Pearson BTEC Level 5 Higher Nationals in Engineering (RQF)

Unit 52: Further Electrical, Electronic and Digital Principles

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Three-phase Theory

2.1 Use of j Notation

When we have circuits containing just resistors then life is so easy in terms of circuit analysis. Most useful circuits also contain capacitors and inductors (usually coils and windings). The introduction of capacitors and inductors into circuits causes 'phase angles' in our calculations. The study of these phase angles is made much easier by introducing complex numbers.

From your level 3 studies you will have come across Inductive Reactance (X_L) and Capacitive Reactance (X_C). These terms are used to quantify the amount of 'opposition' caused by capacitors and inductors to changes in current or voltage. The term 'reactance' is brought about because capacitors cannot be charged or discharged in zero time, and inductors cannot be energised or de-energised in zero time. A good analogy for capacitors is the amount of water in a bathtub. It is impossible to fill a bathtub in zero time, and it's also impossible to empty a bathtub in zero time. The amount of reactance from capacitors and inductors is a function of their manufactured properties and the frequency of operation. Let's review the equations for these reactances…

$$X_L = 2\pi f L \quad [\Omega]$$

$$X_C = \frac{1}{2\pi f C} \quad [\Omega]$$

where;

$X_L = inductive\ reactance\ (measured\ in\ Ohms, \Omega)$

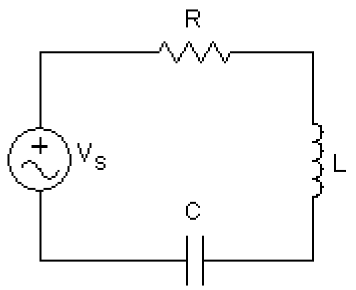
$X_C = capacitive\ reactance\ (measured\ in\ Ohms, \Omega)$

$f = frequency\ (measured\ in\ Hertz, Hz)$

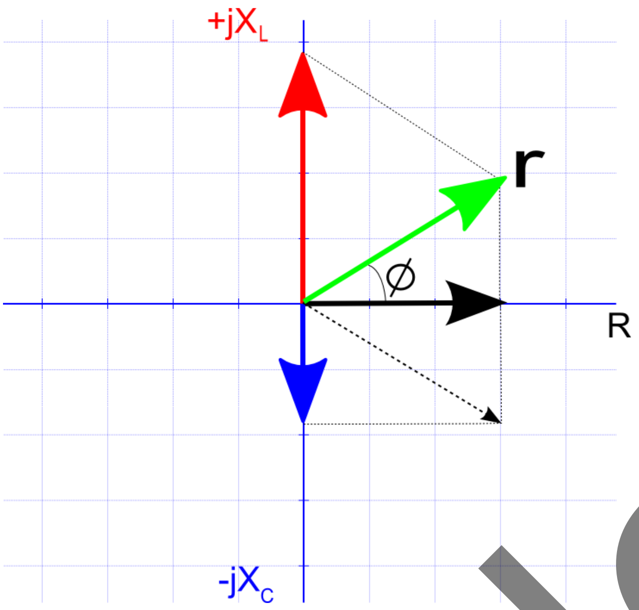
$L = inductance\ (measured\ in\ Henries,$ H)

$C = capacitance\ (measured\ in\ Farads, F)$

Consider the RLC circuit below…



We can draw a phasor diagram for this circuit, as follows…



The black arrow represents resistance. The current through a resistor is always in phase with the voltage across it. We place resistance on the horizontal axis.

The red arrow represents inductive reactance. We see that this *leads* the resistance by 90 degrees ($\pi/2\ rads.$). We name this axis the '+j axis'. Mathematicians tend to designate this the '+i' (imaginary) axis. Engineers do not use i since it clashes with the current symbol, so we use 'j' instead.

The blue arrow represents capacitive reactance. We see that this *lags* the resistance by 90 degrees ($\pi/2\ rads.$). We designate this the '-j' axis.

