Pearson BTEC Level _ Higher Nationals in Engineering (RQF)

# Unit 64: Thermofluids

# Unit Workbook 4

in a series of 4 for this unit

Learning Outcome 4

# Fluid Systems

Sample

edexcel
Approved Centre

# 4.1 Fluid Flow

Fluid flow can be broken down into two terms, either laminar or turbulent. Knowing whether a fluid is laminar or not is very important in figuring out its characteristics.

## 4.1.1 Laminar Flow

Laminar flow is the name given to a "smooth" flow. Consider the pipe shown in Fig.4.1, the arrows in the image show the direction that the fluid travels, and it can be considered that a laminar flow are all parallel to each other. Laminar flow is also simpler to calculate, as it can also be assumed that the flow's velocity, pressure at each point is constant.
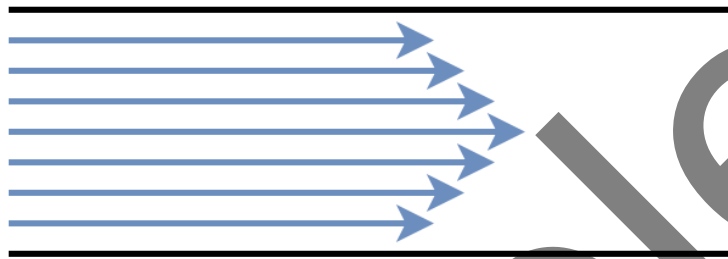


*Figure 4.1: Laminar flow through a pipe*

Laminar flow typically occurs in very small flow channels, where a relatively high viscosity fluid flows slowly. Examples of this are oil through a small pipe, or blood through the capillaries (not arteries or veins). The URL below shows a laminar flow.

https://www.youtube.com/watch?v=9opbBlbXN8c

## 4.1.2 Turbulent Flow

Turbulent flow, on the other hand, is disordered and the flow paths will often cross and mix. The velocity of turbulent flow is constantly changing in direction and magnitude, but the general direction will not change. The constant changes in velocity result in "eddy currents", which are swirls in the flow. Fig.4.2 shows turbulent flow through a pipe.
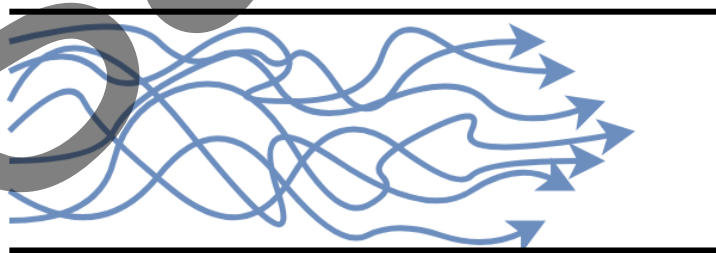


*Figure 4.2: Turbulent flow through a pipe*

There are far more examples of turbulent flow in systems, such as oil transport pipes and blood through arteries and veins. Turbulent flow also includes air over an aerofoil.

## 4.1.3 The Reynold's Number

They Reynold's number (Re) is a dimensionless constant that is used to describe fluid flow. The Reynold's number is calculated using Eq.4.1, and is a ratio of inertia forces to viscous forces.

# 4.2 Head Losses in Pipe

No system is 100% efficient, and pipework is no exception, frictional losses occur as the surface of the water and the surface of the pipe make contact with each other.

