Pearson BTEC Level 5 Higher Nationals in Engineering (RQF)

# Unit 45: Industrial Systems

# Unit Workbook 3

in a series of 4 for this unit

Learning Outcome 3

# Measurement Systems

# 3.1 Transfer Functions

Transfer functions are used in feedback systems to model the equations that will take place in the control system. For this we will consider unit step functions. The unit step function $\mathrm{u(t)}$ can be defined as:

$$\mathrm{u(t)} = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Meaning that until we flip the switch, $\mathrm{u}$ has no value, and once we do flip the switch $\mathrm{u}$ has a value of one.

## 3.1.1 Elements of a Transfer Function

Fig.3.1 shows a completed transfer function diagram, there are three points to note:

- $\mathrm{F(s)}$ – The input signal.
- $\mathrm{H(s)}$ – The overall transfer function of the system (this can be opened up to see a range of smaller transfer functions.
- $\mathrm{Y(s)}$ – The output signal.

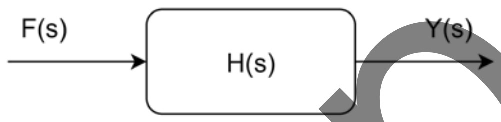



*Fig.3.1: The overall transfer function diagram*

This will create Eq.3.1, a much simpler calculation compared to solving differential equations.

$$\mathrm{H(s)} = \frac{\mathrm{Y(s)}}{\mathrm{F(s)}} = \frac{1}{\mathrm{as^2+bs+c}} \qquad \text{(Eq.3.1)}$$

However, we can expand $\mathrm{H(s)}$ to reveal the multiple transfer functions that take place, noted by $\mathrm{G_1(s)}$ and $\mathrm{G_2(s)}$, shown in Fig.3.2.

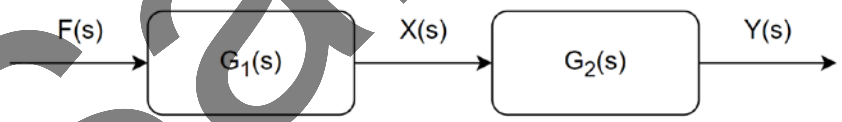



*Fig.3.2: Open loop with two transfer functions*

For the case of Fig.3.2 which is open loop, $\mathrm{H(s)}$ is given as.

$$\mathrm{H(s)} = \mathrm{G_2(s)} \cdot [X(s)] = G_1(s) \cdot G_2(s) \qquad \text{(Eq.3.2)}$$

However, we also have the case of closed loop systems, such as Fig.3.3, which is a negative feedback loop.

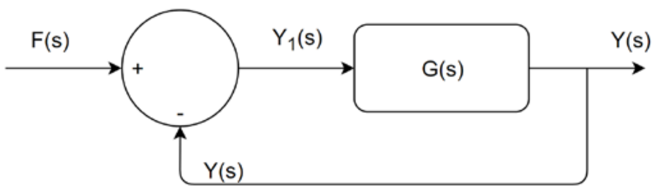



*Fig.3.3: A closed loop, negative feedback system*

The transfer function in the case of Fig.3.3 becomes Eq.3.3 The negative feedback loop means that the denominator is $1 + \mathrm{G(s)}$, a positive feedback loop would require $1 - \mathrm{G(s)}$.

$$\mathrm{H(s)} = \frac{\mathrm{G(s)}}{1+\mathrm{G(s)}} \qquad \text{(Eq.3.3)}$$

With $\mathrm{H(s)}$ calculated we then use Laplace transformations to make $\mathrm{H}(x)$. We then use equation Eq.3.4 to find out the response required to produce the output signal.