The dashed lines represent a graphical method of finding the resultant of these phasors, drawn in green. We term this resultant the *impedance* of the circuit and mark it with 'r' for resultant. This resultant impedance makes an angle with the horizontal axis, marked with ϕ.

The resultant impedance is given the symbol Z for calculation purposes. We see that the green resultant has both horizontal and vertical components. The horizontal contribution is known as the *real* component and the vertical contribution is known as the imaginary component.

We may use Pythagoras' theorem to denote impedance as follows…

$$Z^2 = R^2 + X^2$$

$$\therefore \quad Z = \sqrt{R^2 + X^2} \quad [\Omega]$$

In complex number notation we represent Z as…

$$\boldsymbol{Z = R + j(X_L - X_C)} \quad [\boldsymbol{\Omega}]$$

$$j \times j = j^2 = \sqrt{-1} \times \sqrt{-1} = -1^{0.5} \times -1^{0.5} = -1^{0.5+0.5} = -1^1 = -1$$

$$\frac{6j}{3j} = 2$$

Worked Example 3

A circuit current calculation involves the division of a voltage by an impedance…

$$\boldsymbol{i} = \frac{\mathbf{10 - j30}}{\mathbf{3 + j4}}$$

Determine the value of the current.

To perform such calculations we need to determine the *complex conjugate* of the denominator and then multiply this complex conjugate by both the numerator and denominator. The process is…

$$\frac{complex\ number\ 1}{complex\ number\ 2} = \frac{complex\ number\ 1}{complex\ number\ 2} \times \frac{complex\ conjugate\ of\ number\ 2}{complex\ conjugate\ of\ number\ 2}$$

The use of the complex conjugate actually simplifies our task because the new denominator becomes a purely real number.

The complex conjugate of a complex number is simply the same complex number with the sign on the j term negated. So, we can write…

$$\boldsymbol{i} = \frac{\mathbf{10 - j30}}{\mathbf{3 + j4}} = \frac{(\mathbf{10 - j30})}{(\mathbf{3 + j4})} \times \frac{(\mathbf{3 - j4})}{(\mathbf{3 - j4})}$$

$$= \frac{30 - j40 - j90 - -j^2 120}{9 - j12 + j12 + -j^2 16}$$

We know that $j^2 = -1$ so we may now say…

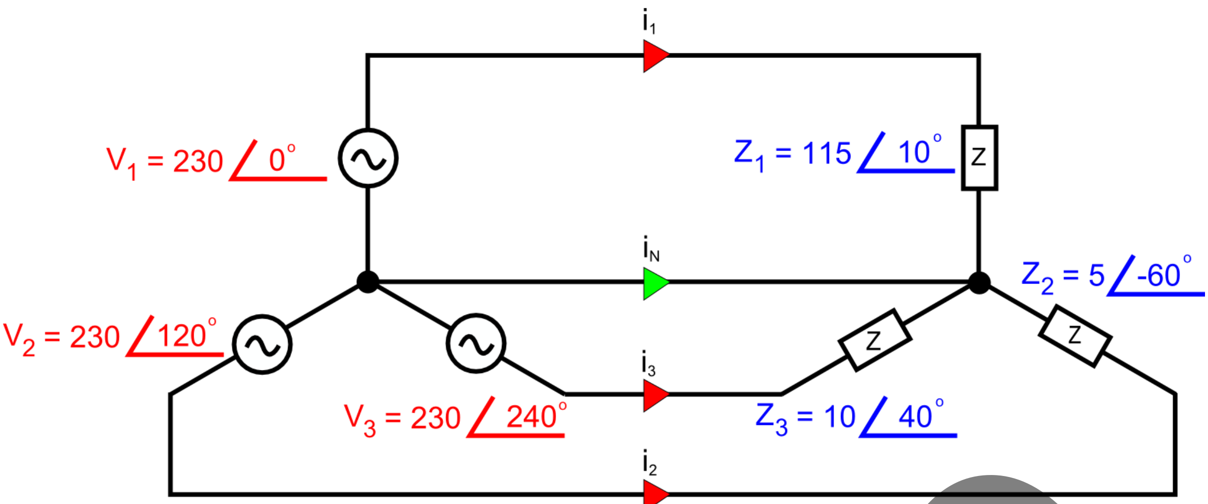
$$= \frac{30 - j40 - j90 - -(-1) \times 120}{9 - j12 + j12 + -(-1) \times 16} = \frac{30 - j40 - j90 - 120}{9 - j12 + j12 + 16} = \frac{-90 - j130}{25}$$