Head losses in particular describe the loss of pressure in the system, consider the pipework in Fig.4.3 below, this method is a simple pressure measurement without the use of electronics. The head losses refer to the height lost at each gauge, the further along the pipe, the more frictional losses occur, this loss in pressure means that the height that the water reaches in the gauge lowers, and as such $h_1 > h_2 > h_3$ in Fig.4.3.
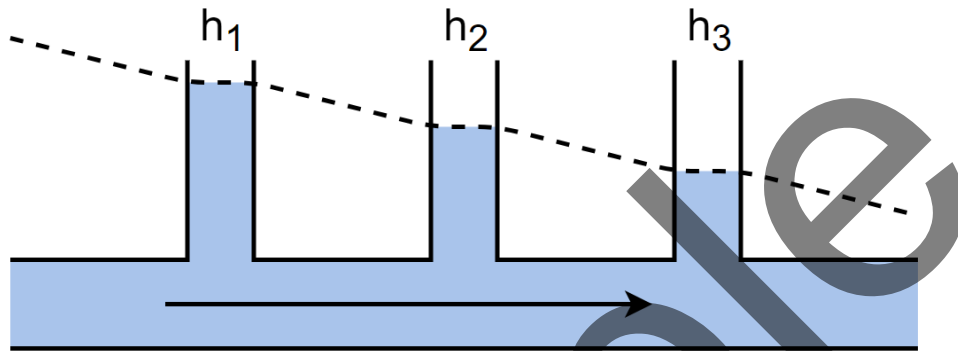


*Figure 4.3: Head losses in pipes*

## 4.2.1 Surface Roughness

Head losses in pipes can have a range of reasons, one important aspect to consider is the relative roughness of the pipe. Fig.4.4 shows the effect surface roughness can have on the flow, the rough surface keeps some of the flow contained within the crevices, while these streamlines can make their way back into the flow, it requires more forces and pressures to get them back on the right track, meaning there are more head losses, compared to the perfectly smooth pipe.
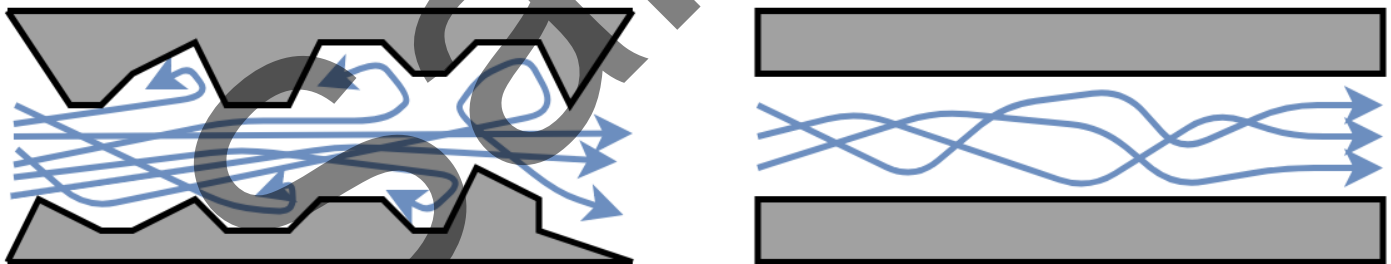


*Figure 4.4: Turbulent flow through a rough (left) and smooth (right) pipe*

The relative roughness of a pipe is defined Eq.4.2, where $k$ is the average height of the surface irregularities (some sources will use $\epsilon$), and $d$ is the diameter of the pipe.

$$\text{Relative Roughness} = \frac{k}{d} \qquad \qquad \text{(Eq.4.2)}$$

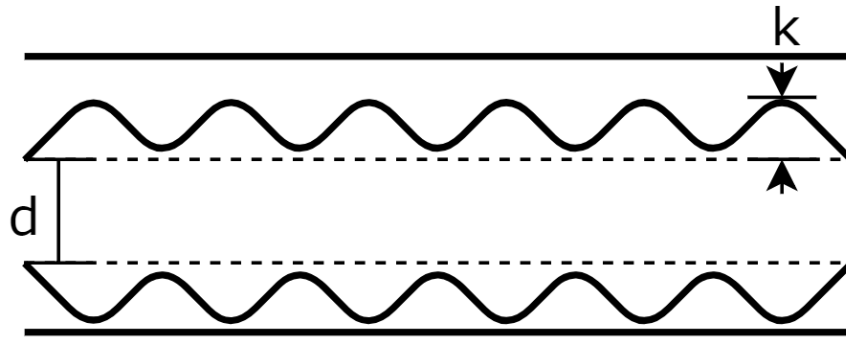Fig.4.5 gives a graphical demonstration of the dimensions.

