

Pearson BTEC Level 5 Higher Nationals in Engineering (RQF)

**Unit 64: Thermofluids**  
**Unit Workbook 1**

in a series of 4 for this unit

Learning Outcome 1

**Thermodynamic  
Systems**

## 1.1 Fundamental Systems

### 1.1.1 Forms of Energy and Basic Definitions.

Thermodynamics is the study of behaviour and dynamics of energy. Energy comes in a wide variety of forms, for example; if your lightbulb was powered by a gas power station, the energy changes several times before coming out of the bulb as light and heat, shown by Fig.1.1. None of the processes will be 100% efficient, and realistically will lose a lot of temperature as heat.



Fig.1.1: An energy flow chart from gas to a lightbulb.

Fig.1.1 shows several different energies, but energy can be categorised into larger groups:

- Kinetic energy is the energy of a moving object, in Fig.1.1, this would be the generator's shaft spinning to generate electrical energy
- Potential energy is the energy that an object has due to its position relative to others, for example, electrical energy moves from high charge to low charge. Or an object on a table has more potential energy than an object on the floor (the potential energy would be converted to kinetic when it drops).
- Internal energy is the energy that is holding the bonds of the molecules together, such as the chemical energy holding a fuel's molecules together.

### 1.1.2 First Law of Thermodynamics

The first law of thermodynamics is simple: "energy cannot be created or destroyed, it can only be transferred from one form to another". The first law of thermodynamics can be expressed as where  $Q$  is the heat of the system,  $W$  is the work  $\Delta U$  is the internal energy change,  $\Delta KE$  is the kinetic energy change and  $\Delta PE$  is the potential energy change.

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

This means that all thermodynamic equations can be expressed as Eq.1.1, where  $c$  is the velocity of the fluid,  $g$  is the acceleration due to gravity, and  $z$  is the height of the system.

$$Q - W = \left( U_2 + \frac{1}{2} mc_2^2 + mgz_2 \right) - \left( U_1 + \frac{1}{2} mc_1^2 + mgz_1 \right) \quad (\text{Eq.1.1})$$

### 1.3.1 Closed Systems and the Non-Flow Energy Equation

A system is defined as either open or closed, and where the system meets its surroundings is called the boundary. A closed system is one that only has an input or output of energy in some form, shown by Fig.1.2, the boundary encloses the entire system, a closed system is typically used when modelling an engine's cylinder. The equation for closed systems (also known as the non-flow energy equation, is shown as Eq.1.2.

$$Q - W = U_2 - U_1 \quad (\text{Eq.1.2})$$

The assumptions when calculating a closed system are:

- The fluid is compressible
- The system is insulated – meaning that heat is not lost to the environment over time (*heat can be put in, or taken out, but it is not slowly lost over time*)

- $\Delta KE$  and  $\Delta PE$  are negligible (= 0)

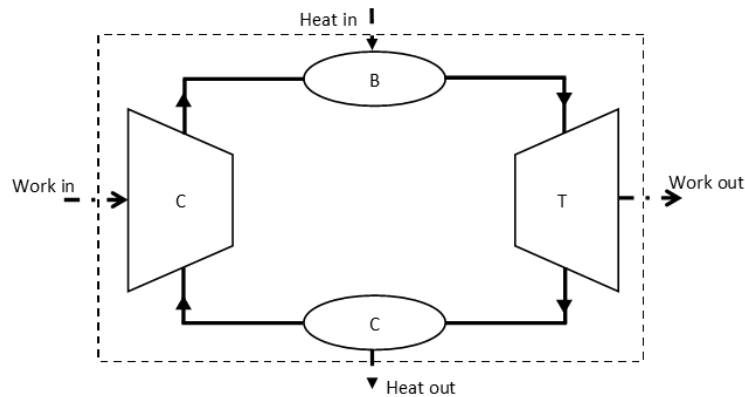


Fig.1.2: A diagram of a closed system

### 1.1.1 Open Systems and the Steady Flow Energy Equation

An open system also has a mass flow, the boundary will surround the equipment, but will also have an intake and exhaust of mass through the boundary. Most thermodynamic systems will use an open system, such as heat pumps and refrigeration cycles. Fig.1.3 shows the boundaries for an open system.

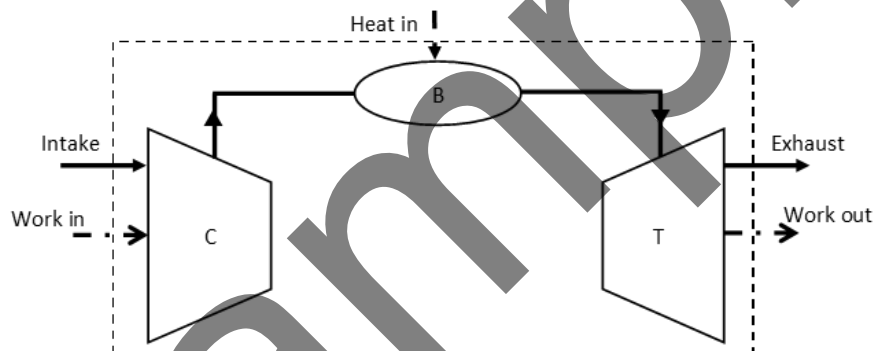


Fig1.3: An open system and its boundaries

Remembering the overall equation of the first law (Eq.1.1), we have:

$$Q - W = \left( U_2 + \frac{1}{2} mc_2^2 + mgz_2 \right) - \left( U_1 + \frac{1}{2} mc_1^2 + mgz_1 \right)$$