$$= (\mathbf{-3.6 - j5.2}) \ \ [A]$$

Such calculations are rather messy, as you can see. Fortunately, using the Polar Form of complex numbers when performing divisions leads to shorter calculations.

Worked Example 4

A circuit current calculation involves the division of a voltage by an impedance…



We notice that phase voltage 1 is entirely across phase impedance 1. We may therefore say…

$$i_1 = \frac{V_1}{Z_1} = \frac{230\angle 0^o}{115\angle 10^0} = 2\angle{-10^o} \quad [A]$$

The situation is similar for the other two phases…

$$i_2 = \frac{V_2}{Z_2} = \frac{230\angle 120^o}{5\angle{-60^0}} = 46\angle 180^o \quad [A]$$

$$i_3 = \frac{V_3}{Z_3} = \frac{230\angle 240^o}{10\angle 40^0} = 23\angle 200^o \quad [A]$$

The sum of these three phase currents is equal to the neutral current (I_N) in this star configuration. Unfortunately, we cannot add Polar quantities (division and multiplication are ok though, as we've seen) so we need to convert each of them into $a + jb$ form. Some quick conversions on the calculator yield…

$$i_1 = \frac{V_1}{Z_1} = \frac{230\angle 0^o}{115\angle 10^0} = 2\angle{-10^o} \equiv 1.97 - j0.35 \;\; [A]$$

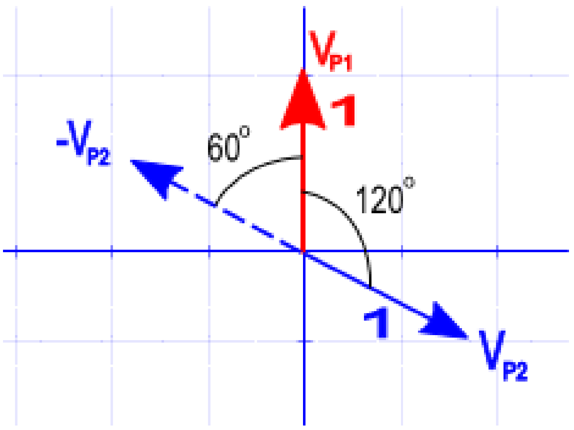
$$i_2 = \frac{V_2}{Z_2} = \frac{230\angle 120^o}{5\angle{-60^0}} = 46\angle 180^o \equiv -46 + j0 \;\; [A]$$

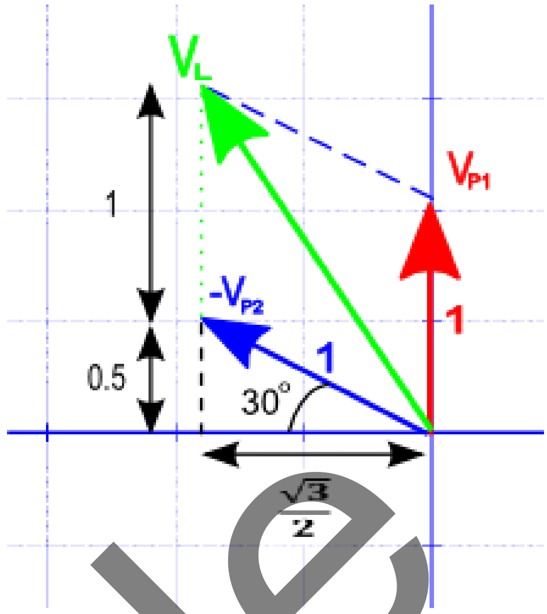
$$i_3 = \frac{V_3}{Z_3} = \frac{230\angle 240^o}{10\angle 40^0} = 23\angle 200^o \equiv -21.61 - j7.87 \;\; [A]$$