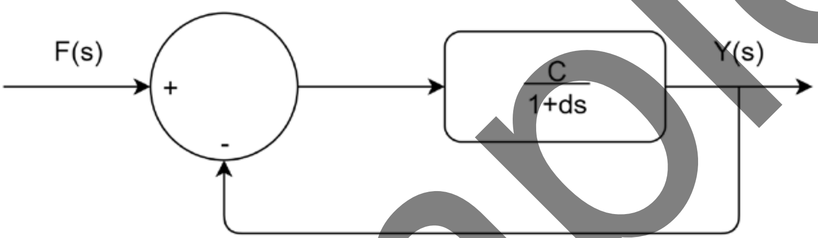
$$\mathrm{y(t)} = \int_0^t u(t-x)H(x)u(t)dx \qquad \text{(Eq.3.4)}$$

While this may look daunting, the equation then simplifies to Eq.3.5

$$\mathrm{y(t)} = \int_0^t H(x)dx \quad t > 0 \qquad \text{(Eq.3.5)}$$

### 3.1.2 Transfer Function Example

An engineering system is modelled by the following block diagram.




In the case of $\mathrm{C} = 1.5$ and $\mathrm{d} = 0.5$, determine the signal response $\mathrm{y(t)}$ when the input function is a unit step function.

We have a negative feedback loop so our equation for $\mathrm{H(s)}$ is:

$$\mathrm{H(s)} = \frac{\mathrm{G(s)}}{1+\mathrm{G(s)}} = \frac{\frac{\mathrm{C}}{1+\mathrm{ds}}}{1+\frac{\mathrm{C}}{1+\mathrm{ds}}} = \frac{\mathrm{C}}{\mathrm{C}+1+\mathrm{ds}}$$

In this case:

$$\mathrm{H(s)} = \frac{1.5}{2.5+0.5\mathrm{s}} = \frac{3}{5+\mathrm{s}}$$

We then use Laplace transformation generates the impulse response $\mathrm{H(t)}$:

$$\mathrm{H(t)} = \mathcal{L}^{-1}\mathrm{H(s)} = \mathcal{L}^{-1}\left\{\frac{3}{5+\mathrm{s}}\right\} = 3\mathrm{e}^{-5\mathrm{t}}\mathrm{u(t)}$$

And so, the response to a step input is given by the integration of $\mathrm{h(t)}$ with $\mathrm{u(t)}$:

$$\mathrm{y(t)} = \int u(t-x)5e^{-7x}u(t)dx$$

$$= \int_0^t 5e^{-7x}dx \quad t > 0$$

# 3.2 Measurement in Practice

People make measurements for many reasons: to make sure an item will fit, to determine the correct price to pay for something, or to check that a manufactured item is within specification. In all cases, a measurement is only useful if it is suitable for the intended purpose. Consider the following questions:

- Do you know how accurate your measurement result is?
- Is this accurate enough?
- How strongly do you trust the result?

These questions relate to the quality of a measurement. When talking about measurement quality, it is important to understand the following concepts:

## 3.2.1 Precision, Accuracy and Uncertainty

Precision is about how close measurements are to one another. Accuracy is about how close measurements are to the 'true value'. In reality, it is not possible to know the 'true value' and so we introduce the concept of uncertainty to help quantify how wrong our value might be. The difference between accuracy and precision is illustrated in Fig.3.4 below. The idea is that firing an arrow at a target is like making a measurement. Accuracy is a qualitative term that describes how close a set of measurements are to the actual (true) value. Precision describes the spread of these measurements when repeated. A measurement that has high precision has good repeatability.


*Fig.3.4: The difference between accuracy and precision*

In practice we are not able to view the target and assess how close to the 'true value' our measurements are. What interests us is the answer to the question "How far from the target could our arrows have fallen?" We also need to ask, "How wrong could we have been?" To answer these questions, we need to look at all the factors that go into making a measurement and how each factor could have affected the final estimate of the answer. The answer to "How wrong are we likely to have been?" is known as the 'measurement uncertainty', and this is the most useful assessment of how far our estimate is likely to lie from the 'true value'. For example, we might say that the length of a particular stick is $200cm$ with an uncertainty of $\pm 1cm$.

## 3.2.2 Don't Confuse Mistakes with Errors!

Measurement scientists use the term 'error' to specify the difference between an estimate of quantity and its 'true value'. The word 'error' does not imply that any mistakes have been made. Where the size and effect of an error are known (e.g. from a calibration certificate) a correction can be applied to the measurement result. If the value of an error is not known, this is a source of uncertainty.