Many mechanical engineering devices involve open systems where the flow can be analysed as being in steady state, such as a gas turbine or a refrigeration cycle. For steady flows undergoing changes in volume, the enthalpy, represented by Eq.1.3 is the most convenient variable to represent that static energy of the flow.

$$h = U + PV \quad (\text{Eq.1.3})$$

For these applications the first law of thermodynamics is expressed as Eq.1.4, known as the Steady Flow Energy Equation.

$$\dot{Q} - \dot{W}_x = \dot{m}_2 \left( h_2 + \frac{1}{2} c_2^2 + gz_2 \right) - \dot{m}_1 \left( h_1 + \frac{1}{2} c_1^2 + gz_1 \right) \quad (\text{Eq.1.4})$$

An important aspect is the introduction of mass flow, the system must follow the conservation of mass. If there is a change of area, speed or density then the mass flow will change. Shown by Eq.1.5, where  $\rho$  is the density of the fluid,  $A$  is the cross-sectional area of the pipe, and  $c$  is the velocity of the fluid.

$$\dot{m} = \rho A c \quad (\text{Eq.1.5})$$

Since the flow can change, or flows can be added or diverted from the pipes, we build Eq.1.6 for mass flow continuity.

$$\sum \dot{m}_{IN} = \sum \dot{m}_{OUT} \quad (\text{Eq.1.6})$$

### Example 3

Two water pipes are converging into one large pipe, the first pipe has an area of  $0.01\text{m}^2$  and its water is flowing at  $160\text{ms}^{-1}$ . The second pipe has an area of  $0.03\text{m}^2$  and its water is flowing at  $20\text{ms}^{-1}$ . The larger pipe has an area of  $0.45\text{m}^2$ . Calculate:

- The mass flow rate of the larger pipe.
- The velocity of the larger pipe.

### Answers:

- a) The continuity of mass equation gives:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \rho A_1 c_1 + \rho A_2 c_2 = 1000(0.01 \cdot 160 + 0.03 \cdot 20) = 2200\text{kg} \cdot \text{s}^{-1}$$

- b) Using Eq.2.5, we can find velocity as

$$\dot{m}_3 = \rho A_3 c_3 \therefore c_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{2200}{1000 \cdot 0.45} = 4.89\text{ms}^{-1}$$

### 1.1.3 Heat and Work

When calculating heat and work, it's important to know the convention for calculations. Consider Eq.1.7 below.

$$Q - W = U_2 - U_1 \quad (\text{Eq.1.7})$$

This is the equation used in a closed system, typically the compression or expansion stroke of a piston (where there is negligible change in kinetic or potential energy). The convention for energy transfer is:

- if heat is transferred from the surroundings into the system, then  $Q$  is **positive**,
- if heat is released from the system into the surroundings, then  $Q$  is **negative**,
- if external work is done **on** the fluid or engine, then  $W$  is **negative**,
- if external work is done **by** the fluid or engine, then  $W$  is **positive**.

### Example 1

Calculate the work of an engine for one stroke if the if the heat is transferred out of the system is  $300\text{kJ/kg}$  and the internal energy has decreased by  $900\text{kJ/kg}$ . State whether work is done on or by the fluid.

$$Q - W = U_2 - U_1$$

Heat is transferred out of the system, so  $Q$  is **negative**, and  $U_2 - U_1$  is also **negative** since there is a decrease in internal energy.

$$-300 - W = -900$$

$$-W = -900 + 300$$

$$-W = -600$$

$$W = 600 \text{ kJ/kg}$$

The work is positive; therefore, work is done **by** the fluid.

Sample

### 1.2.5 Pressure-Volume Diagrams

When analysing the thermodynamics of a system, one of the first steps is to develop its pressure-volume (P – V) diagram. With volume on the x-axis and pressure on the y-axis. Fig.1.4 shows the P – V diagram of a standard air heat engine. The area enclosed by the graph is the work done by the system.

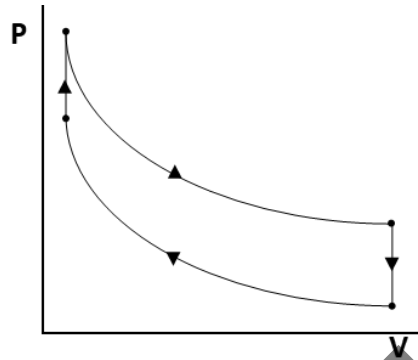


Fig.1.4: P – V diagram of the Otto Cycle heat engine

P – V diagrams can also help show the work output of the system. The work is the useful energy used to create movement, such as a drive shaft of a generator or a car. The area enclosed in the graph is the work done by the system. Work can be calculated using Eq.1.18.

$$W = \int P dV \quad (\text{Eq.1.18})$$

Knowing the overall net work done by the system will give the efficiency of the system in Eq.1.19.

$$\eta = \frac{W_{net}}{Q_{IN}} \quad (\text{Eq.1.19})$$

### 1.2.7 Temperature-Entropy Diagrams

Entropy is classed as the “order” of the molecules in the system, as entropy increases, the molecules become more “disordered”, if there is no entropy change, then the process is considered reversible, meaning that if the same work was put back into the system, it would revert back to its original conditions. A temperature entropy (T – s) diagram is built in a similar way to a P – V diagram, and the area enclosed by a T – s diagram is the heat input to the system.