Adding the complex numbers gives…

$$\boldsymbol{i_N = (1.97 - 46 - 21.61) + j(-0.35 + 0 - 7.87) = -65.64 - j8.22 \equiv 66.15\angle{-172.86^o} \;\; [A]}$$

Let us now consider the voltage between two separate phases. Look at the phasor diagram below…





The diagram on the left illustrates phase 1 in red (V_{P1}). For the sake of analytical simplicity it is given a magnitude of 1 volt, although this could be scaled up to any voltage you like. The solid blue phasor represents phase 2, which is 120 degrees out of phase with phase 1. To work out the difference in voltage between these two phases we must find invert phase 2, giving $-V_{P2}$, shown dashed in blue. Our task is to find the *resultant* of V_{P1} and $-V_{P2}$. This task is performed in the diagram on the right.

The green phasor represents the resultant line voltage, V_L. This is formed by drawing a parallelogram based upon V_{P1} and $-V_{P2}$. To work out the magnitude of the line voltage in green we apply Pythagoras' theorem...

$$|V_L| = \sqrt{\left[(1+0.5)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]}$$

$$\boldsymbol{|V_L| = \sqrt{\left[\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]} = \sqrt{\frac{9}{4}+\frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3} \;\; volts}$$

This then proves that the line voltage (from phase to phase) is $\sqrt{3}$ times the phase voltage. Therefore, if we have a phase voltage of 230 V then the line voltage will be $\sqrt{3} \times 230 = 398.37\ volts$. This figure tends to be rounded to 400 V in the UK since it is impossible to maintain an exact voltage on the distribution system.

In the UK the consumer supply voltage is 230 V +10%/-6%. This means that the phase voltage can rise to 253V and fall to 216.2 V. If we look at the line voltage then it can be as high as 438.2 volts and as low as 374.5 volts. The figure of 438.2 volts for maximum line voltage tends to be rounded to 440 volts for normal everyday use and signage.

Worked Example 6

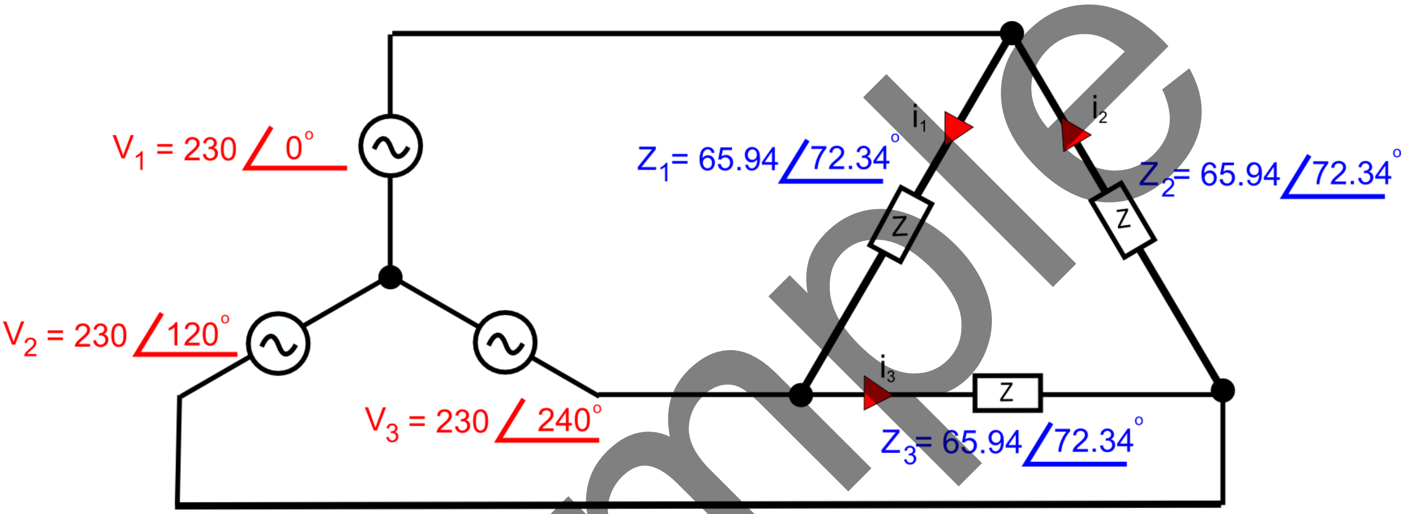
Three identical coils, each of resistance 20Ω and inductance $200mH$, are connected in a Delta configuration to a 230 volt, 50Hz 3-phase supply. Determine the magnitude of each load current.