- **Uncertainty** is the quantification of the doubt about the measurement result and tells us something about its quality.
- **Error** is the difference between the measured value and the true value of the thing being measured.
- **True value** is the value that would be obtained by a theoretically perfect measurement.

So, what is not uncertainty?

- Mistakes made by operators are **NOT** uncertainties – operator mistakes can be avoided by working carefully through a procedure and checking work.
- Tolerances are **NOT** uncertainties – tolerances are acceptance limits chosen for a process or product.
- Accuracy is **NOT** uncertainty – the true value of a measurement is never known.

### 3.2.3 Repeatability and Reproducibility

'Measure twice and cut once.' This popular proverb expresses the need to make sure we have a good measurement before committing to a potentially irreversible decision. It is a concept that you should adhere to. By repeating a measurement many times, a mean (average) value can be calculated. If the repeatability is high, the statistical uncertainty in the mean value will be low. However, if different measuring equipment is used, a different result may be obtained because of errors and offsets in the instruments.

If you take a voltage measurement three times in one minute using the same multimeter, you would expect to get a similar answer each time. Repeatability describes the agreement within sets of measurements where the same person uses the same equipment in the same way, under the same conditions. But if another person had a go at taking the same measurement on different days using different measuring equipment, a wider range of answers would be much less surprising. This is known as 'reproducibility' and describes the agreement within a set of measurements where different people, equipment, methods, locations or conditions are involved.

- **Repeatability** is the closeness of agreement between repeated measurements of the same thing, carried out in the same place, by the same person, on the same equipment, in the same way, at similar times.
- **Reproducibility** is the closeness of agreement between measurements of the same thing carried out in different circumstances, e.g. by a different person, or a different method, or at a different time.

### 3.2.4 Tolerance

Tolerance, also known as 'acceptance criteria'. It is the maximum acceptable difference bet ween the actual value of a quantity and the value specified for it. For example, if an electrical resistor has a specification of $10\Omega$ and there is a tolerance of $\pm 10\%$ on that specification, the minimum acceptable resistance would be $9\Omega$ and the maximum would be $11\Omega$. Many factors can reduce accuracy or precision and increase the uncertainty of your measurement result. Some of the most common are:

- **Environmental conditions** – changes in temperature or humidity can expand or contract materials as well as affect the performance of measurement equipment.
- **Inferior measuring equipment** – equipment which is poorly maintained, damaged or not calibrated will give less reliable results.
- **Poor measuring techniques** – having consistent procedures for your measurements is vital.

# 3.3 Error Analysis

*"Errors using inadequate data are much less than those using no data at all."*

(C. Babbage)

No measurement of a physical quantity can be entirely accurate. It is important to know, therefore, just how much the measured value is likely to deviate from the unknown, true, value of the quantity. The art of estimating these deviations should probably be called uncertainty analysis, but for historical reasons is referred to as error analysis.

## 3.3.1 Significant Figures

Whenever you make a measurement, the number of meaningful digits that you write down implies the error in the measurement. For example, if you say that the length of an object is $0.428m$, you imply an uncertainty of about $0.001m$. To record this measurement as either $0.4$ or $0.42819667$ would imply that you only know it to $0.1m$ in the first case or to $0.00000001m$ in the second. You should only report as many significant figures (S.F) as are consistent with the estimated error. The quantity $0.428m$ is said to have three S.F that is, three digits that make sense in terms of the measurement. Notice that this has nothing to do with the "number of decimal places". The same measurement in centimetres would be $42.8cm$ and still be a three S.F number. The accepted convention is that only one uncertain digit is to be reported for a measurement. In the example if the estimated error is $0.02m$ you would report a result of $0.43 \pm 0.02\ \mathrm{m}$, not $0.428 \pm 0.02\ m$.

Students frequently are confused about when to count a zero as a S.F. The rule is: If the zero has a non-zero digit anywhere to its left, then the zero is significant, otherwise it is not. For example, $5.00$ has three S.F; the number $0.0005$ has only one S.F, and $1.0005$ has five S.F.

## 3.3.2 Absolute and Relative Errors

The absolute error in a measured quantity is the uncertainty in the quantity and has the same units as the quantity itself. For example, if you know a length is $0.428\mathrm{m} \pm 0.002\mathrm{m}$, the $0.002\mathrm{m}$ is an absolute error.