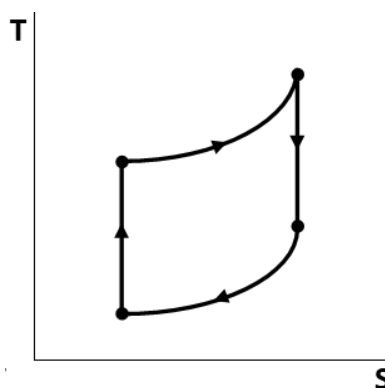


Fig.1.5: A T – s diagram of a heat engine

### 1.2.8 Polytropic Processes

Many systems can be simplified to a polytropic process, in which one term is kept constant, and helps develop a simplified and much easier to calculate system. The polytropic processes are:

- isobaric: constant pressure,
- isochoric: constant volume,
- isothermal: constant temperature,
- adiabatic: no heat transfer,
- isentropic: there is no entropy change and is classed as adiabatic and reversible.

These systems all have a “ratio of compression” noted as  $n$  and it is the power that the volumes are raised to.

**Isobaric:** Charles’ law becomes Eq.1.20. Fig.1.6 shows an isobaric system on a  $P - V$  and  $T - s$  diagram. ( $n = 0$ )

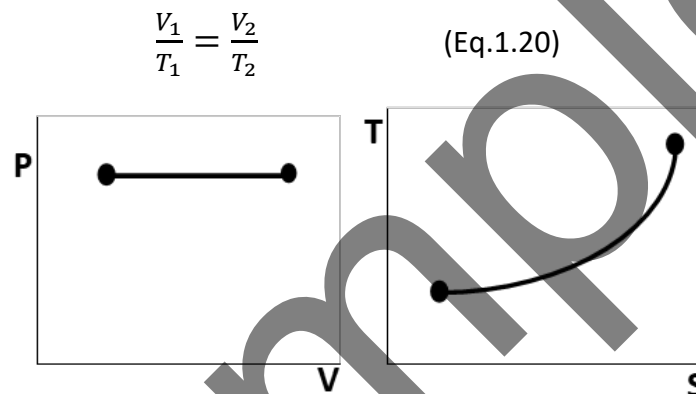


Fig.1.6: Isobaric on a  $P - V$  diagram (left) and  $T - s$  diagram (right).

**Isochoric:** The general gas law simplifies to Eq.1.21. Fig 1.7 shows isochoric actions on both the  $P - V$  and  $T - s$  diagram. ( $n = \infty$ )

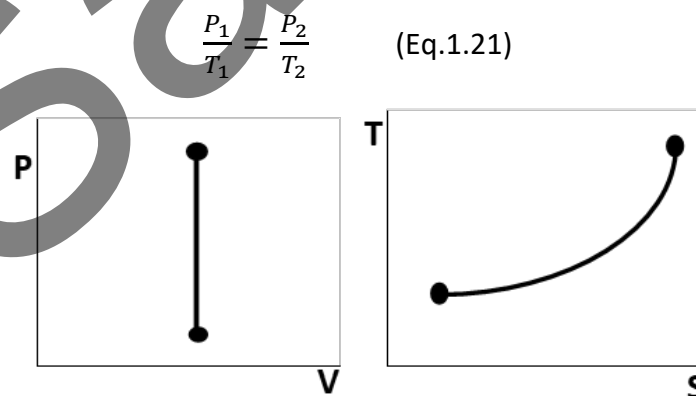


Fig.1.7: Isochoric on a  $P - V$  diagram (left) and  $T - s$  diagram (right).

**Isothermal:** Boyle’s law becomes Eq.1.22. Fig.1.8 shows an isothermal system on a  $P - V$  and on a  $T - s$  diagram. ( $n = 1$ )

$$P_1V_1 = P_2V_2 \quad (\text{Eq.1.22})$$

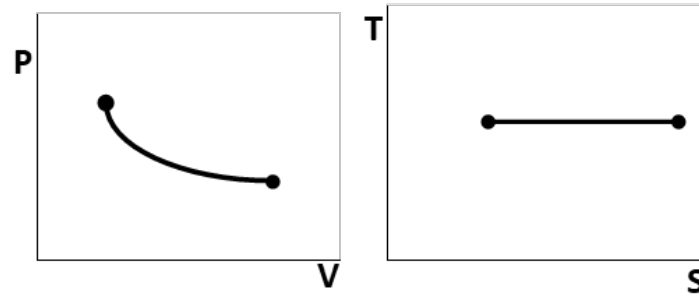


Fig.1.8: Isothermal on a  $P - V$  diagram (left) and  $T - s$  diagram (right).

