Pearson BTEC Levels 5 Higher Nationals in Engineering (RQF)

Unit 39: Further Mathematics

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Matrix Methods

Contents

INTRODUCTION

Solve systems of linear equations relevant to engineering applications using matrix methods

Matrix methods:

Introduction to matrices and matrix notation.

The process for addition, subtraction and multiplication of matrices.

Introducing the determinant of a matrix and calculating the determinant for a 2x2 matrix.

Using the inverse of a square matrix to solve linear equations.

Gaussian elimination to solve systems of linear equations (up to 3x3).

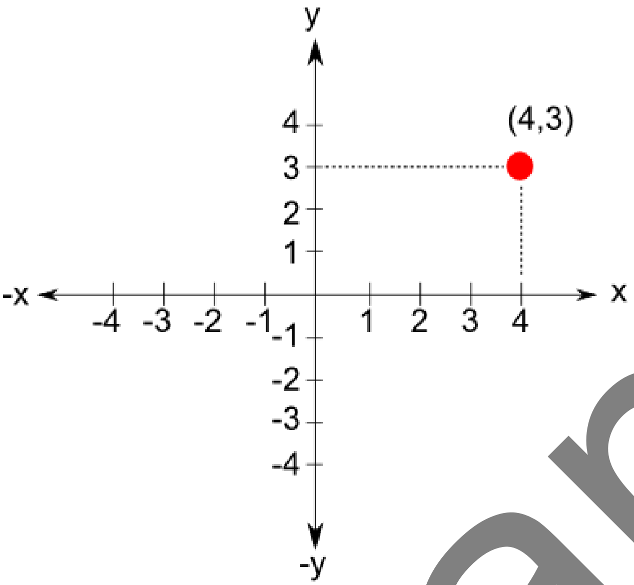
2.1 Vector Notation and Operations

2.1.1 Cartesian Co-Ordinates and Unit Vectors

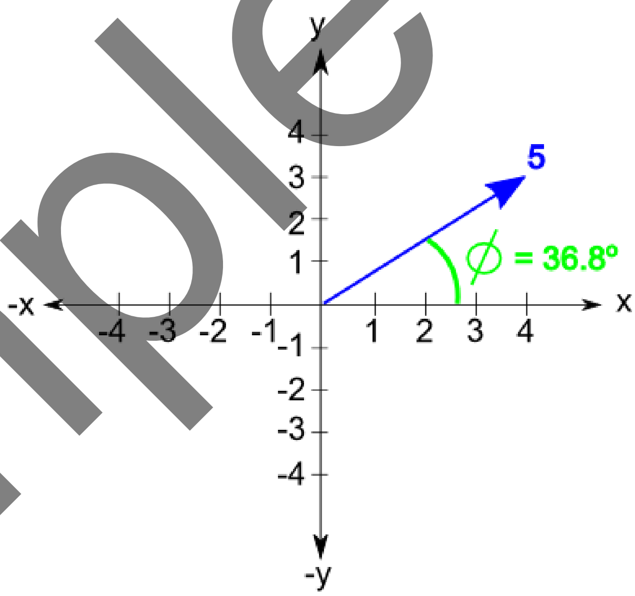
A *Scalar* is a quantity with *magnitude only*. Examples of a scalar are an amount of cash or the age of a person. ***A Vector has both magnitude <u>and</u> direction***. Examples of a vector are the motion of a car or the magnetic field emanating from a bar magnet.

We shall initially examine vectors in two dimensions (north-south and east-west if you like) and then move on to look at vectors in three dimensions (perhaps inside a cube).