The relative error (also called the fractional error) is obtained by dividing the absolute error in the quantity by the quantity itself. The relative error is usually more significant than the absolute error. For example, a $1\mathrm{mm}$ error in the diameter of a skate wheel is probably more serious than a $1\mathrm{mm}$ error in a truck tire. Note that relative errors are dimensionless. When reporting relative errors, it is usual to multiply the fractional error by $100$ and report it as a percentage. So, the absolute error of $0.428\mathrm{m} \pm 0.002\mathrm{m}$ can also be written as a relative error of $0.428m \pm\ 0.467\%$

## 3.3.3 Systematic Errors (Human Error)

Systematic errors arise from a flaw in the measurement scheme which is repeated each time a measurement is made. If you do the same thing wrong each time you make the measurement, your measurement will differ systematically (that is, in the same direction each time) from the correct result. Some sources of systematic error are:

- Errors in the calibration of the measuring instruments.
- Incorrect measuring technique: For example, one might make an incorrect scale reading because of parallax error.
- Bias of the experimenter. The experimenter might consistently read an instrument incorrectly or might let knowledge of the expected value of a result influence the measurements.

Clearly, systematic errors do not average to zero if you average many measurements. If a systematic error is discovered, a correction can be made to the data for this error. If you measure a voltage with a meter that later turns out to have a $0.2\ \mathrm{V}$ offset, you can correct the originally determined voltages by this amount and eliminate the error. Although random errors can be handled routinely, there is no prescribed way to find systematic errors. One must simply sit down and think about all the possible sources of error, and then do small experiments to see if these sources are active. The goal of a good experiment is to reduce the systematic errors to a value smaller than the random errors. For example, a metre stick should have been manufactured such that the millimetre markings are positioned much more accurately than one millimetre.

### 3.3.4 Mistakes

A procedural error that should be avoided by careful attention. These are illegitimate errors and can generally be corrected by carefully repeating the operations.

### 3.3.5 Discrepancies

A significant difference between two measured values of the same quantity, the implication being that the difference between the measured values is greater than the combined experimental uncertainty.

### 3.3.6 Standard Error (Standard Deviation of the Mean)

The sample standard deviation divided by the square root of the number of data points shown by Eq.3.2, where $\sigma^2 = \sum \frac{(x_i - x)^2}{(n-1)}$ is the sample variance.

$$SE = \frac{\sigma}{\sqrt{n}} \qquad \text{(Eq.3.2)}$$

### 3.3.7 Margin of Error

Range of uncertainty. Public opinion polls generally use margin of error to indicate a 95% confidence interval, corresponding to an uncertainty range of $x \pm 2s$.

### 3.3.8 Random errors

Random errors arise from the fluctuations that are most easily observed by making multiple trials of a given measurement. For example, if you were to measure the period of a pendulum many times with a stop watch, you would find that your measurements were not always the same. The main source of these fluctuations would probably be the difficulty of judging exactly when the pendulum came to a given point in its motion, and in starting and stopping the stop watch at the time that you judge. Since you would not get the same value of the period each time that you try to measure it, your result is obviously uncertain. There are several common sources of such random uncertainties in the type of experiments that you are likely to perform:

Uncontrollable fluctuations in initial conditions in the measurements. Such fluctuations are the main reason why, no matter how skilled the player, no individual can toss a basketball from the free throw line through

# 3.3 Data Representation Techniques

## 3.3.1 Tables

Tables are a concise way of recording data, it can show the precise values obtained from the experiments. Using spreadsheet software (such as Microsoft Excel) to record data proves useful as it is easy to take the data and quickly calculate further information for each test, or produce graphical results.

## 3.3.2 Graphical Techniques

Whereas statistics and data analysis procedures generally yield their output in numeric or tabular form, graphical techniques allow such results to be displayed in some sort of pictorial form. They include plots such as scatter plots, histograms, probability plots, spaghetti plots, residual plots, box plots, and many more.