**Isentropic:** The isentropic relationships are shown by Eq.1.23, Eq.1.24, Eq.1.25, and shown on the  $P - V$  and  $T - s$  diagram by Fig.1.9.

$$\frac{T_2}{T_1} = \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} \quad (\text{Eq.1.23})$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad (\text{Eq.1.24})$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{P_2}{P_1} \quad (\text{Eq.1.25})$$

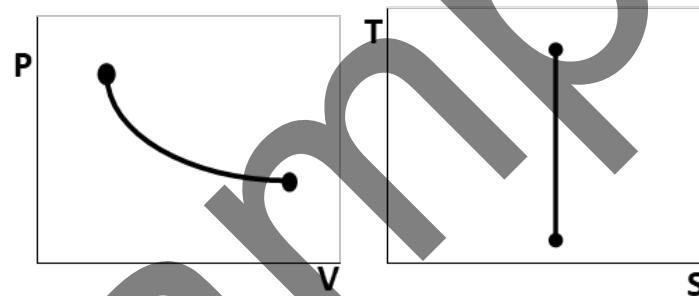


Fig.1.9: Isentropic on a  $P - V$  diagram (left) and  $T - S$  diagram (right).

**Adiabatic:** Without the reversibility of an isentropic process. For adiabatic calculations we need to first find the isentropic temperature change using Eq.1.23, 1.24 or 1.25.

Then using the isentropic efficiency, the real temperature change can be found. The calculation for isentropic efficiency differs for compression,  $\eta_C$ , and expansion,  $\eta_E$ , and are represented by Eq.1.26 and Eq.1.27, respectively.

$$\eta_C = \frac{T_{2S} - T_1}{T_2 - T_1} \quad (\text{Eq.1.26})$$

$$\eta_E = \frac{T_1 - T_2}{T_1 - T_{2S}} \quad (\text{Eq.1.27})$$



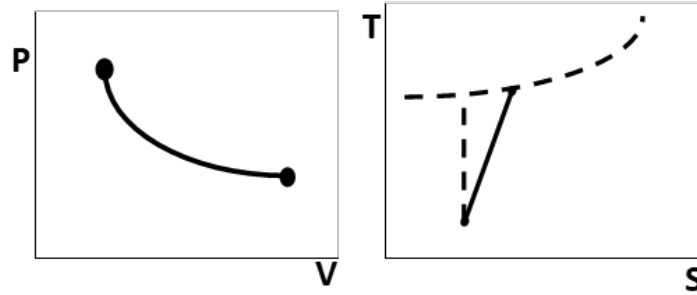


Fig.1.10: Adiabatic on a  $P - V$  diagram (left) and  $T - S$  diagram (right).

### Example 3

A closed system with 20g of gas is heated. The starting volume, pressure and temperature are  $1\text{m}^3$ ,  $0.1\text{MPa}$  and  $293\text{K}$ . Calculate:

- a) Under isochoric heating to  $0.6\text{MPa}$ :
  - a. The temperature of the system
  - b. The index of compression
- b) Under isobaric heating to  $450\text{K}$ :
  - a. The volume of the system
  - b. The index of compression
- c) Under isothermal heating to  $1.1\text{m}^3$ :
  - a. The pressure of the system
  - b. The index of compression
- d) Under adiabatic heating to  $1.1\text{m}^3$  and  $95\text{kPa}$ 
  - a. The index of compression
  - b. The isentropic temperature of the system
  - c. The adiabatic temperature of the system if the isentropic efficiency is 85%.

### Answers:

- 1) Isochoric to  $0.6\text{MPa}$ 
  - a. Temperature of the system

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} \therefore T_2 = \frac{T_1 P_2}{P_1} = \frac{293 \cdot 0.6 \cdot 10^6}{0.1 \cdot 10^6} = 1758\text{K}$$

- b.  $n = \infty$  (Isochoric)

- 2) Isobaric to  $450\text{K}$ 
  - a. Volume of the system

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \therefore V_2 = \frac{V_1 T_2}{T_1} = \frac{1 \cdot 293}{450} = 0.651\text{m}^3$$

- b.  $n = 1$  (Isobaric)

3) Isothermal to  $1.1m^3$

a. Pressure of the system

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \therefore P_2 = \frac{P_1 V_1}{V_2} = \frac{0.1 \cdot 10^6 \cdot 1}{1.1} = 909.1 kPa$$

b.  $n = 1$  (Isothermal)

4) Adiabatic expansion

a. Index of compression

$$P_1 V_1^\gamma = P_2 V_2^\gamma \therefore \left(\frac{V_2}{V_1}\right)^\gamma = \frac{P_1}{P_2}$$

$$\gamma \log \frac{V_2}{V_1} = \log \frac{P_1}{P_2}$$

$$\gamma = \frac{\log(V_2/V_1)}{\log(P_1/P_2)} = \frac{\log(1.1/1)}{\log(0.1 \cdot 10^6 / 95 \cdot 10^3)} = 1.85$$

b. Isentropic temperature

$$\frac{T_{2S}}{T_1} = \frac{P_2^{\frac{\gamma-1}{\gamma}}}{P_1^{\frac{\gamma-1}{\gamma}}} \therefore T_{2S} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 293 \left(\frac{95 \cdot 10^3}{0.1 \cdot 10^6}\right)^{\frac{1.85-1}{1.85}} = 286.2K$$

c. Adiabatic temperature

$$\eta_E = \frac{T_1 - T_2}{T_1 - T_{2S}} \therefore T_2 = T_1 - \eta_E (T_1 - T_{2S}) = 293 - 0.85(293 - 286.2) = 287.2K$$