*Figure 4.5: The measurement of the diameter and height of irregularities in a pipe.*

## 4.2.2  Moody Diagrams

Moody diagrams is a diagram to measure the coefficient of friction in pipes (noted in this workbook as f, some sources will use λ). Calculating the coefficient of friction depends on both the flow in the pipe and its relative roughness.

The equations used for calculating f are:

Laminar:
$$f = \frac{16}{Re}$$
(Eq.4.3)

Turbulent Smooth pipes:
$$f = 0.079 Re^{-0.25}$$
(Eq.4.4)

Turbulent Rough pipes:
$$\frac{1}{\sqrt{f}} = -3.6 log_{10} \left[ \frac{6.9}{Re} + \left( \frac{\epsilon}{3.71d} \right)^{1.11} \right]$$
(Eq.4.5)

These equations are quite long to calculate, with the exception of the laminar equation (Eq.4.3). So alternatively, the Moody chart is available for reference, the Moody chart is a graph that has already plot the values for the friction factor across a range of Reynold's numbers and relative roughness to give a quick (and fairly accurate) estimate. Fig.4.6 shows a Moody chart, the lines show the variation of friction factor at a given relative roughness, but a varying Reynold's number. Most Moody diagrams will also include an absolute roughness value for some materials, the absolute roughness value is a typical estimate for $\epsilon$ for certain materials.

**Example**

Let's say for an example, the Reynold's number of a given flow is $7 \cdot 10^5$, and the absolute roughness is 0.25mm with a diameter of 62.5mm. The friction factor will be $\sim 0.007$ (maybe 0.0071, shown as a red dot in Fig.4.7), or using the Eq.4.6, knowing that the pipe is rough and turbulent.

$$\frac{1}{\sqrt{f}} = -3.6\log_{10}\left[ \frac{6.9}{Re} + \left( \frac{k}{3.71d} \right)^{1.11} \right] = -3.6\log_{10}\left[ \frac{6.9}{7 \cdot 10^5} + \left( \frac{0.25 \cdot 10^{-3}}{3.71 \cdot 62.5 \cdot 10^{-3}} \right)^{1.11} \right] = 11.827$$