Exploratory data analysis (EDA) relies heavily on such techniques. They can also provide insight into a data set to help with testing assumptions, model selection and regression model validation, estimator selection, relationship identification, factor effect determination, and outlier detection. In addition, the choice of appropriate statistical graphics can provide a convincing means of communicating the underlying message that is present in the data to others. Graphical statistical methods have four objectives:

- The exploration of the content of a data set
- The use to find structure in data
- Checking assumptions in statistical models
- Communicate the results of an analysis.

If one is not using statistical graphics, then one is forfeiting insight into one or more aspects of the underlying structure of the data.

## 3.3.3 Scatter Graphs

Scatter graphs are used to show check for any linear correlation, and with the correlation try to confirm a causation. It's always important to see if there are relationships and trends between two quantitative pieces of data. The correlation is the apparent relationship between the two variables, a strong correlation between two variables does not mean there is a causation.

Let's say we run an ice cream van, we record the average temperature over the hour, and record how many sales we make for that hour. Fig.3.5 shows the scatter graph for the data collected over 14 days.

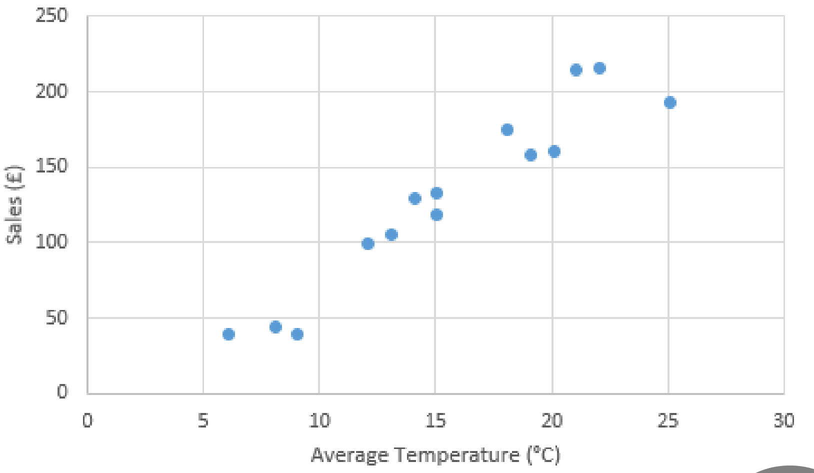



*Fig.3.5: Average temperature compared to ice cream sales*

We can see there is a strong positive correlation between the temperature outside, and the number of ice creams sold, and in this case, we know the causation (everyone wants an ice cream when its hot outside). This would be defined as a strong positive correlation, Fig.3.6 shows different correlations that a graph can show.

