Pearson BTEC Level 5 Higher Nationals in Engineering (RQF)

Unit 52: Further Electrical, Electronic and Digital Principles

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Three-phase Theory

Contents

INTRODUCTION

GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

Purpose Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

Theory Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

Example The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself. Many of the examples given resemble assignment questions which will come your way, so follow them through diligently.

Question Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence. As an Online Learner it is important that the answers to questions are immediately available to you. Contact your Unit Tutor if you need help.

Challenge You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

Video Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

2.1 Use of j Notation

When we have circuits containing just resistors then life is so easy in terms of circuit analysis. Most useful circuits also contain capacitors and inductors (usually coils and windings). The introduction of capacitors and inductors into circuits causes 'phase angles' in our calculations. The study of these phase angles is made much easier by introducing complex numbers.

From your level 3 studies you will have come across Inductive Reactance (X_L) and Capacitive Reactance (X_C). These terms are used to quantify the amount of 'opposition' caused by capacitors and inductors to changes in current or voltage. The term 'reactance' is brought about because capacitors cannot be charged or discharged in zero time, and inductors cannot be energised or de-energised in zero time. A good analogy for capacitors is the amount of water in a bathtub. It is impossible to fill a bathtub in zero time, and it's also impossible to empty a bathtub in zero time. The amount of reactance from capacitors and inductors is a function of their manufactured properties and the frequency of operation. Let's review the equations for these reactances…

$$X_L = 2\pi f L \quad [\Omega]$$

$$X_C = \frac{1}{2\pi f C} \quad [\Omega]$$

where;

$X_L = inductive\ reactance\ (measured\ in\ Ohms, \Omega)$

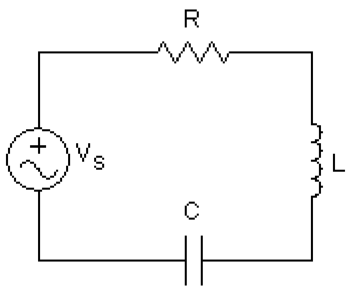
$X_C = capacitive\ reactance\ (measured\ in\ Ohms, \Omega)$

$f = frequency\ (measured\ in\ Hertz, Hz)$

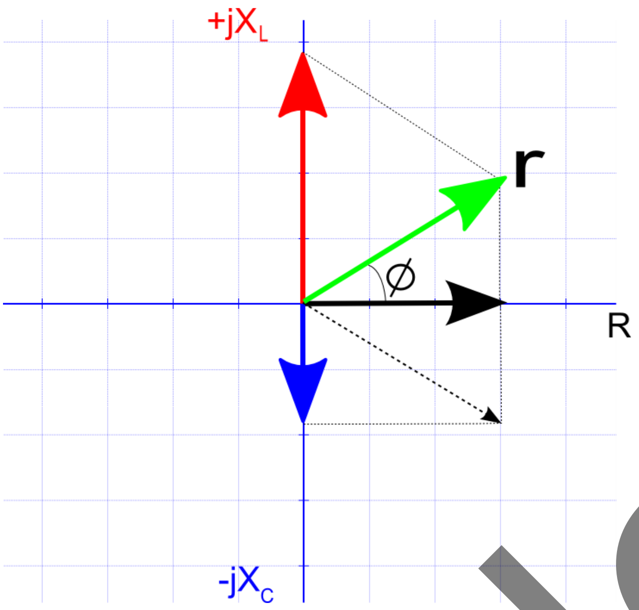
$L = inductance\ (measured\ in\ Henries,$ H)

$C = capacitance\ (measured\ in\ Farads, F)$

Consider the RLC circuit below…



We can draw a phasor diagram for this circuit, as follows…



The black arrow represents resistance. The current through a resistor is always in phase with the voltage across it. We place resistance on the horizontal axis.

The red arrow represents inductive reactance. We see that this *leads* the resistance by 90 degrees ($\pi/2\ rads.$). We name this axis the '+j axis'. Mathematicians tend to designate this the '+i' (imaginary) axis. Engineers do not use i since it clashes with the current symbol, so we use 'j' instead.

The blue arrow represents capacitive reactance. We see that this *lags* the resistance by 90 degrees ($\pi/2\ rads.$). We designate this the '-j' axis.

The dashed lines represent a graphical method of finding the resultant of these phasors, drawn in green. We term this resultant the *impedance* of the circuit and mark it with 'r' for resultant. This resultant impedance makes an angle with the horizontal axis, marked with ϕ.