$$f = \left( \frac{1}{11.827} \right)^2 = 0.00715$$

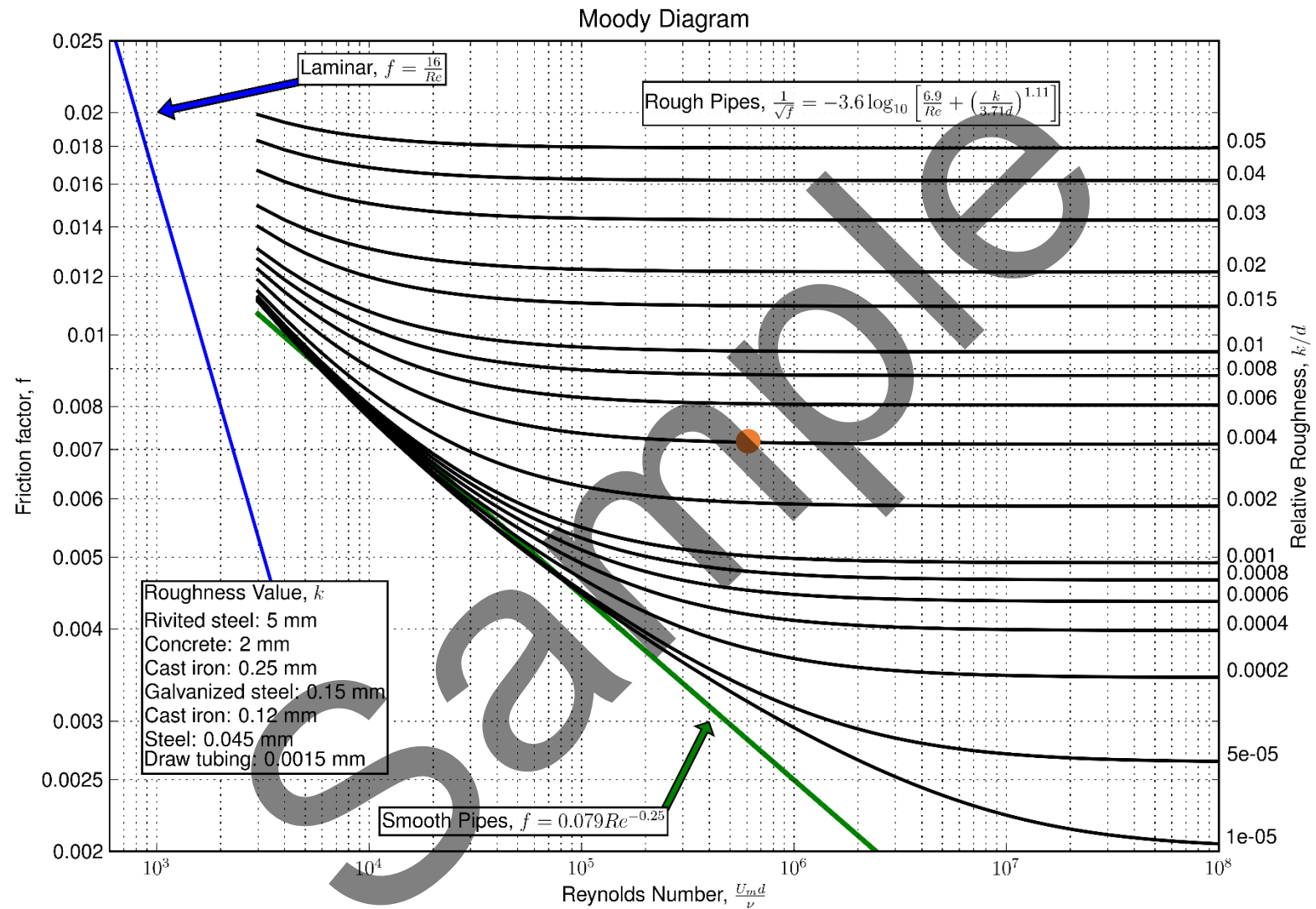Which means the estimate from the Moody diagram is very close.

## Moody Diagram



*Figure 4.7: Moody Diagram*

The chart includes the following annotations:

- Laminar, $f = \frac{16}{Re}$
- Rough Pipes, $\frac{1}{\sqrt{f}} = -3.6 \log_{10}\left[ \frac{6.9}{Re} + \left( \frac{k}{3.71d} \right)^{1.11} \right]$
- Smooth Pipes, $f = 0.079 Re^{-0.25}$

Axis labels:
- Friction factor, f (vertical axis)
- Reynolds Number, $\frac{U_m d}{\nu}$ (horizontal axis)
- Relative Roughness, $k/d$ (right vertical axis)

Roughness Value, $k$
Rivited steel: 5 mm
Concrete: 2 mm
Cast iron: 0.25 mm
Galvanized steel: 0.15 mm
Cast iron: 0.12 mm
Steel: 0.045 mm
Draw tubing: 0.0015 mm

## 4.2.4 The Darcy-Weisbach Equation

The Darcy-Weisbach equation is used to calculate the head losses in a system. The head loss of a system is measured in the height change in metres, and is a valid measurement of pressure drop. The Darcy-Weisbach equation is defined by Eq.4.6:

$$h_f = \frac{flu^2}{2gd} \qquad \text{(Eq.4.6)}$$

Where:

- $h_f$ is the head loss $(m)$
- $f$ is the friction factor
- $l$ is the length of the pipe $(m)$
- $u$ is the velocity of the fluid $(m/s)$
- $g$ is acceleration due to gravity $(m/s^2)$
- $d$ is the diameter of the pipe $(m)$

**Example**

An oil Pipeline is transporting Arabian Light oil at 20°C (v = 10.7mm$^2$/s, ρ = 854kg/m$^3$) over 500m at 1m/s. Calculate the head loss in the pipe if the pipe's diameter is 20cm with a relative roughness of 0.008.