## 1.3 The Brayton Cycle

The Brayton cycle (*otherwise known as the Joule Cycle*) is a thermodynamic process that is a simplified form of the systems used in electricity generating power plants. The components of the Brayton cycle are shown below in Fig.1.11. The connection between the turbine and the compressor is a shaft, as the gases spin the turbine and generate work, this work will also be used to power the compressor.

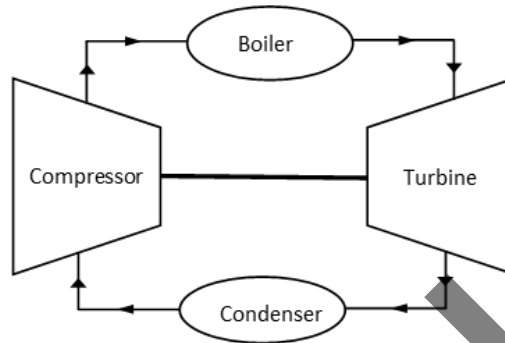


Figure 1.11: The Brayton Cycle

Analysis of the cycle typically starts at the before the gas enters the compressor. Where isentropic compression takes place. The effect a compressor will have on the  $P - V$  diagram and the  $T - s$  diagram is shown in Fig.1.12.

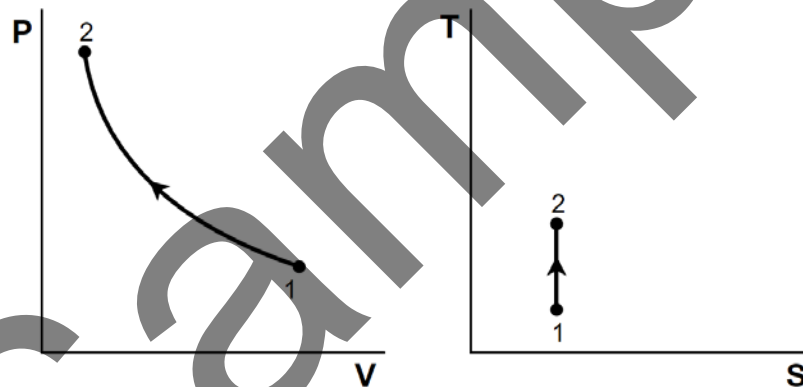


Figure 1.12: Compression stage of the Brayton cycle

Once leaving the compressor the gas moves to the boiler, which is a constant pressure heat addition, represented on  $P - V$  and  $T - s$  diagrams by Fig.1.13.

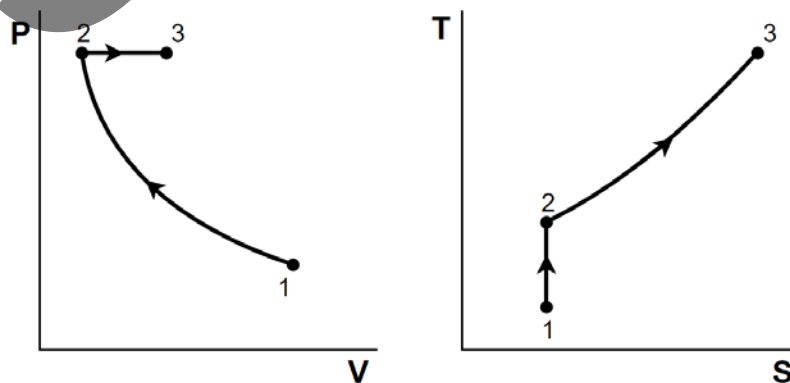


Figure 1.13: The  $P - V$  (left) and  $T - s$  (right) diagrams of a gas moving through a boiler after compression

The gas then moves to the turbine, where the gas undergoes isentropic expansion, and work is done on the turbine shaft. The turbine shaft will likely be connected to the compressor, and in the case of a power plant, a generator. Fig.1.14 demonstrates the turbine's effect on the Brayton cycle.

