Pearson BTEC Levels 5 Higher Nationals in Engineering (RQF)

Unit 39: Further Mathematics

Unit Workbook 3

in a series of 4 for this unit

Learning Outcome 3

Graphical & Numerical Methods

Contents

INTRODUCTION

Approximate solutions of contextualised examples with graphical and numerical methods

Graphical and numerical methods:

> **Standard curves of common functions, including quadratic, cubic, logarithm and exponential curves.**
>
> **Systematic curve sketching knowing the equation of the curve.**
>
> **Using sketches to approximate solutions of equations.**
>
> **Numerical analysis using the bisection method and the Newton–Raphson method.**
>
> **Numerical integration using the mid-ordinate rule, the trapezium rule and Simpson's rule.**

3.1 Graphical Techniques

3.1.1 Curve Sketching Strategies

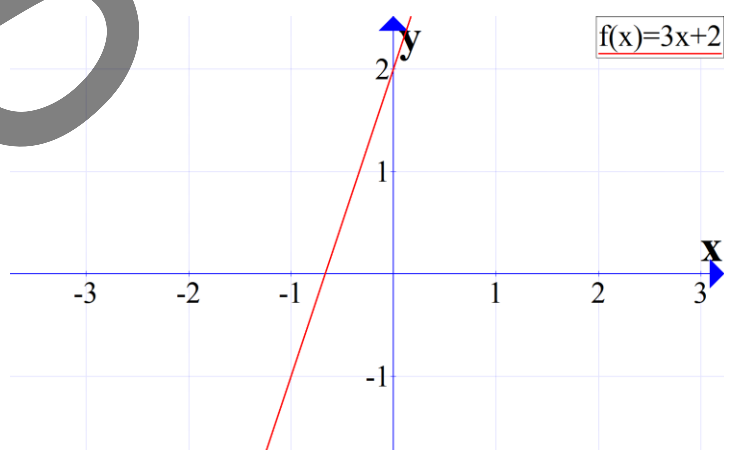
When examining the expression for a function it is usually a good idea to be able to sketch the function roughly. There are a number of ways to go about this, as follows;

- Let the independent variable (usually x or t) be zero and see what value that gives for the dependent variable (usually y or voltage or current). This will give the point where the function crosses the vertical axis. Always a good starting point in curve sketching.
- Let the dependent variable be zero and possibly then determine the roots of the equation – i.e. at which point(s) does it cross the horizontal axis?
- If the highest power of the independent variable is 1 then the function is a straight line.
- Check if it is an EVEN function i.e. does $f(x) = f(-x)$? If this is the case then the function is symmetrical about the vertical axis. A common example is $y = \cos(x)$.
- Check if it is an ODD function i.e. does $f(-x) = -f(x)$? If this is the case then the function is symmetrical about the origin. A common example is $y = \sin(x)$.
- Differentiate the function to determine any turning point. If possible, differentiate again to determine whether the turning point is a maxima or minima (peak/trough) by checking the sign on the result. Negative sign means a maxima, positive sign means a minima.
- A quadratic equation (i.e. where the highest power of the independent variable is 2) will have one turning point. A parabola will be produced.
- Determine any asymptotes (explained in a while).

3.1.2 Curve Sketching Examples

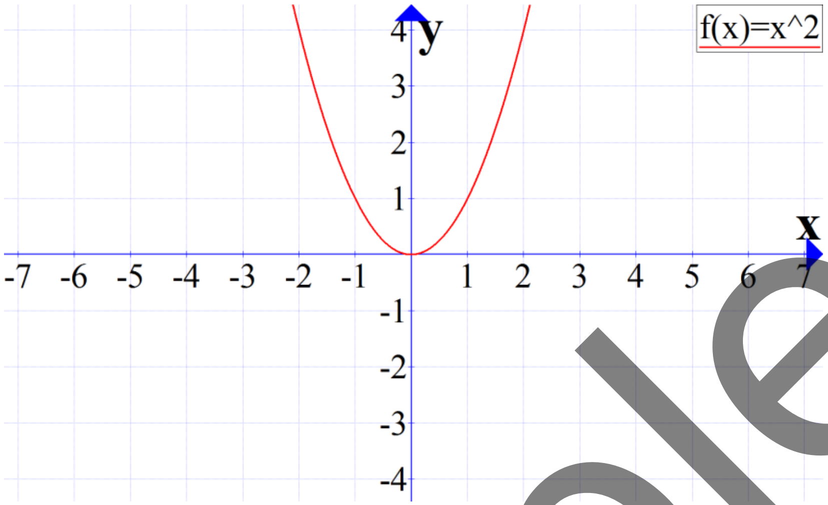
Note: In these examples Graph software is used to generate the plots. ***Your curve sketches will be drawn by hand for the assignment***.

Linear function



- Letting $x = 0$ makes $y = 2$ which is where the function crosses the y axis
- Letting $y = 0$ gives $3x + 2 = 0 \quad \therefore \quad x = -2/3$ which is where the function crosses the x axis
- The slope of the function is the coefficient of x, which is 3

Quadratic Function



- Since $y = x^2$ (raising to a power in Graph needs the ^ symbol, as indicated on the legend) then letting $x = 0$ gives $y = 0$. So, the curve meets the axes at (0, 0) – i.e. the origin.
- Since the function is a quadratic (highest power is 2) then there is one turning point. To find out what type it is we find the sign of the 2nd differential. Differentiating once gives $y' = 2x$ and differentiating again gives $y'' = 2$. The sign here is positive, indicating a local minima (as indicated in section 2.1.1).
- Also, since the function is quadratic, we have a parabola.