**Answer**

$$\text{Re} = \frac{uL}{v} = \frac{1 \cdot 500}{10.7 \cdot 10^{-6}} = 4.67 \cdot 10^7$$

Flow is turbulent, and from the Moody diagram, the friction factor f can be estimated as 0.0089, meaning that h$_f$ can be calculated as:

$$h_f = \frac{flu^2}{2gd} = \frac{(0.0089)(500)(1)^2}{2(9.81)(0.2)} = 1.13\text{m}$$

By using the equation to find f, *we will also need to find the absolute roughness of the system (*k*), which is:*

$$0.008 \cdot 0.2 = 0.0016$$

$$\frac{1}{\sqrt{f}} = -3.6\log_{10}\left[\frac{6.9}{\text{Re}} + \left(\frac{k}{3.71d}\right)^{1.11}\right] = -3.6\log_{10}\left[\frac{6.9}{4.67 \cdot 10^7} + \left(\frac{0.0016}{3.71 \cdot 0.2}\right)^{1.11}\right] = 10.6542$$

$$f = \frac{1}{10.6542^2} = 0.00881$$

Meaning head loss is:

$$h_f = \frac{flu^2}{2gd} = \frac{0.0088(500)(1)^2}{2(9.81)(0.2)} = 1.12\text{m}$$

Again, an almost miniscule difference over 500m of pipe.

# 4.3 Turbines

Hydraulic Machines are systems that will use the force of water to do work (or vice versa). The earliest examples of this are the traditional water wheels which date back to roughly 4000 B.C, which were used for crop irrigation, water transport and grinding grain. As technology advanced, water wheels were used to power sawmills, bellows and textile mills, before being replaced with fuel powered engines in a lot of applications.

Turbines use the movement of fluid to generate work, while pumps do work to move fluid. Turbines can be classed as either an impulse turbine, or a reaction turbine. Reaction turbines move as a result of a reaction force with the fluid acting on it, while impulse turbines are pushed using an impulse (the rate of change of momentum) from the fluid.

## 4.3.1  Pelton Wheel

The Pelton wheel was developed in the 1800s during the American gold rush. This system is an impulse turbine, and consists of a series of buckets connected to an impeller. The fluid is fed into a into a nozzle, the pressure of the discharge is dictated by the position of the spear rod. As the fluid hits the buckets, it will create a moment acting around the impeller, which will force it to rotate, the buckets will discharge the fluid as they rotate. Fig.4.9 shows a Pelton wheel.
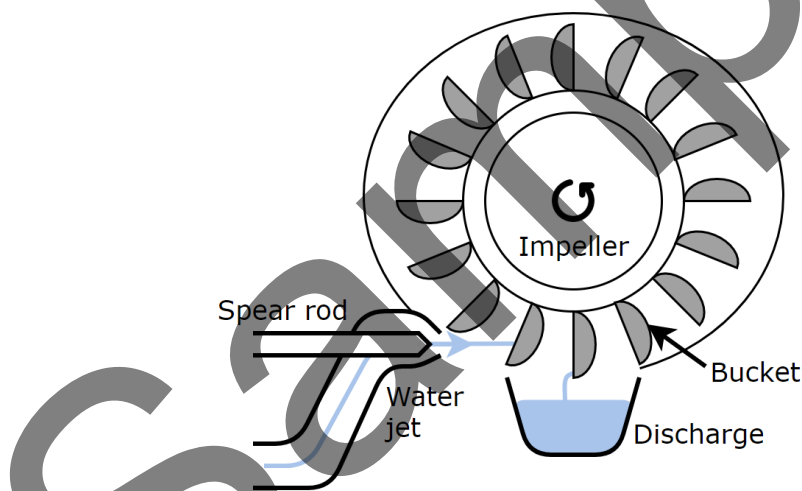