The first step here is to determine the load impedance on each phase…

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 20\pi = 62.83\Omega$$

$$\therefore \quad Z_L = 20 + j62.83 \quad \equiv 65.94\angle 72.34^o \ [\Omega]$$

The circuit is shown below.



We already know that the line voltage magnitude is $\sqrt{3}$ times the phase voltage…

$$Line\ voltage\ magnitude = \sqrt{3} \times 230 = 398.37\ V$$

We also notice that each load impedance has a line voltage connected. We simply need to divide our magnitudes for voltage and impedance to find the magnitude of each load current…

$$\boldsymbol{Magnitude\ of\ each\ load\ current} = \frac{\mathbf{398.37}}{\mathbf{65.94}} = \mathbf{6.04\ A}$$

There are two common ways to measure the total effective power dissipated in the loads. The two-wattmeter method is employed for *balanced* loads. The three-wattmeter method is employed for balanced or *unbalanced* loads.

Worked Example 7

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Draw a star-star 230V, 50Hz, 3-phase system with BALANCED loads of $20\angle 30^o$ [Ω] on the TINA-TI simulator. Use both the two-wattmeter and three-wattmeter methods to determine the total effective power dissipated in the load system.

Video **Useful starter video on TINA-TI**

The TINA-TI free simulator is available here. Before we draw the two measurement circuits we need to know how to represent a load impedance of $20\angle 30^o$ [Ω] at 50Hz.

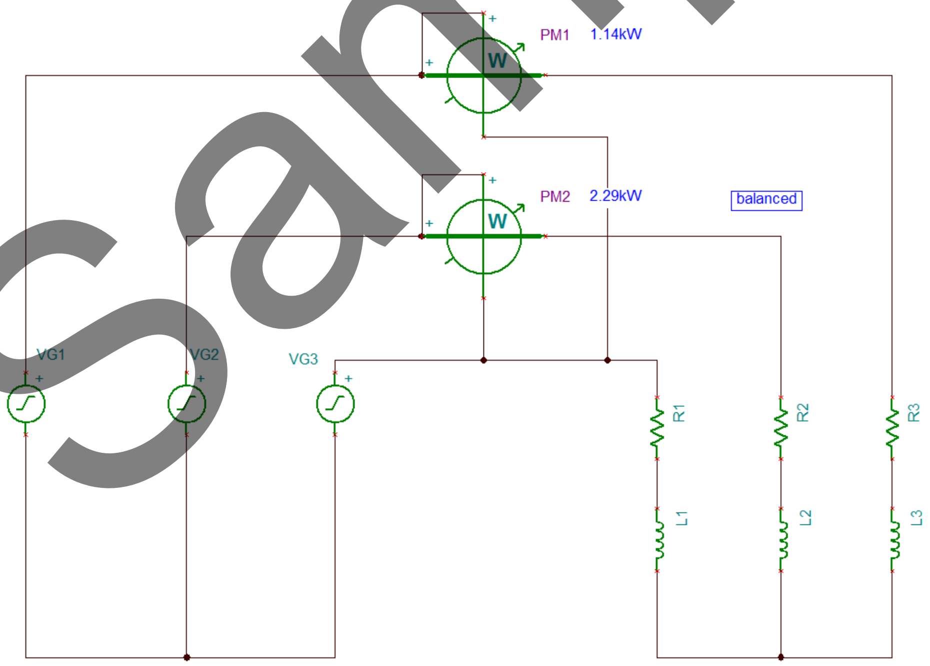
Convert the Polar form of the given impedance into its complex number form using a calculator…

