Pearson BTEC Level 5 Higher Nationals in Engineering (RQF)

Unit 64: Thermofluids

Unit Workbook 3

in a series of 4 for this unit

Learning Outcome 3

Viscosity

3.1 Viscosity

Viscosity is a fluid's resistance to deformation under shear stresses.

Viscosity is an important property of any fluid, as it also helps determine their behaviour and motion against solid boundaries (such as pipes, gears, sliding contacts etc.). The viscosity is determined by the inter-molecular friction that is seen when one layer slides over the other. Or to put it simply, ***viscosity is how runny the fluid is***. The higher the viscosity, the thicker the fluid is.

It is very important to note that viscosity is temperature dependent, when considering a shortlist of fluids to a given application, it is vital that the temperature of the system is also considered.

3.1.1 Dynamic Viscosity

The dynamic viscosity the fluid's resistance to flow when an external force is applied. Dynamic viscosity can be though of as the tangential force per unit area required to move one plane of fluid with respect to another. The velocity between layers of a laminar fluid moving in straight parallel lines for a Newtonian fluid can be seen in Fig.3.1.

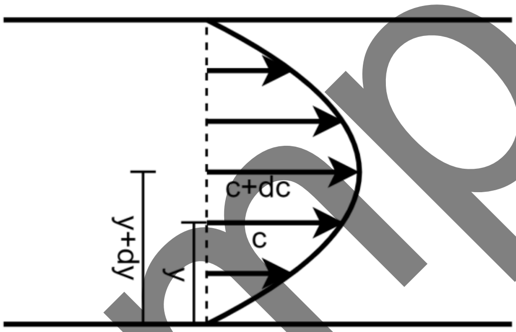

Figure 3.1: Velocity between layers of a laminar fluid

The shear stress τ can be defined by Eq.3.1, where μ is the dynamic viscosity, c is the velocity of the fluid, y is the height from the surface. dc/dy is also known as the "shear rate".

$$\tau = \mu \frac{dc}{dy} \qquad \text{(Eq.3.1)}$$

The SI units for dynamic viscosity is $\text{Pa} \cdot \text{s}$, the values used are typically very low (e.g., the dynamic viscosity of water at $20°\text{C}$ is $0.0010005\ \text{Pa} \cdot \text{s}$. More commonly the units that are used are the Poise, or centipoise, where $10\text{P} = 1\text{Pa} \cdot \text{s}$, therefore the dynamic viscosity of water at $20°\text{C}$ is 0.010005P or 1.0005cP.

3.1.2 Kinematic Viscosity

Kinematic viscosity is the fluid's resistive flow under its own weight (no external forces are applied, just gravity). The substance with the highest kinematic viscosity is tar pitch which, despite appearing to be a solid and even shatters when it is hit with a hammer, is actually an incredibly viscous liquid, and will drip roughly once every ten years. The experiment widely recognised as the longest running in the University of Queensland in Australia is analysing the drip of tar pitch which began in 1927. Since the drip occurs around once every ten years, it has never actually been seen; the last time it did drip, the webcam failed and missed it.

Kinematic viscosity v can be calculated using Eq.3.2, where ρ is the density of the fluid

$$v = \frac{\mu}{\rho} \qquad \text{(Eq.3.2)}$$

It is not just the University of Queensland conducting this experiment, Trinity College in Dublin also have their own experiment, which has been running since 1944. In July 2013 Trinity College managed to record the drop on video. The URL below shows the only drop that has been recorded.

https://www.youtube.com/watch?v=k7jXjn7mIao

The SI units for kinematic viscosity are given as $\mathrm{m^2/s}$; however, due to the low numbers that are generally used (e.g., the kinematic viscosity of water at $20°C$ is $0.0000010023\mathrm{m^2/s}$), more commonly the units are Stokes or centistokes, where $1\mathrm{cSt} = 1 \cdot 10^{-6}\mathrm{m^2/s}$. Therefore, the kinematic viscosity of water at $20°\mathrm{C}$ is $1.0023\mathrm{cSt}$.