*Figure 4.9: Pelton wheel diagram*

## 4.3.2  Kaplan Turbine

The Kaplan turbine are reaction turbines that typically use adjustable blades; a schematic of a vertical-axis Kaplan turbine can be seen in Fig.4.10, Kaplan turbines can also be found in an "s-turbine" configuration. The fluid flow into the system is controlled using inlet vanes. Altering the position of the vanes means that the fluid will hit the rotors at the appropriate angle for the highest efficiency, this can also be achieved using the rotor blade's pitch; the rotor pitch will be angled almost flat during low flow, and heavily pitched at high flow (~45°). The nose cone of the system is used to prevent the formation of rope vortices, which can cause large pressure fluctuations in the system, which can be detrimental to the blades. The draft tube's diameter slowly tapers off after the fluid has passed through the turbine blades in order to reduce the pressure of the fluid and extract all the kinetic energy from the fluid to put into the turbine.
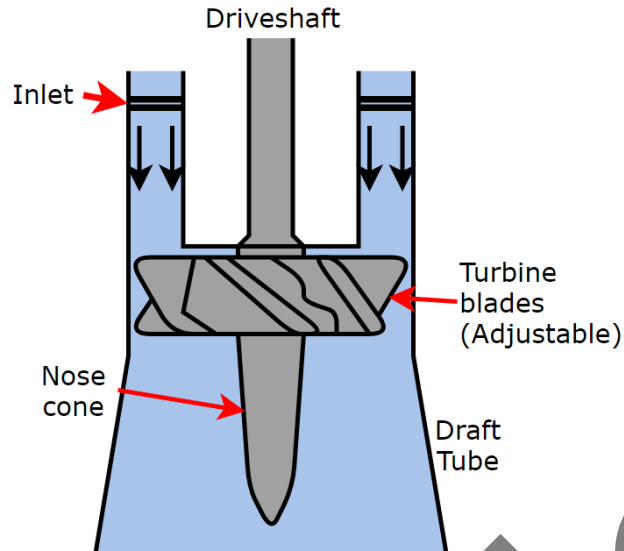
*Figure 4.10: Kaplan Turbine*

The Kaplan turbine can have a number of configurations:

- Full Kaplan (Adjustable inlet vanes and turbine pitch)
- Semi-Kaplan
  - o Adjustable inlet vanes – fixed turbine pitch
  - o Fixed inlet vanes – adjustable turbine pitch
- Simple propeller turbine (Fixed inlet vanes – fixed turbine pitch)

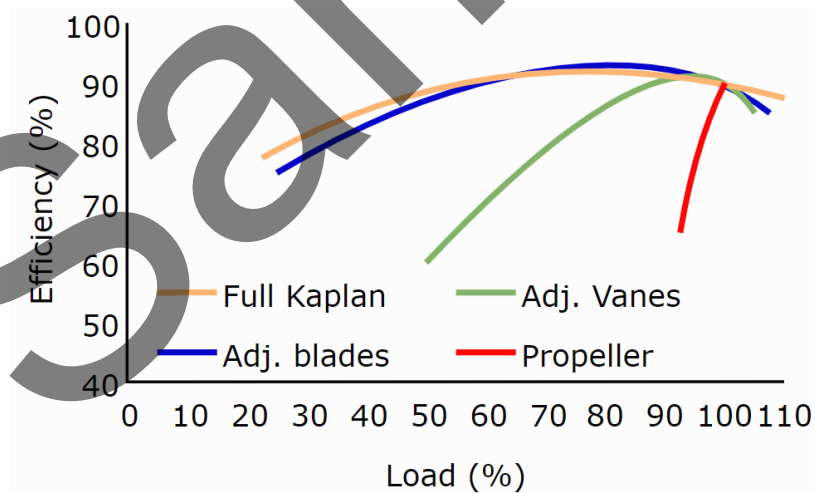Figure 4.11 shows an estimated efficiency curve for the four systems.



