



Contents

NTRODUCTION	3
GUIDANCE	3





INTRODUCTION

Apply practical and computer-based methods to design and test a measurement system

- System modelling and analysis:
 - The use of transfer functions to help predict the behaviour and constancy of an industrial process, including accuracy, resolution and tolerances, repeatability and stability, sensitivity and response time.
 - o Dealing with error and uncertainty in industrial systems.
 - Use of computer packages in measurement and control, and dealing with uncertainty and errors in systems.

GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;





3.1 Transfer Functions

Transfer functions are used in feedback systems to model the equations that will take place in the control system. For this we will consider unit step functions. The unit step function u(t) can be defined as:

$$\mathbf{u}(\mathbf{t}) = \begin{cases} 0 & t < 0\\ 1 & t \ge 0 \end{cases}$$

Meaning that until we flip the switch, u has no value, and once we do flip the switch u has a value of one.

3.1.1 Elements of a Transfer Function

Fig.3.1 shows a completed transfer function diagram, there are three points to note:

- F(s) The input signal.
- H(s) The overall transfer function of the system (this can be opened up to see a range of smaller transfer functions.
- Y(s) The output signal.



Fig.3.1: The overall transfer function diagram

This will create Eq.3.1, a much simpler calculation compared to solving differential equations.

$$H(s) = \frac{Y(s)}{F(s)} = \frac{1}{as^2 + bs + c}$$
 (Eq.3.1)

However, we can expand H(s) to reveal the multiple transfer functions that take place, noted by $G_1(s)$ and $G_2(s)$, shown in Fig.3.2.



For the case of Fig.3.2 which is open loop, H(s) is given as.

 $H(s) = G_2(s) \cdot [X(s)] = G_1(s) \cdot G_2(s)$ (Eq.3.2)

However, we also have the case of closed loop systems, such as Fig.3.3, which is a negative feedback loop.



Fig.3.3: A closed loop, negative feedback system



The transfer function in the case of Fig.3.3 becomes Eq.3.3 The negative feedback loop means that the denominator is 1 + G(s), a positive feedback loop would require 1 - G(s).

$$H(s) = \frac{G(s)}{1+G(s)}$$
 (Eq.3.3)

With H(s) calculated we then use Laplace transformations to make H(x). We then use equation Eq.3.4 to find out the response required to produce the output signal.

$$y(t) = \int_0^t u(t-x)H(x)u(t)dx$$
 (Eq.3.4)

While this may look daunting, the equation then simplifies to Eq.3.5

$$y(t) = \int_0^t H(x) dx \quad t > 0$$
 (Eq.3.5)

3.1.2 Transfer Function Example

An engineering system is modelled by the following block diagram.



In the case of C = 1.5 and d = 0.5, determine the signal response y(t) when the input function is a unit step function.

We have a negative feedback loop so our equation for H(s) is:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{\frac{C}{1 + ds}}{1 + \frac{C}{1 + ds}} = \frac{C}{C + 1 + ds}$$

In this case:

$$H(s) = \frac{1.5}{2.5 + 0.5s} = \frac{3}{5 + s}$$

We then use Laplace transformation generates the impulse response H(t):

$$H(t) = \mathcal{L}^{-1}H(s) = \mathcal{L}^{-1}\left\{\frac{3}{5+s}\right\} = 3e^{-5t}u(t)$$

And so, the response to a step input is given by the integration of h(t) with u(t):

$$y(t) = \int u(t-x)5e^{-7x}u(t)dx$$
$$= \int_0^t 5e^{-7x}dx \quad t > 0$$



$$= \left[-\frac{5}{7} e^{-7x} \right]_0^t = -\frac{5}{7} [e^{-7t} - 1]$$





3.2 Measurement in Practice

People make measurements for many reasons: to make sure an item will fit, to determine the correct price to pay for something, or to check that a manufactured item is within specification. In all cases, a measurement is only useful if it is suitable for the intended purpose. Consider the following questions:

- Do you know how accurate your measurement result is?
- Is this accurate enough?
- How strongly do you trust the result?

These questions relate to the quality of a measurement. When talking about measurement quality, it is important to understand the following concepts:

3.2.1 Precision, Accuracy and Uncertainty

Precision is about how close measurements are to one another. Accuracy is about how close measurements are to the 'true value'. In reality, it is not possible to know the 'true value' and so we introduce the concept of uncertainty to help quantify how wrong our value might be. The difference between accuracy and precision is illustrated in Fig.3.4 below. The idea is that firing an arrow at a target is like making a measurement. Accuracy is a qualitative term that describes how close a set of measurements are to the actual (true) value. Precision describes the spread of these measurements when repeated. A measurement that has high precision has good repeatability.



In practice we are not able to view the target and assess how close to the 'true value' our measurements are. What interests us is the answer to the question "How far from the target could our arrows have fallen?" We also need to ask, "How wrong could we have been?" To answer these questions, we need to look at all the factors that go into making a measurement and how each factor could have affected the final estimate of the answer. The answer to "How wrong are we likely to have been?" is known as the 'measurement uncertainty', and this is the most useful assessment of how far our estimate is likely to lie from the 'true value'. For example, we might say that the length of a particular stick is 200cm with an uncertainty of $\pm 1cm$.

3.2.2 Don't Confuse Mistakes with Errors!

Measurement scientists use the term 'error' to specify the difference between an estimate of quantity and its 'true value'. The word 'error' does not imply that any mistakes have been made. Where the size and effect of an error are known (e.g. from a calibration certificate) a correction can be applied to the measurement result. If the value of an error is not known, this is a source of uncertainty.