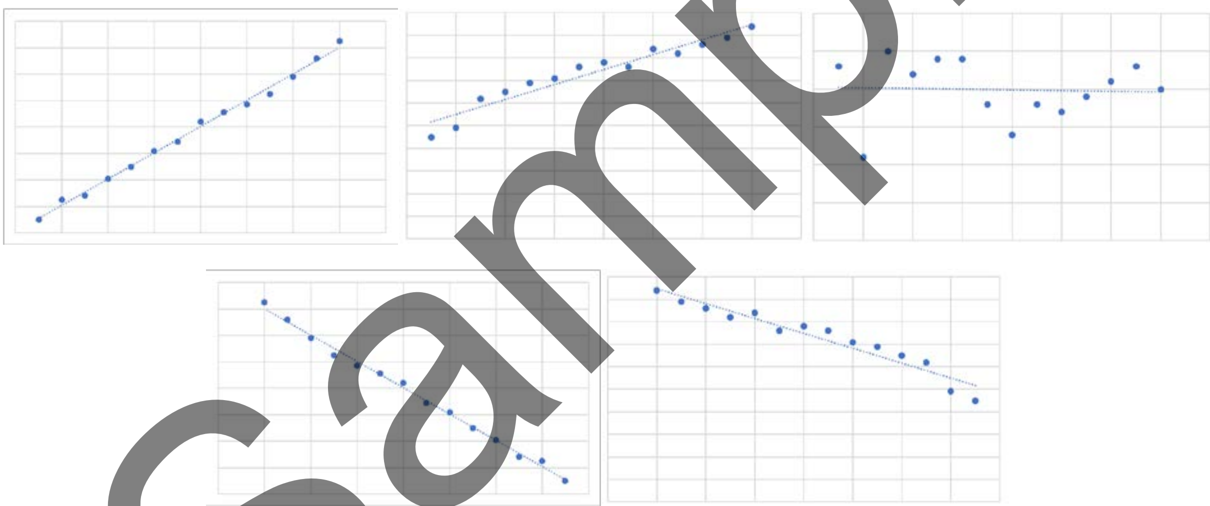


*Fig.3.6: Possible correlations (clockwise from top left): Strong positive, weak positive, no correlation, weak negative, and strong negative*

Sometimes there can be too much data to realise there is a correlation, so it can be important to use the equations discussed in Unit 2: Engineering Maths.





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**Example**

Consider the following 15-point data set:

*Table 3.1: Resistance of a component compared with current drawn*

| Resistance (Ω) | Current Drawn (A) |
|---:|---:|
| 1 | 4 |
| 5 | 2 |
| 1 | 8 |
| 6 | 2 |
| 8 | 4 |
| 4 | 3 |
| 3 | 6 |
| 5 | 5 |
| 8 | 4 |
| 7 | 2 |
| 9 | 1 |
| 4 | 3 |
| 3 | 5 |
| 2 | 8 |
| 1 | 7 |

Recalling the general equation for a straight-line graph, shown with Eq.3.3.

$$y = mx + c \qquad \text{(Eq.3.3)}$$

Eq.3.4 shows the equation with linear regression analysis variables:

$$y = a_1 x + a_0 \qquad \text{(Eq.3.4)}$$

Use Eq.3.5 and 3.6 to calculate $a_0$ and $a_1$, where N is the number of points.

$$\sum y_i = a_0 N + a_1 \sum x_i \qquad \text{(Eq.3.5)}$$

$$\sum x_i y_i = a_0 \sum x_i + a_1 \sum x_i^2 \qquad \text{(Eq.3.6)}$$

Table 3.2 below gives more information needed to calculate $a_0$ and $a_1$.

*Table 3.2: $x_i, y_i, x_i y_i, x_i^2$ and their respective totals*

| $x_i$ | $y_i$ | $x_i y_i$ | $x_i^2$ |
|---:|---:|---:|---:|
| 1 | 4 | 4 | 1 |
| 5 | 2 | 10 | 25 |
| 1 | 8 | 8 | 1 |
| 6 | 2 | 12 | 36 |
| 8 | 4 | 32 | 64 |
| 4 | 3 | 12 | 16 |
| 3 | 6 | 18 | 9 |
| 5 | 5 | 25 | 25 |
| 8 | 4 | 32 | 64 |
| 7 | 2 | 14 | 49 |
| 9 | 1 | 9 | 81 |
| 4 | 3 | 12 | 16 |
| 3 | 5 | 15 | 9 |
| 2 | 8 | 16 | 4 |
| 1 | 7 | 7 | 1 |
| 67 | 64 | 226 | 401 |

With the following information we can put the appropriate values into Eq.1.3 and 1.4 to create two simultaneous equations.

$$64 = \mathrm{a}_0(15) + \mathrm{a}_1(67) \qquad [1]$$

$$226 = \mathrm{a}_0(67) + \mathrm{a}_1(401) \qquad [2]$$

Multiply [1] by 67 and [2] by 15 to get [3] and [4]

$$4288 = 1005\mathrm{a}_0 + 4489\mathrm{a}_1 \qquad [3]$$

$$3390 = 1005\mathrm{a}_0 + 6015\mathrm{a}_1 \qquad [4]$$

Subtract [3] from [4]

$$-898 = 1526\mathrm{a}_1 \therefore \mathrm{a}_1 = \frac{-898}{1526} = -0.588$$

$$\therefore \mathrm{a}_0 = \frac{64 - 67(-0.588)}{15} = 6.89$$

The equation for linear regression is therefore:

$$\mathrm{y} = -0.588\mathrm{x} + 6.89$$

The scatter graph with the line of regression is shown by Fig.3.7.