*Figure 4.11: Efficiency against load of various Kaplan turbines*

## 4.3.3  Francis Wheel

The Francis wheel is also a reaction turbine. Hoover Dam in Nevada, USA uses 17 Francis turbines powered by flow of the Colorado River.  A schematic of a Francis wheel turbine is shown in Fig.4.12.  While the Pelton wheel and Kaplan turbines both used an axial inflow to the turbine (straight line), Francis turbines use a radial inflow, but the flow ends up axial once leaving the turbine. The fluid flows in and is slowly guided in to the

propellers of the turbine by a series of vanes dotted around the runner, as the fluid is flowing through the runner, its pressure and angular momentum decreases, these reductions provide a reaction on the runner and will rotate the driveshaft at the centre of the runner. As the fluid loses its speed in the runner, it will flow into the draft tube underneath. Like the Kaplan turbine, the draft tube tapers outwards to maximise the energy extracted by the turbine.
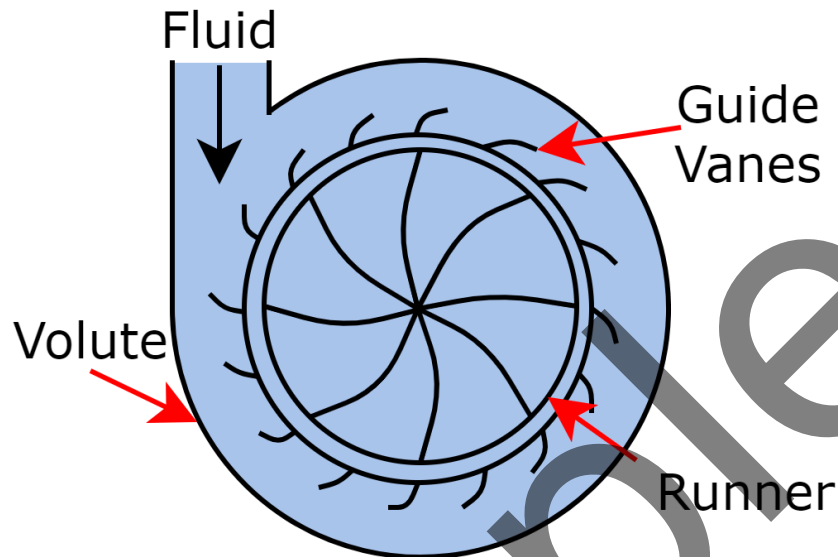


*Figure 4.12: A Francis wheel turbine*

# 4.4 Pumps

## 4.4.1 Centrifugal Pump

A centrifugal pump is one that uses rotation to move the fluid. It has a very similar shape and design to the Francis wheel turbine. The fluid is brought into the system through the centre of the impeller (which typically, is not the centre of the volute) and constant rotation through the impeller blades will direct the fluid to push out through the discharge pipes. Due to the shape of the and position of the impeller, as the distance between the impeller and the volute increase, there is a build up in pressure in the system which will push the fluid out of the system.
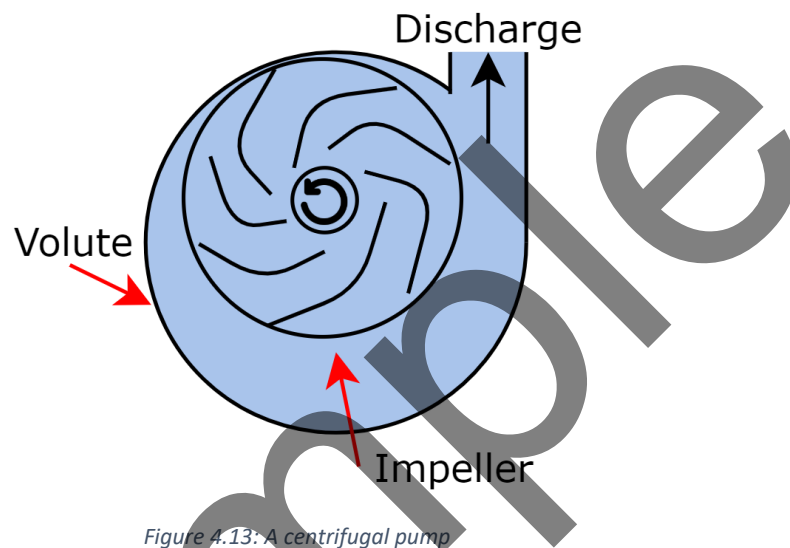


