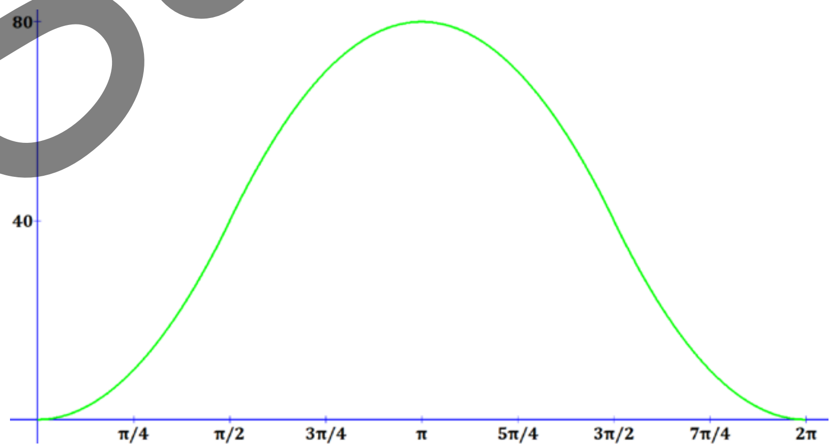
$\frac{dy}{d(2\pi)} = 0$ and $y(2\pi) = 0$

$$\frac{dy}{d(2\pi)} = c(2\pi) = \frac{320(2\pi)}{\pi^2} + h = 0 \therefore h = -\frac{2(320)}{\pi} = -\frac{640}{\pi}$$

$$y(2\pi) = \frac{320(2\pi^2)}{2\pi^2} + \frac{-640(2\pi)}{\pi} + i = 0$$

$$i = -4(160) + 2(640) = 640$$

And the displacement function is $y(\pi) = \frac{160(x)^2}{\pi^2} - \frac{320x}{\pi} + 640$ which looks like



c) Simple harmonic motion acceleration is expressed as $\frac{d^2y}{dx^2} = cCos(x)$, integrating gives $\frac{dy}{dx} = cCos(x) + j$ and displacement is $y = -cCos(x) + jx + k$

The boundary conditions are $y(0) = 0, \frac{dy}{d(0)} = 0$

$$\frac{dy}{d(0)} = cSin(0) + j = 0 \therefore j = 0$$

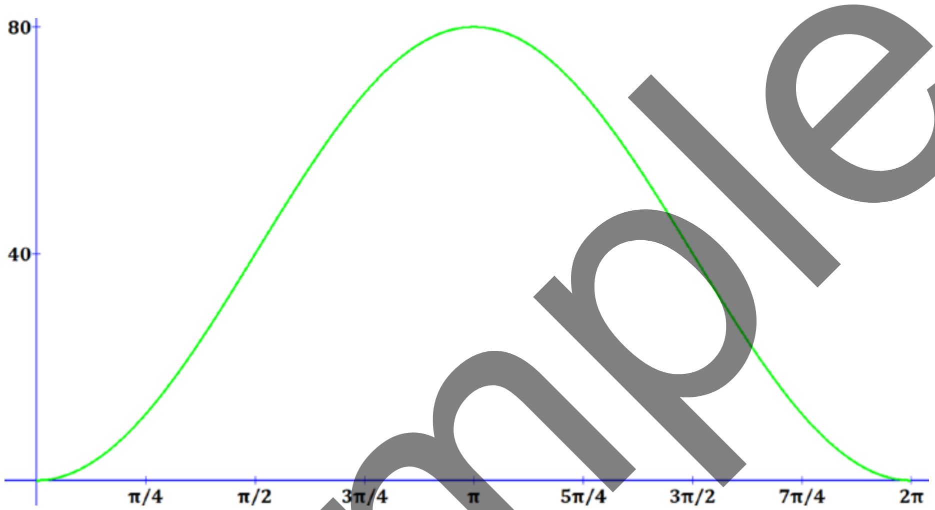
$$y(0) = -cCos(0) + k = 0 \therefore k = c$$

We know that $k = c$, but c is still unknown, so we need another condition, such as $y(2\pi) = 80$

$$-cCos(\pi) + c = 80$$

$$c + c = 80 \therefore c = 40$$

The displacement function is therefore $y = -40Cos(x) + 40$ which looks like the graph below



4.3 Rotating Systems

4.3.1 Static Balancing Rotating Mass Systems

Purpose

An imbalance in a rotating mass system can cause problems within any system. The rotating mass will produce a vibrating force, which can resonate and cause a lot of damage. It's important to balance any moments or forces in the system.