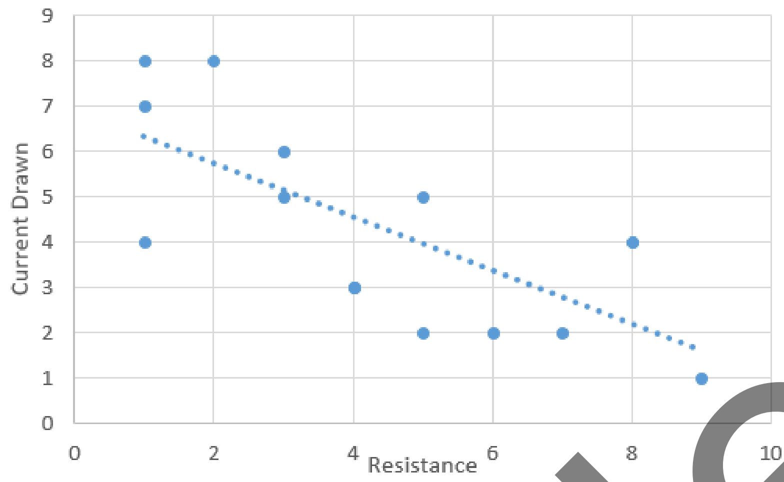


*Fig.3.7: Scatter graph and linear regression line for the Example*

However, just because there is a correlation, does not mean there is a causation. In Fig.3.5 we know that on a hot day everyone loves an ice cream, so we can link that causation. What if data was gathered to "compare the height of a person and the number of times they have been skydiving" and the data obtained just happens to look like Fig.3.8 below. It could be said that "from the data obtained, more people who have skydived were tall" but it is impossible to say that "people skydive because they're tall", unless further experimentation is done to prove causation.

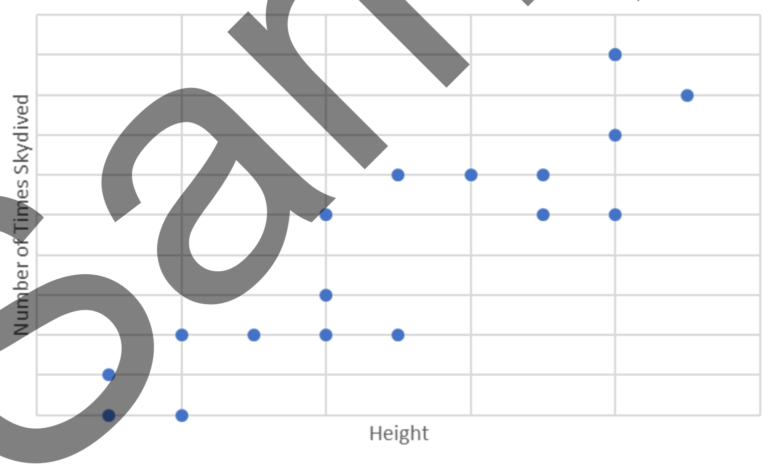


*Fig.3.8: Height compared to number of times skydived*



## 3.3.4 Histograms

Histograms are similar to bar charts, but are used to cover a range of values, instead of a singular value and can be used to show a probability distribution graphically, Fig.3.9 shows a histogram for the production rate per hour over a 24-hour period, the data is shown in Table 3.3. It is possible that the histogram can create a normally distributed graph, recalling from Unit 2: Engineering Maths, it produces the bell curve that is used to calculate the probability using "Z values".

*Table 3.3: Frequency of achieved production rates over a 24-hou3.3.3r period.*

| Production rate (products/hour) | Frequency |
|---|---|
| 1-10 | 5 |
| 11-20 | 3 |
| 21-30 | 4 |
| 31-40 | 8 |
| 41-50 | 3 |
| Total | 23 |