$$20\angle 30^0 \equiv 17.32 + j10$$

We see that the j part is positive so we are looking for an inductor. To determine the value of the inductor we proceed as follows…

$$X_L = 2\pi f L = 10 \quad \therefore \quad L = \frac{X_L}{2\pi f} = \frac{10}{2\pi \times 50} = 0.032\ H$$

So each load consists of a resistor of 17.32Ω and a series inductor of 0.032 H. We then place these into our star load and construct the measurement circuits. The simulation is started by clicking **ANALYSIS->AC ANALYSIS->CALCULATE NODAL VOLTAGES**…



Here we can see that the neutral current will comprise of $i_2 + i_3$. Let's do the calculation…

$$i_N = i_2 + i_3 = \frac{V_2}{Z_2} + \frac{V_3}{Z_3} = \frac{230\angle 120^o}{5\angle -60^o} + \frac{230\angle 240^o}{10\angle 40^0} = 46\angle 180^o + 23\angle 200^o$$

Since we cannot directly add Polar numbers we must convert to complex number form, do the addition, then convert back to Polar form…

$$(-46 + j0) + (-21.61 - j7.87) = -67.61 - j7.87 \equiv 68.1\angle -173.4^0 \quad [A]$$

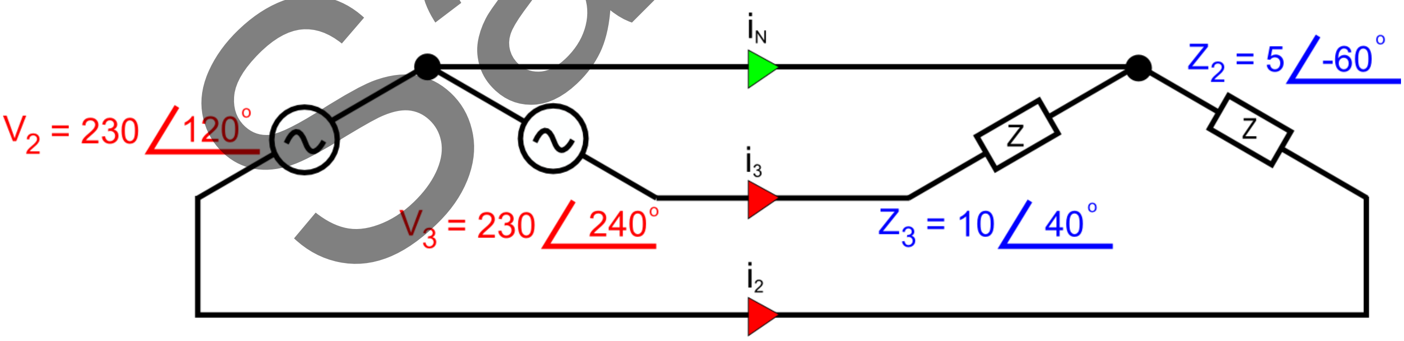
Note: When you see a load impedance with a negative angle then the reactive component involved is a capacitance rather than an inductance. This is handy to know when you are checking your calculations on the TINA-TI simulator.

Challenge

Reproduce the above results on the TINA-TI simulator.

Worked Example 9

What about the situation where we have impedance 1 developing an open circuit? In this case we would like to determine the current flowing in impedance 2. Let's look at the scenario again…



The current flowing in Z_2 would simply be given by V_2/Z_2. We may therefore easily calculate…

$$i_2 = \frac{V_2}{Z_2} = \frac{230\angle 120^o}{5\angle -60^o} = 46\angle 180^o = -46\ A$$

When checking these calculations on the simulator you may get a negative sign for current, rather than an expected positive quantity. It depends which way around you have placed your supply voltage. Try it and see.