Theory

Consider two weighted plates on a rotating mass, like the one shown in Fig.4.4. The two black circles indicate where the point mass of the plates can be measured from. If the shaft is static (not rotating), then the point will rotate under gravity, meaning that the shaft is not balanced. A realistic example of a weighted shaft is a crankshaft.

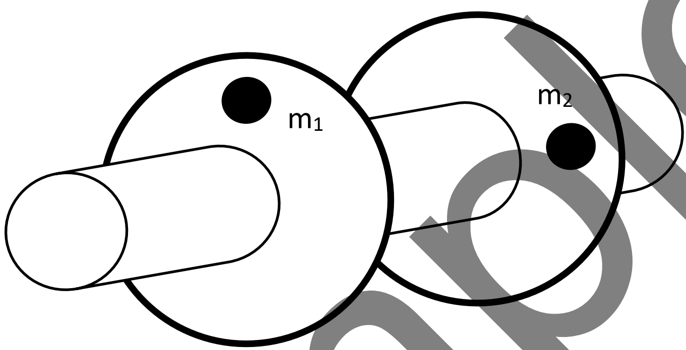


Fig.4.4: A weighted shaft

When statically balancing a shaft, it is helpful to draw a force polygon by introducing ficticious "centrifugal forces". Let's say that the distances of the masses from the centre of the shaft is "r", we can construct the force polygon, or an "mr" polygon. Fig.4.5 shows the force polygon about the shaft and determines the static forces acting on the shaft. When just considering the black arrows of m_1r_1 and m_2r_2, the polygon is "open" and unbalanced.

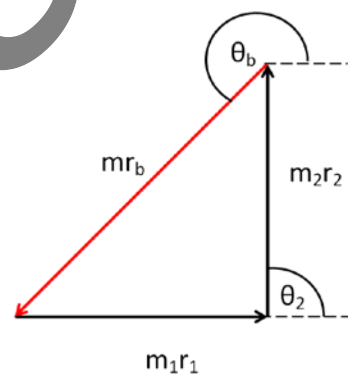


Fig.4.5: Force (mr) polygon for balancing Fig4.4, in this case, θ is the angle relative to m_1

The red line indicates the the angle, mass and distance from the centre of the shaft that the balancing force must be placed to keep the shaft statically balanced and the polygon "closed".

Example

$$mr_4 = \sqrt{0.0470^2 + 0.003^2} = 0.0471$$

We can now calculate ν:

$$\nu = Sin^{-1}1\left(\frac{0.0470}{0.0471}\right) = 86.3°$$

Which gives θ as:

$$\theta = 180 + 86.3 = 266.3°$$

4.3.2 Dynamic Balancing Rotating Mass Systems

Purpose

We have just covered static balancing. But this does not work when the shaft is rotating, this will cause problems if any of the planes are not in the centre of the shaft. The shaft in Fig.4.4 will not be balanced dynamically, when the system is rotating it can cause a moment about the centre of the shaft. One option to remove this is by statically balancing each plane on the shaft, but this is not always possible. Another option is to create a balancing moment on a new plane on the shaft, but this is ill-advised if the shaft is flexible.

Theory

Let's consider the shaft in Fig.4.4. A side view of the shaft is shown in Fig.4.6. The method for balancing dynamically is similar to balancing statically, but the masses are multiplied by their distance to the centre-line of the shaft.