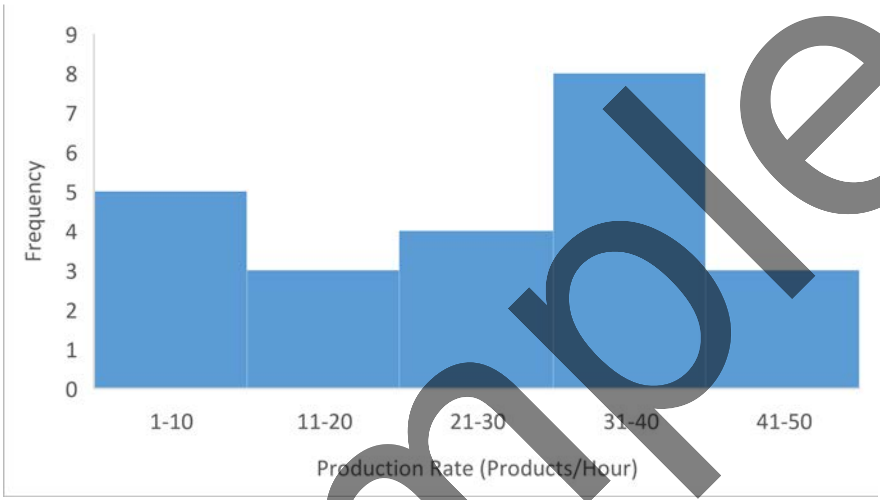


*Fig.3.9: Histogram example*

## 3.3.5 Spaghetti Plots

Spaghetti plots are normally associated with weather forecasts, to analyse the possible paths that a storm may follow. But they can also be used in engineering analysis. They can also be used for "parametric sweeps" of a component, by applying a range of values to "sweep through" and gather a variety of results. Consider the damped mass-spring system shown in Fig.3.10, where $m$ is the mass of the body, $k$ is the spring constant, and c is the damping coefficient.

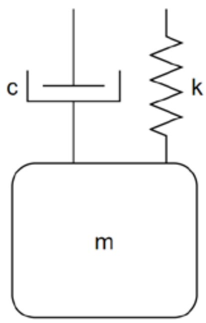


*Fig.3.10: Damped mass-spring system*

What would be the result if a parametric sweep was applied to c analysing 4 different values. The graphical readout would look like Fig.3.11, giving a spaghetti plot.

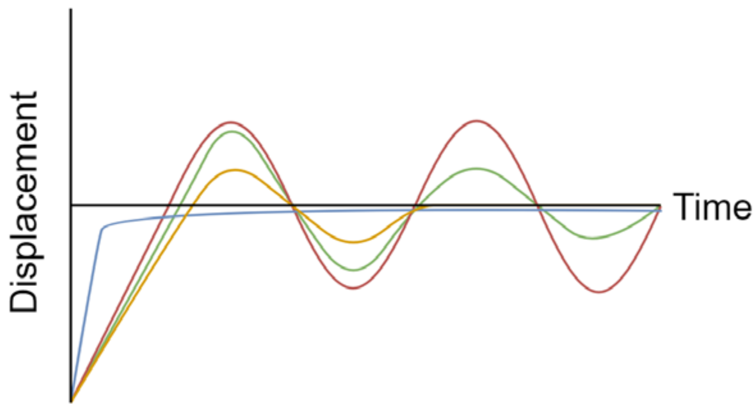


*Fig.3.11: Spaghetti plot of a parametric sweep for a damped system.*

### 3.3.6 Box Plot

A box plot (commonly referred to as the "Box and Whiskers" diagram) are used to display the maximum, minimum, median and also the inter-quartile range. This can be used to interpret the spread of the data; if the box is small, then the majority of the data is densely packed, and if the whiskers are long, then there is a there will be a large difference between the majority of the data, which could suggest a statistical anomaly. Fig.3.12 shows a box plot example.

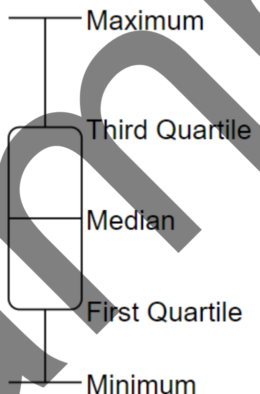


*Fig3.12: Box plot example*

### 3.3.7 Residual Plot

Residual plots are used to develop regression models to map trendlines. If the plot appears to look random about the horizontal axis, then a linear model is used. However, if there is a notable U-shape in the graph, then a non-linear model would be used.





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