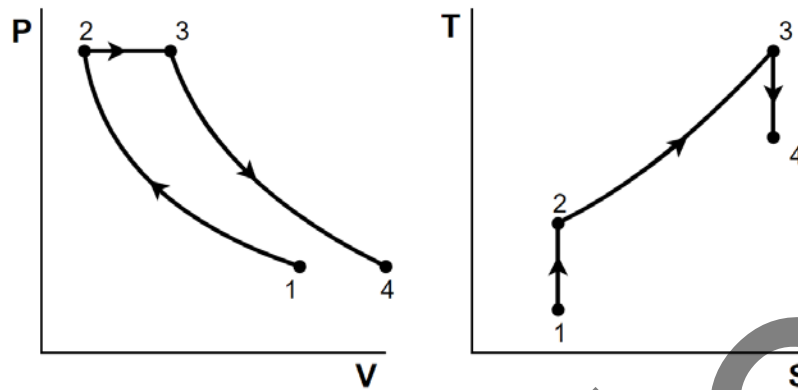


Figure 1.14:  $P - V$  (right) and  $T - s$  (left) of the Brayton cycle after the turbine

After the turbine, the gas can go one of two ways. The system could be considered to be open, in which case the gas could just be expelled out of the system and the resultant cycle will look as Fig.1.14. Or the system could be closed, in which case the fluids will have to pass through a condenser, which will be a constant pressure heat rejection back to the starting point before the compressor (point 1). The impact of a compressor, and the completed closed Brayton cycle is shown in Fig.1.15.

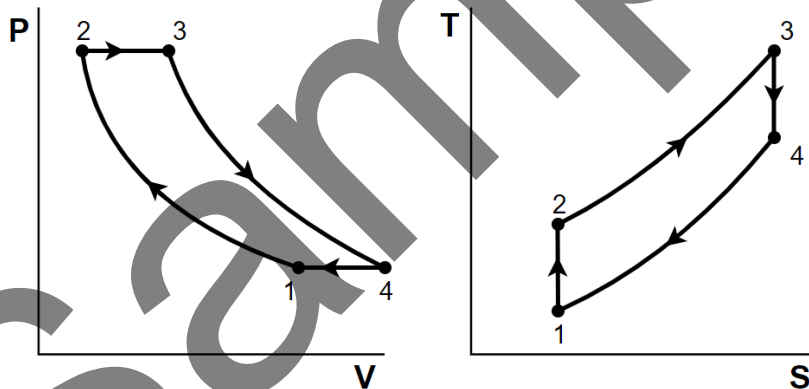


Figure 1.15: The complete Brayton cycle

## 1.4 Two-Phase Fluids

In *Unit 13: Fundamentals of Thermodynamics and Heat Engines* an important assumption at every step was the fluid in the thermodynamic processes was an “ideal gas”. In reality however, the working fluid should be treated as a “pure substance”. This means that the system can (and most probably will) exist in more than one phase (solid/liquid/gas) or even as a mixture of all three.

### 1.4.1 Phase Transitions

Consider heating water and maintaining atmospheric pressure (1 bar) from 30°C. When the fluid reaches boiling point (in this case 100°C) then the system does not immediately switch to completely gas, the two phases can exist in equilibrium with each other.

*It is worth noting that the freezing points and boiling points can change when pressure changes. For example, IUPAC states that while the boiling point for water at sea level is 100°C, the boiling point at the summit of Mt. Everest (8848m above sea level) is 70°C. Hence why it is important to consider constant pressure for this example.*

Fig.1.16 shows the effect of heat addition to water at constant pressure. The phase change (which happens between Stage 2 and Stage 4) is accompanied by a large absorption or release of energy, which can exist in the form of work and/or heat, in this case, work is done on the piston during phase change as the piston rises. This is a result of either the breaking or forming of intermolecular bonds.

When phase changes at constant pressure and temperature occur, the heat transferred per unit mass is known as the latent heat.

- Latent heat of fusion is for melting or freezing
- Latent heat of vaporisation is for boiling or condensing

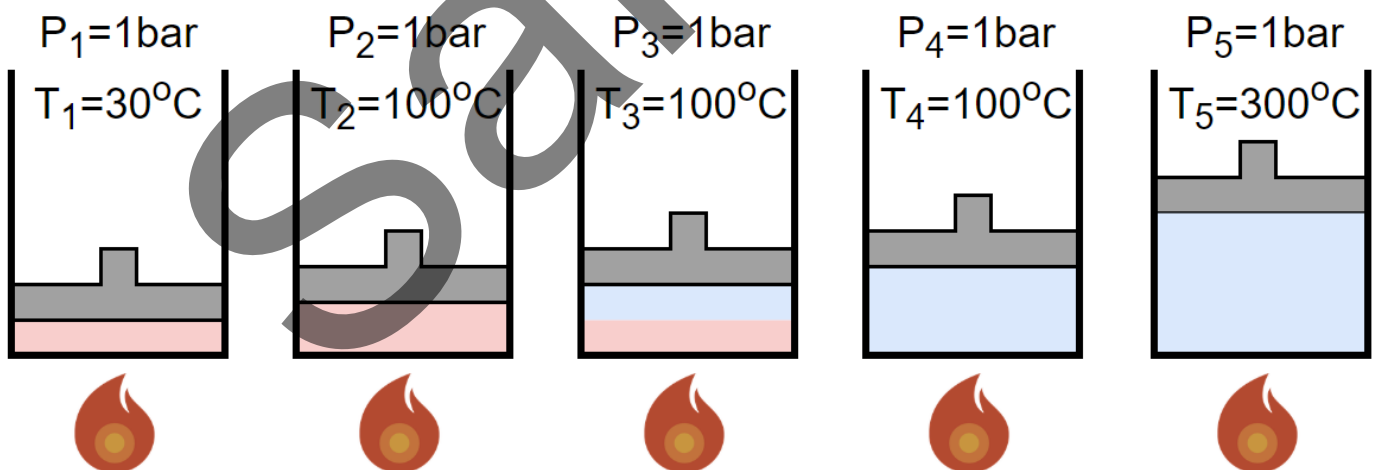


Figure 1.16: Heat addition and phase transition.

Between Stages 1 and 2, no phase change has happened as of yet and the system is completely liquid. Between Stages 2 and 4, the system slowly changes towards vapour, temperature does not rise as the heat energy input into the system will be used to break the intermolecular bonds of the liquid to form gaseous vapour. At Stage 4, the entire system is vapour, and temperature will once again begin to increase, like

Stage 5. The Volume-temperature graph of the system can be seen in Fig.1.17. Stage 2 is the maximum to keep a saturated liquid. At point 4 is the minimum point for saturated vapour, and anything in between is the saturated mixture.

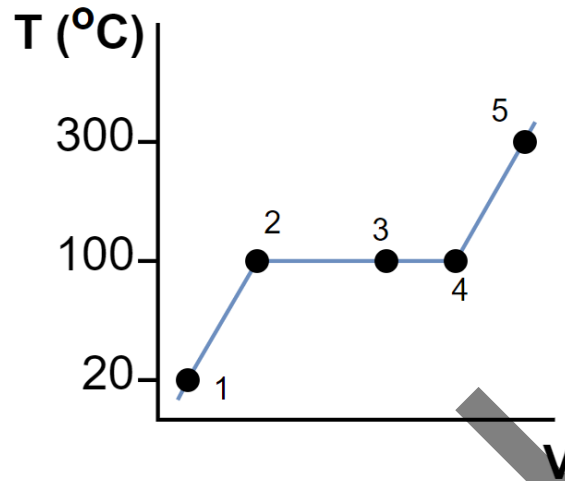


Figure 1.17:  $T - v$  graph of constant pressure heating.

**What if this experiment was conducted across a range of pressures?** It would be possible to connect all the saturated liquid points together, and also connect all the saturated vapour points together, and the result would be Fig.1.18.

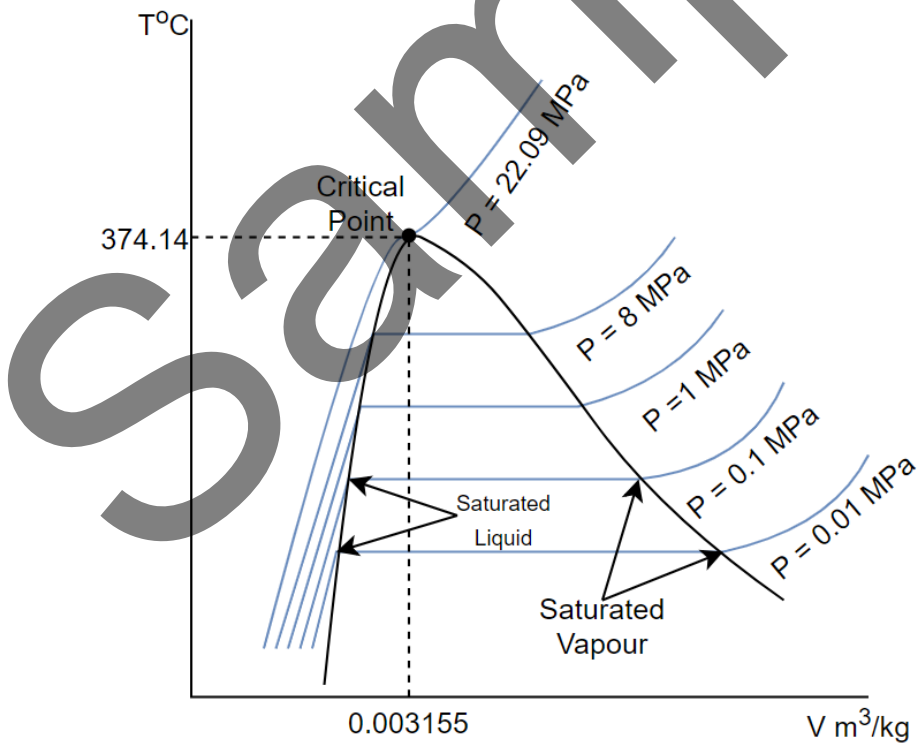


Figure 1.18:  $T - v$  graph of constant pressure heating of water across a range of pressures

The point where the saturated liquid and the saturated vapour lines intersect is known as the *critical point*. The critical point is where there is no phase change, and also no distinction between liquid and vapour. For water, the critical point is:

$$T = 647.3K = 674.14^{\circ}C$$

$$P = 221.2 \text{ bar} = 22.09 \text{ MPa}$$

$$v = 0.00317 \text{ m}^3/\text{kg}$$

*Note: These values will vary between sources*

The pattern shown in Fig.1.18 can also translate to a  $P - V$  diagram, and  $T - S$ , shown in Fig.1.19 and 1.20, respectively. Generally, the region to the left of the saturated liquid line and below the critical point is the “compressed liquid” or “sub-cooled liquid” region. The region to the right of the saturated vapour line and below the critical point is the “superheated vapour region”. Anything above the critical point is considered a gas. The region between the saturated liquid line and saturated vapour line has a number of names: wet region; saturation region; two-phase region; liquid-vapour mixture region.

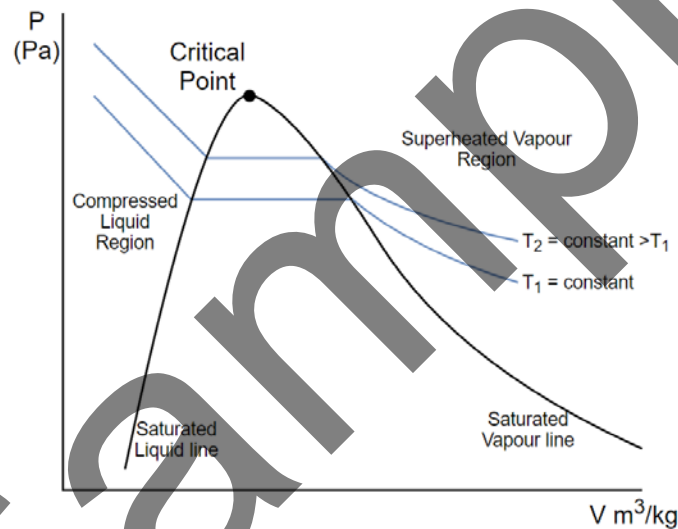


Figure 1.19:  $P - V$  diagram showing the saturation line of water

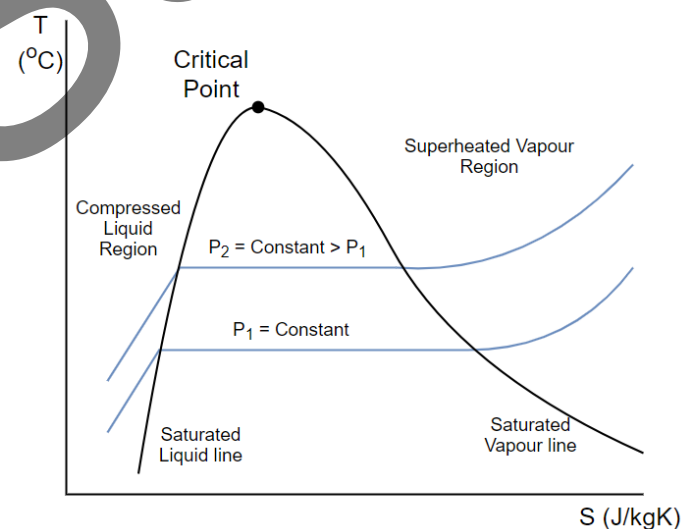


Figure 1.20:  $T - s$  diagram showing the saturation line of water

## 1.4.2 Liquid-Vapour Mixtures

While anywhere on the line, or outside of it are considered to be saturated (either 100% liquid or 100% vapour) Anywhere underneath the saturation line is a mixture of vapour and liquid. The values for a number of properties, specific volume  $v$ , internal energy  $u$ , enthalpy  $h$ , entropy  $s$ , will alter between the liquid phase and the vapour phase of the fluid. The subscript f is used to denote the liquid property of the substance ( $u_f$  for example) and subscript g for the vapour phase ( $u_g$ ).

The difference in the fluid and gas property is given as Eq.1.28 (for specific volume):

$$v_{fg} = v_g - v_f \quad (\text{Eq.1.28})$$

This also applies to the other properties discussed in this section:

$$u_{fg} = u_g - u_f$$

$$h_{fg} = h_g - h_f$$

$$s_{fg} = s_g - s_f$$

## 1.4.3 The Dryness Fraction

It is usually more convenient to know the average property of the mixture, instead of the values for the liquid and vapour independently. To calculate the average value, we need to know the mass fraction of the vapour phase, known as the “dryness fraction”. The dryness fraction ( $x$ ) is calculated using Eq.1.29, where  $m_f$  and  $m_g$  are the mass of the fluid and vapour, respectively.

$$x = \frac{m_g}{m_f + m_g} \quad (\text{Eq.1.29})$$

The average value for a given property, relating to the dryness fraction is given by Eq.1.30 (for specific volume):

$$v_{av} = v_f + xv_{fg} \quad (\text{Eq.1.30})$$

Which can also be used to calculate other average properties:

$$u_{av} = u_f + xu_{fg}$$

$$h_{av} = h_f + xh_{fg}$$

$$s_{av} = s_f + xs_{fg}$$

Rearranging for these equations gives Eq.1.31:

$$x = \frac{v_{av} - v_f}{v_{fg}} = \frac{u_{av} - u_f}{u_{fg}} = \frac{h_{av} - h_f}{h_{fg}} = \frac{s_{av} - s_f}{s_{fg}} \quad (\text{Eq.1.31})$$

## 1.4.4 Property Tables

There are a lot of books available to find the thermodynamic properties of water, refrigerants and cryogenes. These will predominantly cover the superheated and saturation regions, with some books also covering the sub-cooled region. The books will cover the liquid and gas phases, but will typically avoid including solids,