Let's look at the Cartesian co-ordinate system…



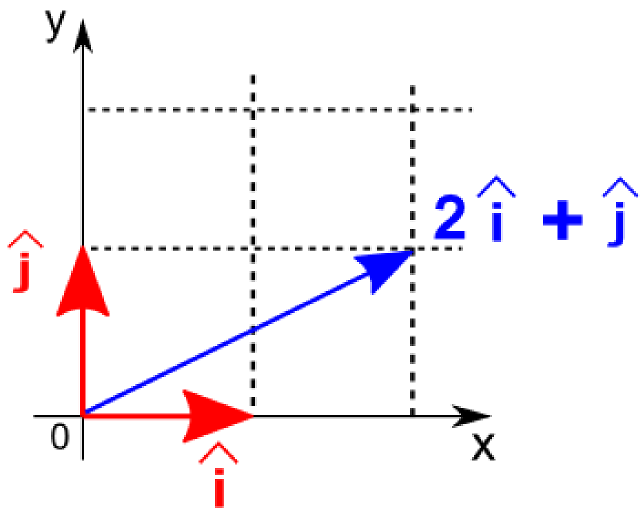
Point on a Cartesian plane



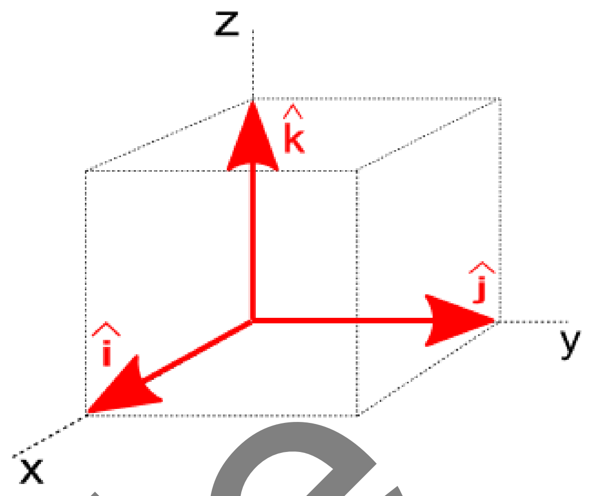
Vector representation of the same point

The point in red sits in a Cartesian co-ordinate system. It has a horizontal component ($x = 4$) and a vertical component ($y = 3$). We may represent the same point as a vector (in blue). A little bit of Pythagoras will yield a length of 5 for this vector and an angle of 36.8^{o} from the horizontal reference plane. Should a car be travelling at 5 mph in a direction 4 miles due east and 3 miles due north then it may be represented as per the figure on the right i.e. a vector quantity.

A Unit Vector is a scaled version of a vector, always having a magnitude (length) of 1. To find the unit vector version of a starting vector we simply divide the vector by its original magnitude. In our example above, the magnitude is 5 so we divide 5 to give a unit vector magnitude of 1. The horizontal component of this unit vector will then be 4/5 = 0.8 and the vertical component will be 3/5 = 0.6. A quick check using Pythagoras will give magnitude = sqrt(0.8^2 + 0.6^2) = 1. Unit vectors will be very useful in our calculations, especially when we work in three dimensions. Let's look at a unit vector co-ordinate systems in two and three dimensions…



Unit Vectors along 2-dimensional axes



Unit Vectors along 3-dimensional axes

The 2-D drawing shows our normal x and y axes in black. If we take a vector along the x axis and give it a length of 1 then this will be a unit vector along x which is called $\hat{\imath}$. Along the y axis we may produce a unit vector called $\hat{\jmath}$. The blue vector has a horizontal component of $2\hat{\imath}$ and a vertical component of $1\hat{\jmath}$ or, just $\hat{\jmath}$, to save some ink. The 3-D drawing on the right shows these unit vectors again, plus another one ($\hat{k}$) to represent the third dimension. These unit vectors will be extensively used throughout this workbook and in your third assignment.

2.1.2 Types of Vector and Vector Representation

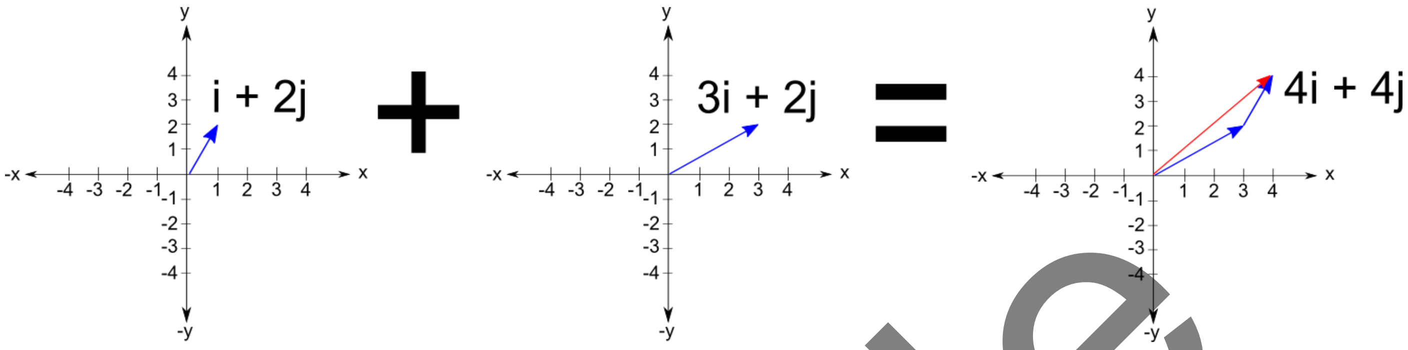
There are many types of vector. Some of the most notable ones are…