3.1.3 The Importance of Viscosity

3.1.3.1 Lubrication

The application of viscosity is most commonly seen in lubrication. It has recently been discovered that lubrication dates back to ancient Egypt. Fig.3.2 shows a wall painting from the tomb of Djehutihotep, a Nomarch (official) of the twelfth dynasty of Egypt (~ 1900 B.C). Notice the person on top of the sled pouring a liquid in front of it. Most Egyptologists believed that this was nothing more than a ritual, however, recent studies have shown that by adding water to the sand reduces the force required to pull an object is reduced by 50%. The mixture of water and sand increased the viscosity of the water and also eliminated the possibility that sand would simply form a heap in front of the sled. However, this still had to be delicately controlled, adding too much water to the to the sand results in a loss of stability in the ground, which would cause the statue to sink; too little water would mean that there is no real difference to the situation than if the sand was just dry.

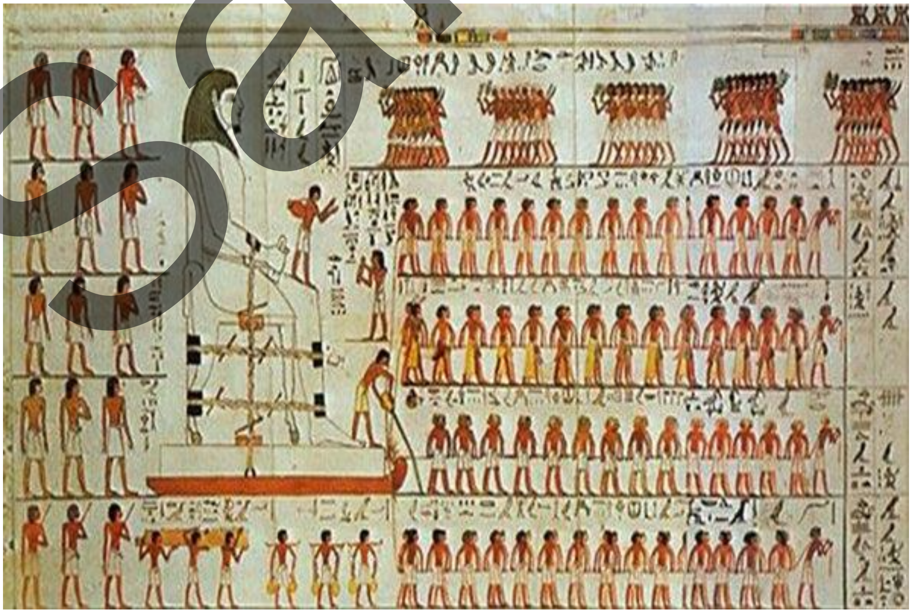

Figure 3.2: The wall painting in the tomb of Djehutihotep showing the earliest known use of lubrication

3.2 Viscometers

Viscometers are used to measure the viscosity of the fluid, and there are several types that exist.

3.2.1 Capillary Viscometers

Otherwise known as u-tube or glass viscometers, shown in Fig.3.3. These are the most common viscometers, they are cheap and relatively easy to use, and best suited for transparent or translucent liquids. The method is simple, use suction to bring the fluid up to the start mark (or ideally further past it). Once the suction is removed, the fluid will start to flow downwards.

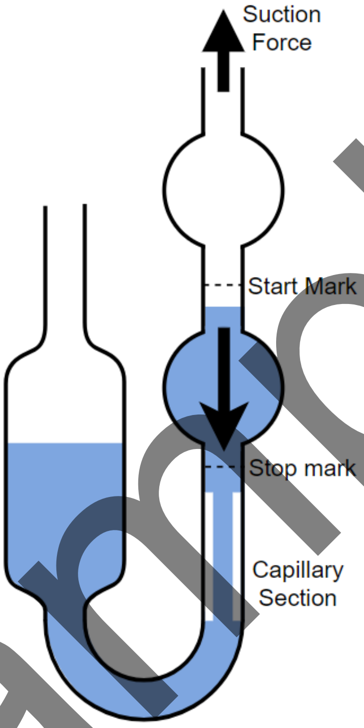

Figure 3.3: Capillary viscometers

The viscosity is measured by calculating the time it takes for the fluid to pass from the start mark to the stop mark. The equation used to calculate the kinematic viscosity of the fluid is given by Eq.3.3, where t is the time taken for the fluid to pass between the two marks, and K is the capillary constant of the viscometer, which is calibrated by measuring a reference liquid of known viscosity.

$$v = K_c \cdot t \qquad \text{(Eq.3.3)}$$

The falling sphere viscometer experiences similar problems to the capillary viscometer, as it relies on visual cues, it cannot use opaque fluids.