The resultant impedance is given the symbol Z for calculation purposes. We see that the green resultant has both horizontal and vertical components. The horizontal contribution is known as the *real* component and the vertical contribution is known as the imaginary component.

We may use Pythagoras' theorem to denote impedance as follows…

$$Z^2 = R^2 + X^2$$

$$\therefore \quad Z = \sqrt{R^2 + X^2} \quad [\Omega]$$

In complex number notation we represent Z as…

$$\boldsymbol{Z = R + j(X_L - X_C)} \quad [\boldsymbol{\Omega}]$$

Worked Example 1

The series combination of a 100Ω resistor and a 10mH inductor form an impedance. If the circuit frequency is 10kHz determine the impedance of the circuit in complex number form.

We have the real part already. It is 100Ω (the value of the resistor). We calculate the imaginary part of the circuit as follows…

$$X_L = 2\pi f L = 2\pi \times 10^4 \times 10 \times 10^{-3} = 628.3\Omega$$

We denote a complex number as follows…

$$\boldsymbol{complex\ number = real\ part + j(imaginary\ part)}$$

So we may answer the question by writing…

$$\boldsymbol{Z = (100 + j628.3)\ \ [\Omega]}$$

Worked Example 2

If a complex number is given by $\boldsymbol{Z = (100 + j628.3)\ [\Omega]}$ find its Polar Form.

The Polar Form is given by the length of the resultant impedance (represented by r and the green phasor above) and the associated angle (ϕ) of the resultant impedance, as follows…

$$Z = r\angle\phi \quad [\Omega]$$

Basic trigonometry tells us that…

$$r = \sqrt{R^2 + {X_L}^2} = \sqrt{100^2 + 628.3^2} = 636.2\ \Omega$$

$$\phi = tan^{-1}\left\{\frac{X_L}{R}\right\} = tan^{-1}\left\{\frac{628.3}{100}\right\} = 80.96^o$$

So the Polar Form is given by…

$$\boldsymbol{Z = 636.2\ \angle 80.96^o \quad [\Omega]}$$

Quite often we need to add, subtract, multiply and divide complex numbers. We may also like to multiply and divide Polar numbers. Let's look at the mechanics of these operations.

We define j as the square root of -1…

$$j = \sqrt{-1}$$

$$j + j = 2j$$

$$5j - 3j = 2j$$

$$j \times j = j^2 = \sqrt{-1} \times \sqrt{-1} = -1^{0.5} \times -1^{0.5} = -1^{0.5+0.5} = -1^1 = -1$$

$$\frac{6j}{3j} = 2$$

Worked Example 3

A circuit current calculation involves the division of a voltage by an impedance…

$$\boldsymbol{i} = \frac{\mathbf{10 - j30}}{\mathbf{3 + j4}}$$

Determine the value of the current.

To perform such calculations we need to determine the *complex conjugate* of the denominator and then multiply this complex conjugate by both the numerator and denominator. The process is…

$$\frac{complex\ number\ 1}{complex\ number\ 2} = \frac{complex\ number\ 1}{complex\ number\ 2} \times \frac{complex\ conjugate\ of\ number\ 2}{complex\ conjugate\ of\ number\ 2}$$

The use of the complex conjugate actually simplifies our task because the new denominator becomes a purely real number.

The complex conjugate of a complex number is simply the same complex number with the sign on the j term negated. So, we can write…

$$\boldsymbol{i} = \frac{\mathbf{10 - j30}}{\mathbf{3 + j4}} = \frac{(\mathbf{10 - j30})}{(\mathbf{3 + j4})} \times \frac{(\mathbf{3 - j4})}{(\mathbf{3 - j4})}$$

$$= \frac{30 - j40 - j90 - -j^2 120}{9 - j12 + j12 + -j^2 16}$$

We know that $j^2 = -1$ so we may now say…

$$= \frac{30 - j40 - j90 - -(-1) \times 120}{9 - j12 + j12 + -(-1) \times 16} = \frac{30 - j40 - j90 - 120}{9 - j12 + j12 + 16} = \frac{-90 - j130}{25}$$

$$= (\mathbf{-3.6 - j5.2}) \quad [A]$$

Such calculations are rather messy, as you can see. Fortunately, using the Polar Form of complex numbers when performing divisions leads to shorter calculations.

Worked Example 4

A circuit current calculation involves the division of a voltage by an impedance…