- **Co-initial vectors** their starting points are the same
- **Co-terminal vectors** their ending points are all the same
- **Coplanar vectors** they lie in the same plane
- **Null vector** has direction but no magnitude (advanced stuff)
- **Unit vectors** we now know a bit about these
- **Resultant vector** result of adding two vectors together
- **Like vectors** they act in the same direction
- **Unlike vectors** they are parallel but act in opposite directions
- **Axial vectors** act along one of the axes
- **Space vector** exists within 3-D space

Representation of vectors is performed by using multiples of the unit vector along each axis. The previous sketch represented a vector in this way.

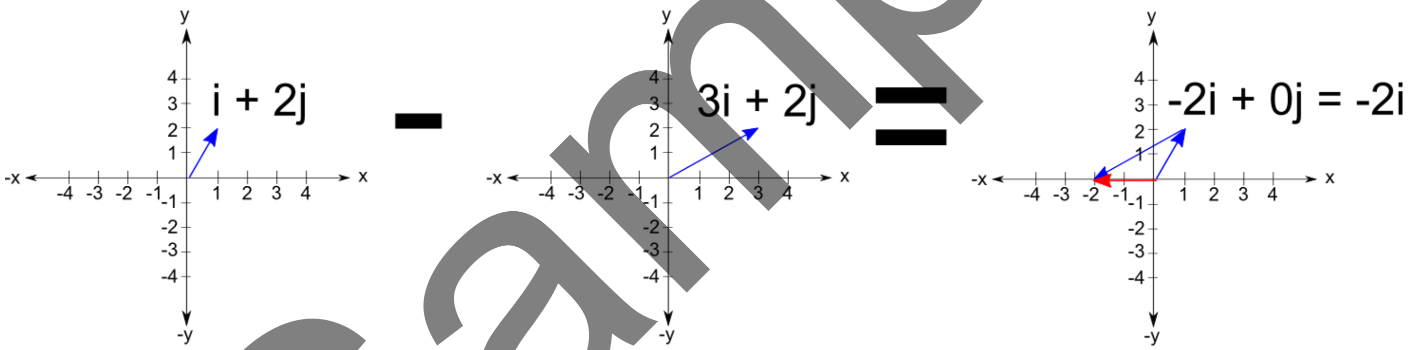
2.1.3 Addition and Subtraction of Vectors

When adding vectors all we need to do is to sum their horizontal components to arrive at the horizontal component for the resultant vector. Likewise, if we add the vertical components of two vectors then we arrive at the vertical component of the resultant vector. Let's see this pictorially…



The drawing shows the ***resultant vector*** in red. The action of adding vectors produces a resultant. The easy way is to place the vectors in a chain and then the resultant starts at the beginning of the chain and end at the end of the chain (the diagram on the right shows the two blue vectors chained together).

Subtraction of vectors is no more complicated than reversing the direction of the vector which is preceded by the '-' sign. An example…



After reversing the subtracted vector (after all a '-' sign does mean to reverse something) we do the same as vector addition – chain the vectors and draw the resultant to complete the triangle.

2.1.4 Multiplication by a Scalar

It was mentioned at the start of this section that a scalar is just a number – it has no direction. If we wish to multiply a vector by a scalar (i.e. scale the vector – make it bigger or smaller) then we just multiply the vector's magnitude by the scalar, leaving the angle alone.

2.1.5 Graphical Methods

The main graphical method has been covered in previous sections. Just be wary of labelling for axes, as you might see a,b,c or x,y,z or i,j,k or some other system used. Vectors can exist in more than three dimensions – I dare you to draw a 4-D vector! When we discover matrices we will see that 4-D, 5-D and more