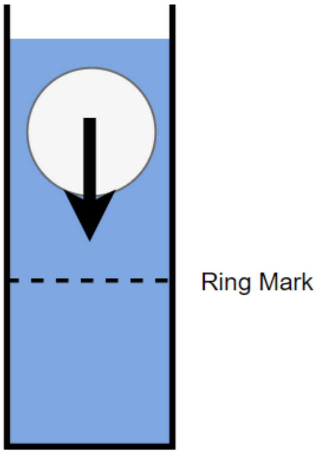

Figure 3.4: Falling sphere viscometer

Example 2

An engine oil company is using a new falling sphere viscometer to test a new range of lubricants they have been developing. The ball constant needs to be calculated before the viscosity of the lubricant can be tested. The testers have decided to use water to calibrate the viscometer, keeping a constant temperature at 20° means that the dynamic viscosity is known to be $1.0005\ cP$ and its density is $998\mathrm{kg/m^3}$. The ball they used has a density of $3040\mathrm{kg/m^3}$, and passes the mark in $2.46s$ in water, while it takes $1.89s$ in the oil. Assuming $\mathrm{F} = 1.0$ and the density of the oil is $790\mathrm{kg/m^3}$, calculate:

a) The ball constant K_b
b) The kinematic viscosity of the oil

Answers:

a) We know all variables to find $\mathrm{K_b}$ using water

$$\mathrm{K_c} = \frac{\mu}{\mathrm{F} \cdot \mathrm{t}(\rho_1 - \rho_w)} = \frac{1.0005 \times 10^{-4}}{1.0 \cdot 2.46(3040 - 998)} = 1.99 \times 10^{-8}\mathrm{Pa} \cdot \mathrm{m^3/kg}$$

b) With $\mathrm{K_c}$ calculated its possible to calculate the dynamic viscosity of the oil:

$$\mu = t(\rho_1 - \rho_2)K_c \cdot F = 1.89(3040 - 790)(1.99 \times 10^{-8}) \cdot 1.0 = 8.46 \times 10^{-5} Pa \cdot s = 0.846cP$$

The kinematic viscosity is therefore:

$$\mathrm{v} = \frac{\mu}{\rho_2} = \frac{8.46 \cdot 10^{-5}}{790} = 1.07 \cdot 10^{-7}\mathrm{m^2/s} = 0.107\mathrm{cSt}$$

3.2.3 Rotational Viscometers

Rotational Viscometers are reliant on the measurement of torque on a vertical stand to determine the viscosity of a liquid. Where the past two viscometers relied on gravity, for more viscous fluids, gravity if not a strong enough driving force to complete the experiment (think about using a capillary viscometer for tar pitch), and so rotational viscometers employ a motor to add a rotational driving force. Rotational viscometers can apply two different principles to calculate viscosity:

- Couette Principle
- Searle Principle

Viscometers also follow measure torque using two different systems.

- Servo systems
- Spring systems

3.2.3.1 Couette Principle

The Couette principle relies on a bob to be suspended in a container filled with the test fluid. In this case, the driving force is acting on the container itself, meaning that the bob is the stationary frame of reference in the system (shown in Fig.3.5). This design avoids any problems with turbulent flow, but it is rarely used in commercial applications as it can be difficult to ensure that the container is well insulated and sealed in the rotating cup.