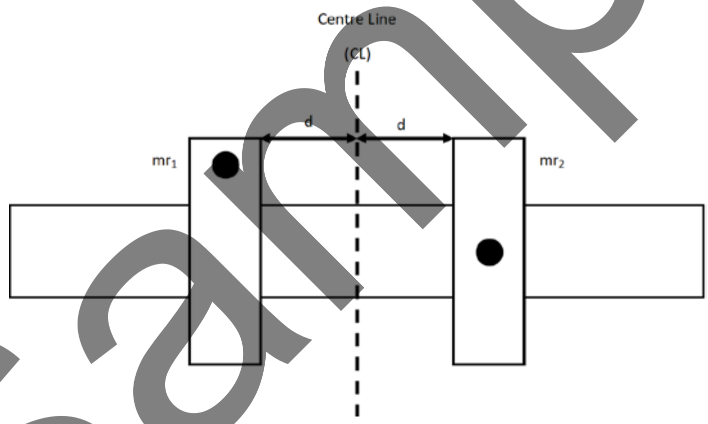


Fig.4.6: A side view of a Fig.4.4

The moment polygon of the shaft will look like Fig.4.7 below, you will notice that mr_2d is in the opposite direction, compared to mr_2 in Fig.4.5. This because the direction is a vector, and because it is left of the centre-line, the value is negative. Again, if the black arrows do not close the polygon, it is unbalanced and needs a balancing moment to close the polygon.

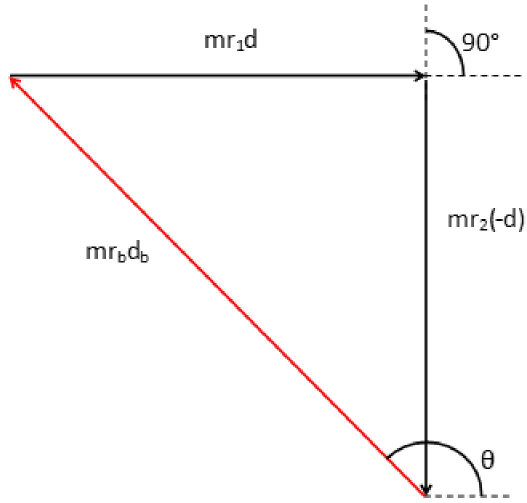
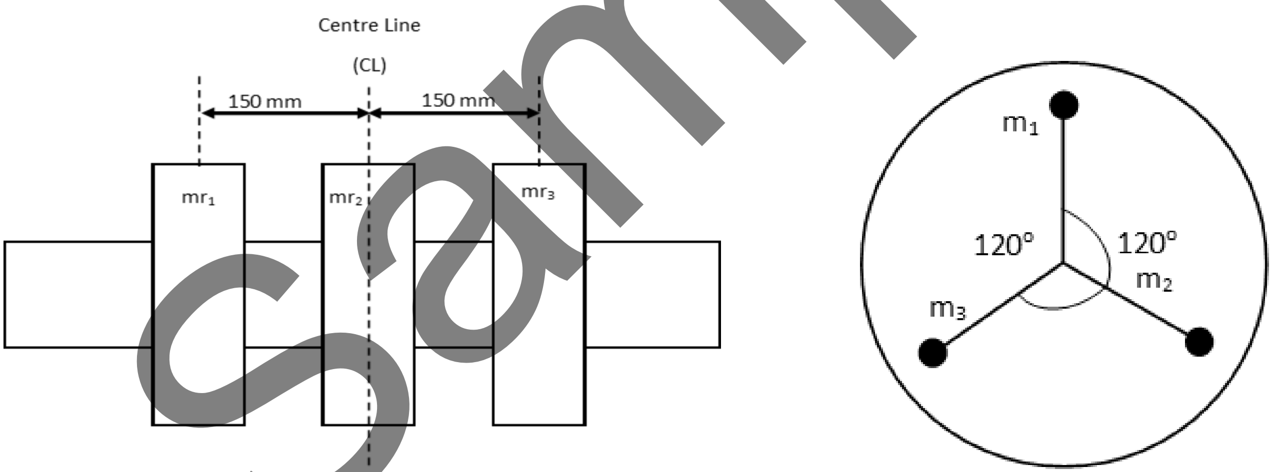


Fig.4.7: Moment polygon for the two-plane shaft from Fig.4.4

Example

The diagrams below show a simplified schematic of an unbalanced crankshaft, with all measurements required to balance the moments about the shaft. The mass of each crank is 500 g and 150 mm away from the centre of the shaft. The designer wants to balance the shaft by adding another crank of the same mass and radius to the shaft. Calculate the distance from the centre-line of the shaft that this additional crank must be placed.



Answer:

The moment polygon is drawn below **Taking left as positive for d, means mr_3d is negative, and $mr_2d = 0$**. Like static balancing, if the arrows and angles are drawn to scale, measuring the length and angle will give the correct answer. However, the trigonometry for the answer is: