



2.1 Power Cycles

2.1.1 The Brayton Cycle

So far, the only power cycle discussed has been the Brayton cycle, which usually considers that the working fluid moving through the system is a gas, and that it does not go through any phase change. In this workbook, power cycles that use a liquid will be analysed, and because of the high temperatures, these liquids will likely involve a phase change.

2.1.2 Second Law of Thermodynamics with Heat Engines

The second law of thermodynamics is a series of observations that concerns the way things flow as time progresses forward. Typical observations are "water flows from high to low", and "heat flows from hot to cold". In the context of heat engines, however, the second law can be summed up as: "No heat engine can be 100% efficient".

2.1.3 Carnot Cycle

The Carnot cycle is a theoretical heat engine design, that is meant to be the ideal operating system of a heat engine. It consists of four closed processes:

1-2: Fig.4.1 shows the first stage of the Carnot cycle, and its effect on the T - s and P - V diagram. As an isentropic system $\Delta Q = \Delta s = 0$.



2-3: Fig.2.2 represents the second stage, the isothermal process means that there is a heat input, but the process also produces a work output.



Approved Centre

3-4: Fig.2.3 shows the isentropic expansion of the system, as with stage 1-2, $\Delta Q = \Delta s = 0$.



Figure 2.3: Stages 1-2-3-4 of the Carnot cycle

4-1: The final stage, isothermal compression, completes the Carnot cycle, illustrated by Fig.2.4



We know, from the previous learning objectives, that $Q_{net} = W_{net}$, we can calculate the thermal efficiency of the system as Eq.2.1:

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_{cold}\Delta s_{cold}}{T_{hot}\Delta s_{hot}}$$
(Eq.2.1)

Since $\Delta s_{cold} = \Delta s_{hot}$, then thermal efficiency can be reduced to Eq.4.2:

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$$
(Eq.2.2)

This gives the **Carnot efficiency**, the ideal efficiency of an engine that cannot be attained in practical systems.

Example 1

What is the maximum possible efficiency of an engine where $T_{cold} = 50K$ and $T_{hot} = 320K$?

$$\eta_{th} = 1 - \frac{50}{320} = 0.844$$

Example 2

A claim that a new engine has been developed with a thermal efficiency of 75%. It draws in air at $10^{\circ}C$ and its exhaust releases gas at $680^{\circ}C$. Comment on whether a system such as this is possible.

Answer: Remembering to convert the temperatures to K



$$\eta_{\it th} = 1 - \frac{273 + 10}{273 + 680} = 0.703 = 70.3\%$$

The maximum possible efficiency is 70.3%, the claim cannot possibly be true.

2.1.3 Vapour Cycles

The most common fluid used in power cycles is water, due to its abundancy, price and chemical stability, and while open cycle steam engines are outdated and obsolete; closed cycle steam turbines are one of the largest contributions to electricity generation. A steam power generator is shown in Fig.2.5.

Starting at point 1 in Fig.2.5, water enters the pump to be compressed before it enters the boiler, where it is heated to steam (phase change) before moving to the turbine. The steam is then expanded in the turbine, which will output work to turn the shaft (which will be connected to a generator to produce electricity). The steam is then sent to the condenser, where it is turned back into water before moving back to the pump.

The advantages of using water/steam vapour systems compared to that of just gas is the higher energy density of the vapour, it also requires less work to increase the pressure of a liquid than a gas, meaning that less work is required at the pump. Together, amongst other factors, contributes to a higher thermal efficiency, closer to that of the Carnot cycle.

By using the steady flow energy equation (SFEE), and assuming that there is negligible change in Kinetic and Potential energy means that each process can be simplified to Eq.2.3.

$$q + w_x = h_{out} - h_{in} \tag{Eq.2.3}$$

Applying Eq.2.3 to each process in the system, and assuming the turbine and pump are isentropic gives Eq.2.4 to 2.7.

$$q_{in} = h_3 - h_2$$
 (Eq.2.4)
 $w_{out} = h_3 - h_4$ (Eq.2.5)

- $q_{out} = h_4 h_1$ (Eq.2.6)
- $w_{in} = h_2 h_1$ (Eq.2.7)

Using Eq.2.8:

$$Tds = dh - vdP \tag{Eq.2.8}$$

And the assumption that the pump is isentropic, Eq.2.8 simplifies to Eq.2.9.

$$dh = vdP$$
 (Eq.2.9)

Substituting Eq.2.9 into 2.5 gives Eq.2.10:

$$w_{in} = \int_1^2 v dP \qquad (Eq.2.10)$$

In the case of a vapour power cycle, the working fluid will often stay in the liquid phase throughout the compression stage. If this is the case, then the specific volume is small enough to neglect and Eq.2.10 simplifies further to Eq.2.11.

(Eq.2.11)

$$w_{in} = v(P_2 - P_1)$$

The thermal efficiency of the heat engine is given as Eq.2.12:

$$\eta_{th} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{1_{out}}{q_{in}}$$
(Eq.2.1)

2.1.4 Whole Plant Efficiency

While the thermal efficiency of the heat engine has been calculated, this is not the overall efficiency of the plant. Taking a closer look into the boiler in Fig.2.6, q_{in} is dependent on the heat transfer between the combustion products and the working fluid. So, the efficiency of the boiler also needs to be calculated.

Figure 2.5: An expanded view of a power plant boiler

Calculating the efficiency of the boiler requires analysing the *ideal* boiler and the *real* boiler.

2.1.4.1 Ideal Boiler

The ideal boiler assumption is that the fuel and air enter the boiler at the standard state temperature $T_0 = 25^{\circ}$ C, and once complete combustion occurs the products leave the boiler at 25°C. From the SFEE, the heat into the cycle for an ideal scenario is given as Eq.2.13, where $h_{a|0}$, $h_{f|0}$ and $h_{p|0}$ are the enthalpies at 25°C.

$$\dot{Q}_{in|ideal} = \dot{m}_f h_{f|0} + \dot{m}_a h_{a|0} - \left(\dot{m}_a + \dot{m}_f\right) h_{p|0} = \dot{m}_f [-\Delta H_0]$$
(Eq.2.13)

Under these conditions, the heat in per unit mass is simplified to the Lower Calorific Value (LCV = $-\Delta H_0$) of the fuel.

2.1.4.2 Real Boiler

The real boiler does not assume that the products of combustion leave the boiler at 25° C and instead leave at a higher temperature $T_{exhaust}$, the heat into the system is calculated as Eq.2.14:

$$\dot{Q}_{in|real} = \dot{m}_f h_{f|0} + \dot{m}_a h_{a|0} - (\dot{m}_a + \dot{m}_f) h_{p|exhaust}$$
 (Eq.2.14)

The equation no longer simplifies to the LCV, this is because some of the heat produced is used to warm the unburnt air in the boiler to $T_{exhaust}$, instead of all the heat moving into the working fluid of the heat engine. The efficiency of the boiler is given as a ratio of heat in under the real condition to the heat in under the ideal condition. The boiler efficiency is given as Eq.2.16, where AFR is the air-fuel ratio that is put into the boiler and $c_{p|p}$ is the specific heat for constant pressure of the products.

$$\eta_{\rm B} = \frac{\dot{Q}_{in|real}}{\dot{Q}_{in|ideal}} = \frac{\dot{Q}_{in|real}}{\dot{m}_f [-\Delta H_0]} = 1 - \frac{AFR + 1[h_{p|exhaust} - h_{p|0}]}{[-\Delta H_0]}$$
$$\cong 1 - \frac{c_{p|p}(AFR+1)(T_{exhaust} - T_0)}{[-\Delta H_0]}$$
(Eq.2.15)

The overall efficiency of the plant can be calculated as Eq.2.16:

$$\eta_{\text{plant}} = \frac{\dot{w}_{\text{out}} - \dot{w}_{\text{in}}}{\dot{m}_{\text{f}} [-\Delta H_0]} = \eta_{\text{th}} \eta_{\text{B}}$$
(Eq.2.16)

2.2 Two-Phase Fluids Through the Carnot Cycle

Fig.2.6 shows the *ideal* Carnot cycle for a heat engine on a T - s diagram, notice that it only occurs under the saturation curve.

Figure 2.6: The ideal Carnot cycle for a heat engine under the saturation curve

When a fluid changes from liquid to vapour (stage 2-3) or back again (stage 4-1), this will occur at constant temperature as latent heat will be absorbed or released. Also considering that pumps and turbines can operate at almost isentropic conditions; then it is theoretically possible to match the Carnot cycle.

2.2.1 Carnot Cycle Issues

There are issues that are not considered in the "ideal" cycle, such as:

- Isentropic compression (stage 1-2) takes place with a saturated liquid-vapour mixture, as discussed in Section 2.1.3, a lot more work is required to compress a gas than it is a liquid. Difficulty in designing compressors to handle two phase mixtures means that it is much more preferable to condense the fluid completely before compression starts.
- Isentropic expansion (stage 3-4) reduces the dryness fraction of the fluid when under the saturation curve. Water droplets would have a huge contribution to the erosion of the turbine blades and so a high dryness fraction needs to be considered (x > 0.9).
- Hydrocarbon combustion in boilers will produce a much higher temperature than is required for the working fluid to remain under the saturation curve.

With these considerations, the most effective power cycle is the Rankine cycle.

2.3 Rankine Cycle

2.3.1 The Ideal Rankine Cycle

The Rankine cycle is the closest practical cycle to the Carnot efficiency, when comparing to other cycles such as the Otto, Diesel or Brayton. Figure 2.7 shows the ideal Rankine cycle, with respect to the saturation curve, on a T - s diagram.

The four processes of the Rankine cycle are:

- 1-2: Isentropic compression in the liquid phase
- 2-3: Constant pressure heat addition to produce a superheated vapour
- **3-4:** Isentropic expansion in a turbine
- 4-1: Constant pressure heat rejection to fully condense the fluid

In comparison to the Carnot cycle, the Rankine cycle reduces the efficiency of the system as more heat must be rejected to produce, and the starting temperature is much lower than the Carnot cycle. However, while there is a small reduction in efficiency, there is a significant reduction in work input as it is a liquid that is compressed in the pump.

Another difference is the introduction of superheated vapour in the process, the result of this is that when expansion occurs in the turbine, there will still be a high dryness fraction and the damage to the turbine is minimised.

2.3.2 Real Rankine Cycles

In practice, however the system will look slightly different, the ideal cycle assumes that the compression and expansion processes are isentropic, but there will inevitably be some irreversibility in the systems, and will be modelled as adiabatic. There is also the consideration of the boilers and the condensers, which will also

experience small pressure drops during heating and cooling. A comparison of the ideal and real Rankine cycles can be seen in Fig.2.8.

Example 1

Consider a power plant operating on a simple ideal Rankine cycle. Steam enters the turbine at 3MPa and $350^{\circ}C$, it is then condensed at a pressure of 10kPa. Determine the thermal efficiency of the cycle.

Answer:

At state 1 we know that the system is using saturated water and $P_1 = 10$ kPa. Using the tables:

$$h_1 = h_5 = 192 kJ/kg$$

 $h_1 = v_f = 0.00101 m^3/kg$

At state 2 we know that $P_2 = P_3 = 3MPa$ and also $s_2 = s_1$

$$w_{in} = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{kJ/kg}$$

$$h_2 = h_1 + w_{in} = 192 + 3.02 = 195.02 kJ/kg$$

At state 3 we know that $P_3 = 3MPa$, $T_3 = 350^{\circ}C = 623K$, which means that the water is a superheated vapour. From the tables:

$$h_3 = 3117 kJ/kg$$

 $s_3 = 6.744 kJ/kgK$

At state 4, we know that $P_4 = 10$ kPa, $s_4 = s_3$, using the appropriate tables we know that:

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.744 - 0.649}{7.5} = 0.812$$

2.4 Improving the Rankine Cycle

While the Rankine system is close to the Carnot efficiency, the scale that steam generators operate on, with some of the largest power plants in the UK rated between 1000 - 2000MW, means that any option to improve efficiency is desired. Fig.2.9 shows the Digest of UK's Energy Statistics (DUKES) breakdown of electricity generation. As you can see, fossil fuels accounted for over 50% of electricity generation in 2016, improving the efficiency will save a substantial amount of money in fuel, and also reduce CO_2 emissions.

Figure 2.9: DUKES breakdown of electricity generation in 2015 and 2016

2.4.1 Lowering Condenser Pressure

By lowering the condenser pressure, the saturation temperature, and hence the temperature that heat is rejected in the cycle. Consider Fig.2.10, which shows the effect of the Rankine cycle with a lower condenser pressure on the T - s diagram; the area bound by the cycle signifies an increase in heat, but remembering the first law means that this can also translate to an increase in w_{net} . The reduction in condenser pressure will require a small increase in work and heat input into the system, but these can be considered to be quite small compared to the increase in work output.

Figure 2.10: Comparing Rankine cycles using a decreased condenser pressure

The limit to the condenser pressure is dependent on the medium used to cool the condenser (this may be ambient air or water). A requirement for any heat exchanger is that there must be a minimum difference in temperature between the cooling medium and the cycle's working fluid.

For an example, if the cooling medium is 15° C and the working fluid is going to be cooled to 30° C. Then the condenser pressure will need to be 4.25kNm⁻² (0.0425 bar). Since this is significantly lower than atmospheric pressure, the condenser needs to be designed against air-leaks and the high volumes of the volumes in the plant.

Another consideration is the entropy of the vapour when it enters the turbine. If the working fluid leaves the turbine at such a low pressure, it is possible that the dryness fraction is reduced too far, and the turbine will be damaged from water droplets.

2.4.2 Increasing Boiler Pressure

This is more or less the reverse of lowering the condenser pressure. By increasing the boiler pressure, the required temperature will require less heat input, saving on fuel for the system. There is also the compromise for the change in net work to consider, while reducing the overall entropy change for the system means that there is a loss of w_{net} between Stages 3-4, the increase in boiler pressure will compensate accordingly. Fig.2.10 shows the effects of increasing the boiler pressure.

Similar to one of the constraining problems with the condenser, the change in entropy before entering the turbine will provide a serious problem as temperature drops, as it will not be long before the vapour-liquid mixture is too wet, and the turbine blades are damaged.

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = \frac{2180}{3488} = 0.372$$

From this, we can see that efficiency has improved and there is a higher dryness fraction (turbines will suffer less wear)

b) In this case, only state 1 is the same. Looking at State 2 (15MPa and $s_2 = s_1$):

$$h_2 = 207 kJ/kg$$

State 3 (15MPa and 600°C):

$$h_3 = 3583 kJ/kg$$

State 4 (10kPa and $s_4 = s_3$):

$$x_4 = 0.804$$

 $h_4 = 2115 kJ/kg$

Using these values, we can deduce:

$$q_{in} = h_3 - h_2 = 3583 - 207 = 3376$$
kJ/kg
 $q_{out} = h_4 - h_1 = 2115 - 192 = 1923$ kJ/kg

The thermal efficiency is therefore:

$$\eta_{th} = 1 - \frac{q_{out}}{q_{ih}} = 0.43$$

Again, the efficiency of the system is improved, but the dryness fraction has been reduced

3.4.4 Reheating

Reheating is a more complex solution improving the efficiency of the system. The system has two turbines, the first will cause a small drop in temperature, before it sent back to the boiler for reheating, before heading back into a second turbine. Fig.2.13 shows a Rankine cycle with a re-heating stage.

Figure 2.13: Re-heating Rankine cycle

Answer:

a) Looking at stage 6 of the cycle ($P_6 = 10kPa$):

We also know that the dryness fraction will be 1 - 0.104 (1 - moisture content):

 $x_6 = 0.896$

With this information:

$$s_6 = s_f + x_6 s_{fg} = 0.6942 + 0.896(7.4996) = 7.3688 kJ/kgK$$

 $h_6 = h_f + x_6 h_{fg} = 191.81 + 0.896(2392.1) = 2335.1 kJ/kg$

We are also assuming that the system is isentropic $s_5 = s_6 = 7.3688$ kJ/kgK and that $T_5 = 600$ °C, using the steam tables, we can find that $P_5 = 4$ MPa

b) State 1 (10kPa and saturated liquid):

$$h_1 = h_5 = 192 \text{kJ/kg}$$

 $v_1 = v_f = 0.00101 \text{m}^3/\text{kg}$

State 2 (15 MPa, $s_2 = s_1$):

$$w_{in} = v_1(P_2 - P_1) = 0.00101(15000 - 10) = 15.14 \text{kJ/kg}$$

State 3 (15MPa, 600°C):

$$h_3 = 3583 \text{kJ/kg}$$
$$s_3 = 6.67 \text{kJ/kgK}$$

State 4 (4MPa, $s_4 = s_3$):

$$m_4 = 3155$$
kJ/kg $T_4 = 375$ °C

With all this, we can show that:

$$q_{in} = (h_3 - h_2) + (h_5 - h_4) = 3896 \text{kJ/kg}$$
$$q_{out} = h_6 - h_1 = 2143 \text{kJ/kg}$$
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 0.45$$

3.4.5 Feed-Water Regeneration

In steam power plants the temperature of the compressed liquids (commonly known as the feed-water) during the compression stage can also be used to raise the temperature of the boiler, and therefore increase the efficiency of the system. Fig.2.15 shows the regeneration schematic of the system, looking at the exit of the turbine, it can be seen that the steam takes two paths.

Path 7 - 1 - 2 is the standard cycle, the fluid leaves the turbine and heads to the condenser before it reaches Pump 1. Path 6 however, is a short bypass of the condenser and Pump 1, this is fluid that has not been completely expanded by the turbine. They are broken off as fractions, noted as y and 1 - y.

In order for both paths to mix together again. They must be the same pressure (hence why Pump 1 is required). The mixed feed-water is then pumped into a second compression stage.

The T - s diagram of the system can be seen in Fig.2.16. The process reduces the heat input required by the system, and while there is a reduction in work output from the turbine (since full expansion of all fluid has not been achieved) the overall efficiency of the system increases.

Figure 2.16: T - s diagram of a Rankine cycle with feed-water regeneration

The fraction y can be defined using Eq.2.19, where \dot{m}_5 and \dot{m}_6 are the respective mass flow rates at stages 5 and 6.

$$y = \frac{\dot{m}_5}{\dot{m}_6}$$
 (Eq.2.19)

Using this, the following equations can be derived:

$$q_{in} = h_5 - h_4$$
 (Eq.2.20)

$$q_{out} = (1 - y)(h_7 - h_1)$$
 (Eq.2.21)

$$w_{out} = (h_5 - h_6) + (1 - y)(h_6 - h_7)$$
 (Eq.2.22)

$$w_{pump1} = v_1(P_2 - P_1)$$
 (Eq.2.23)

$$w_{pump2} = v_3(P_4 - P_3)$$
 (Eq.2.24)

 $w_{in} = (1 - y)w_{pump1} + w_{pump2}$ (Eq.2.25)

It's also possible to calculate the fraction y using the enthalpies at the feed-water heater, shown by Eq.2.26.

$$\sum_{in} \dot{m}h = \sum_{out} \dot{m}h \tag{Eq.2.26}$$

(Eq.2.27

(Eq.2.28)

Which is expanded out as Eq.2.27:

$$yh_6 + (1 - y)h_2 = h_3$$

Rearranging gives Eq.2.28:

$$y = \frac{h_3 - h_2}{h_6 - h_2}$$

As more stages of feed-water regeneration are added, the efficiency of the system increase. Large power plants will use as many as eight feed-water heaters.

Example 4

Consider a steam plant operating on an ideal Rankine cycle that is used on feed-water heater. Steam enters the turbine at 15MPa and 600° C and is condensed at a pressure of 10kPa. Some steam, however, leaves the turbine at a pressure of 1.2MPa and enters the feed-water heater. Determine:

- a) The fraction of steam extracted from the turbine
- b) The thermal efficiency of the system

Answer:

a) State 1 (10kPa, saturated liquid)

$$h_1 = h_f = 192 \text{kJ/kg}$$

 $v_1 = v_f = 0.00101 \text{m}^3/\text{kg}$

State 2 (1.2MPa, $s_2 = s_1$):

$$w_{in}^1 = v_1(P_2 - P_1) = 0.00101(1200 - 10) = 1.2kJ/kg$$

 $h_2 = h_1 + w_{in}^1 = 192 + 1.2 = 193.2kJ/kg$

State 3 (1.2MPa, saturated liquid):

$$\label{eq:h3} \begin{split} h_3 &= h_f = 798 \text{kJ/kg} \\ v_3 &= v_f = 0.00113 \text{m}^3/\text{kg} \end{split}$$

State 4 (15MPa, $s_4 = s_3$):

$$\begin{split} w_{in}^2 &= v_3(P_4 - P_3) = 0.00113(15000 - 1200)15.7 \text{kJ/kg} \\ h_4 &= h_3 + w_{in}^2 = 798 + 15.7 = 813.7 \text{kJ/kg} \end{split}$$

State 5 (15MPa, 600°):

$$h_{5} = 3583 kJ/kg$$

$$s_{5} = 6.67 kJ/kgK$$
State 6 (1.2MPa, $s_{6} = s_{5}$):

$$h_{6} = 2860 kJ/kg$$

$$T_{6} = 218^{\circ}C$$
State 7 (10kPa, $s_{7} = s_{5}$):

$$x_{7} = \frac{s_{7} - s_{f}}{s_{fg}} = \frac{6.67 - 0.649}{7.5} = 0.802$$

$$h_{7} = h_{f} (+x_{7} + h_{fg} = 2115 kJ/kg)$$
Using this information:

$$y = \frac{h_{3} - h_{2}}{h_{6} - h_{2}} = 0.227$$
b)

$$q_{inr} = h_{5} - h_{4} = 2769 kJ/kg$$

$$q_{out} = (1 - y)(h_{7} - h_{1}) = 1487 kJ/kg$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{out}} = 0.46$$

$$\eta_{\rm th} = 1 - \frac{q_{\rm out}}{q_{\rm in}} = 0.$$

3.5 Combined Cycle Gas Turbines

This unit has discussed using both the Rankine and Brayton cycle as methods to generate electricity. However, it is also worth considering the difference in temperatures, the Brayton cycle (which operates with gas) can exit the turbine at $\sim 600^{\circ}$ C, while the maximum temperature of most steam cycles will also reach $500 - 600^{\circ}$ C. So, by this logic, is it not possible to use the remaining heat from the gas to heat the steam?

The Combined Cycle Gas Turbines (CCGT) system does just that. Fig.2.17 shows the schematic of a CCGT, which are becoming more common for their high thermal efficiencies of over 50%.

From Fig.2.17, the overall heat input is at the combustion chamber (stage 6 - 7), the heat input into the system, once the gas has been expanded, it moves into the heat recovery steam generator (HRSG), which will be the heat input for the steam cycle. The steam is expanded before it is finally condensed, which is the overall Q_{out} for the CCGT system. The T - s diagram of the CCGT cycle is shown in Fig.2.18, it is worth noting that the entropies will not typically line up in this fashion, it is to show that heat is transferred between systems.