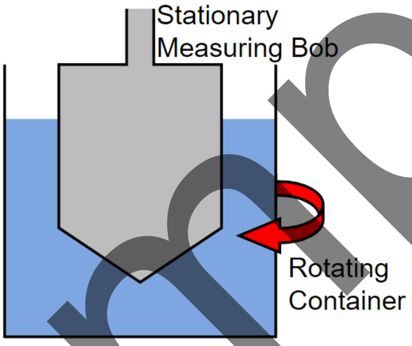

Figure 3.5: Couette principle rotational viscometer

3.2.3.2 The Searle Principle

The Searle principle holds the container stationary, and instead spins the measuring bob (as can be seen in Fig.3.6). In this case, the viscosity is proportional to the motor torque hat is required for turning the bob against the resistive viscous forces of the fluid. These are much more common viscometers; however, the measuring bob must be kept at a low enough velocity to ensure that the flow in the container does not become turbulent.

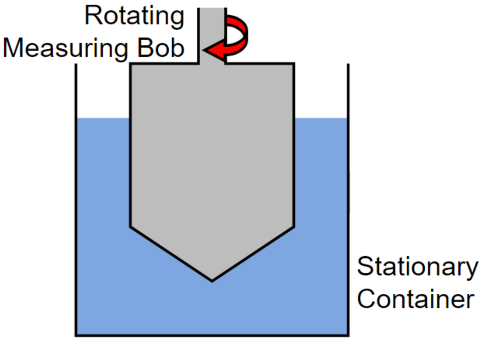

Figure 3.6: Searle principle rotational viscometer

3.2.3.3 Servo Devices

Servo systems use a servo motor to drive the main shaft, which will turn the measuring bob. Tachometers or high resolution digital encoders measure the rotational speed. The current drawn by the motor is proportional to the torque caused by the viscosity of the test fluid, meaning viscosity can be calculated using the rotational speed of the servo and current demand.

Servo systems allow a larger measuring range and are more robust, and allow for a greater torque and speed range than with the spring devices

3.2.3.4 Spring Devices

Spring systems use calibrated springs set by the manufacturer, with each spring designed to cover a range of viscosity (between 1cP to $1 \times 10^8\text{cP}$). A spring rotates on a shaft, the shaft is attached the to the system. As the system rotates, the viscous forces in the fluid generate a deflection force in the spring. This deflection is proportional to the torque caused by the test fluid's viscosity.

Spring devices are generally cheaper than their servo counterparts, they are also more accurate at low speeds and viscosities, as friction and bearing losses in the servo will impact the measurement.

3.2.3.5 Rotational Viscometer Equations

The shear rate and shear stress of the fluid is given by Eq.3.6 and Eq.3.7, respectively; where ω is the rotational velocity in rad/s, R_c is the radius of the container in metres, R_b is the radius of the bob in metres, h is the height of the bob in metres, and T is the measured torque in Nm. The dimensions of the system are shown in Fig.3.7.

$$\frac{dc}{dy} = \frac{2\omega R_c^2}{(R_c^2 - R_b^2)} \qquad \text{(Eq.3.6)}$$

$$\tau = \frac{M}{2\pi R_b^2 h} \qquad \text{(Eq.3.7)}$$

With these values in calculated, dynamic viscosity can be calculated using Eq.3.1.

$$\tau = \mu \frac{dc}{dy}$$

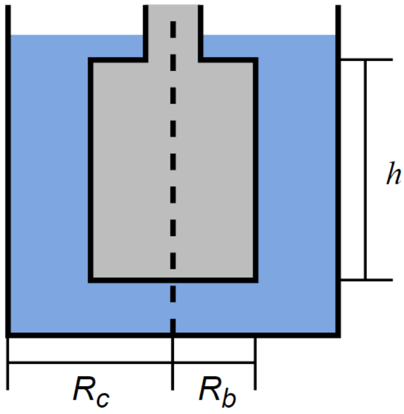

Figure 3.7: Rotational viscometer dimensions

Example 3

A rotational viscometer has a measuring bob of diameter 10cm and height 20cm, in a container of diameter 15cm. The bob is spun at 2500rpm to test the viscosity of a fluid. The torque reading from the system is quoted as 0.05Nm. Determine:

a) The shear rate
b) The shear stress
c) The dynamic viscosity

Answers:

a) The shear rate is given as:

$$\frac{dc}{dy} = \frac{2\omega R_c^2}{(R_c^2 - R_b^2)}$$

We need to find ω, R_c, R_b in the appropriate dimensions

$$\omega = 2500\text{rpm} = 261.8\ \text{rad/s}$$

$$R_c = 0.5d_c = 7.5\text{cm} = 0.075\text{m}$$

$$R_b = 0.5d_b = 5\text{cm} = 0.05\text{m}$$

With this information:

$$\frac{dc}{dy} = \frac{2\omega R_c^2}{(R_c^2 - R_b^2)} = \frac{2(261.8)(0.025)^2}{(0.075^2 - 0.05^2)} = 942.5\ \text{s}^{-1}$$

b) Shear stress is given as:

$$\tau = \frac{M}{2\pi R_b^2 h}$$

h in the appropriate dimension is:

$$h = 20\text{cm} = 0.2\text{m}$$

and therefore τ is:

$$\tau = \frac{M}{2\pi R_b^2 h} = \frac{0.05}{2\pi(0.05)^2(0.2)} = 15.9\ \text{Pa}$$

c) The dynamic viscosity is therefore:

$$\mu = \tau \div \frac{dc}{dy} = 15.9/942.5 = 0.0169\text{Pa}\cdot\text{s}$$
$$= 0.169\text{P}$$
$$= 16.9\text{cP}$$

3.2.4 Orifice Viscometers

Orifice viscometers are used to in the oil industry because of their simplicity and ease of use. The system consists of a reservoir, an orifice and a receiver. The method is simple, the sample fluid is poured into the reservoir, which is temperature controlled in a water bath. Once the sample fluid has reached the desired temperature (that of the water bath), a valve at the base of the reservoir is opened and the time taken for a specific amount of sample fluid to flow out of the orifice is measured. While the industry has several types of orifice viscometers, this workbook will only look at the Saybolt and Redwood viscometers.

Other orifice viscometers include:

- *Engler viscometers*
- *Ford viscosity cup viscometer*
- *Shell viscosity cup viscometer*
- *Zahn cup viscometer*

3.2.4.1 Saybolt Viscometer

A schematic of the Saybolt viscometer is shown in Fig.3.8. A practical system would most likely have a thermometer in both the water bath and the sample fluid reservoir, as a sure way to make sure that the temperature is controlled, something that isn't accurately controlled in comparison to the capillary, falling sphere or rotational viscometers. Since this system analyses the flow rate of the fluid with only a force due to gravity acting on the fluid, the Saybolt viscometer calculates the kinematic viscosity.

The Saybolt viscometer gives its own unit of viscosity, "Saybolt seconds". This is the time it takes for $60\mathrm{ml}$ to pour into the receiver, while it is not as scientific as $\mathrm{Pa}\cdot\mathrm{s}$, it is a valid measurement of viscosity. Most standards analyse the viscosity in Saybolt seconds at $100^{\circ}\mathrm{F}$, a reasonable estimate for a given temperature can be found using Eq.3.8, where v_{T} is the Saybolt kinematic viscosity at the desired temperature T is the desired temperature, and $v_{100^{\circ}\mathrm{F}}$ is the Saybolt kinematic viscosity at $100^{\circ}\mathrm{F}$.

$$v_T = v_{100^{\circ}F}\left(1 + \frac{1}{16400}(T - 100)\right) \qquad \text{(Eq.3.8)}$$

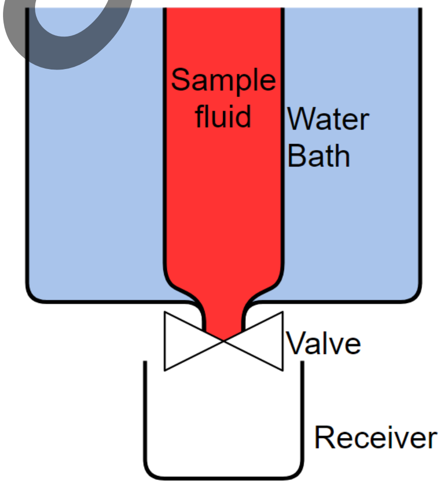

Figure 3.8: Saybolt viscometer