*Figure 4.13: A centrifugal pump*

## 4.4.2 Reciprocating Pump

Reciprocating pumps are piston mechanisms that use pressure differentials to move fluid. A common use of this system is to extract water from mines to prevent flooding. The process of the system can be broken down into several stages.

$\theta = 0°$: The crank and piston system are considered to be top dead centre (TDC), this is the maximum reach of the piston, in this position, the intake valve is open, and the discharge valve is closed.
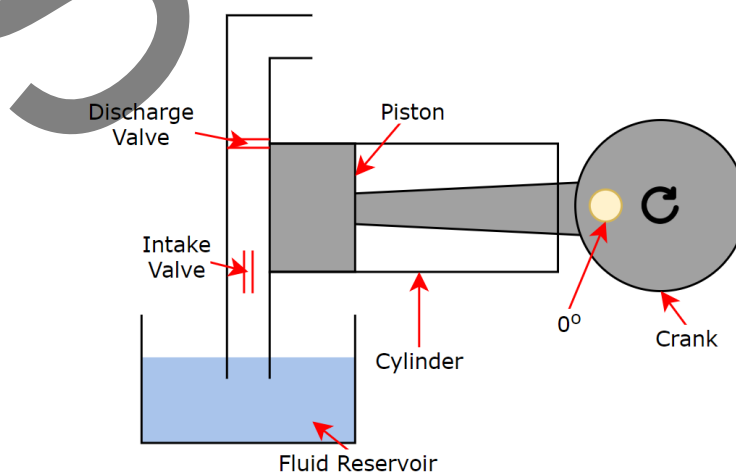


*Figure 4.14: A reciprocating pump at TDC*

# 4.5 Dimensional Analysis

We can create the equations we use in maths by looking at their dimensions. Which can be broken down into three distinct dimensions:

- Length [L]
- Mass [M]
- Time [T]

If an equation is dimensionally consistent (RHS=LHS) then the equation maybe accurate **(This does not guarantee it is accurate)**. Let's take a look at Newton's 2$^{nd}$ Law. $F = ma$, we can break this down into the three variables and their dimensions.

$$F = [N] = [M][L][T]^{-2}$$

$$m = [M]$$

$$a = [L][T]^{-2}$$

So, when we put this into the equation we get.

$$[M][L][T]^{-2} = [M] \cdot [L][T]^{-2}$$

Which shows the equation for the Newtons 2$^{nd}$ law as dimensionally consistent.

The same can be said for dimensions, when we calculate Volume, V, we multiply the length, l, by the width, w, by the depth, d. Shown in the Equation below

$$V = l \cdot w \cdot d$$

The table below shows the different units of each variable, and the dimensional form.

| Variable | Units | Dimensional Form |
|----------|-------|------------------|
| $V$ | m$^3$ | $[L]^3$ |
| l | m | [L] |
| w | m | [L] |
| d | m | [L] |

So the equation, in dimensional form, is:

$$[L]^3 = [L] \cdot [L] \cdot [L]$$

Which we simplify further to

$$[L]^3 = [L]^{(1+1+1)}$$

Or, just looking at the powers, we can see that:

$$3 = 1 + 1 + 1$$

We can therefore show that this is dimensionally equal.

From [3]:

$$A = 2 \qquad [4]$$

Sub [4] into [1]:

$$1 = 1.5(2) + B \therefore B = -2$$

Sub [4] into [2]:

$$1 = 0.5(2) + C \therefore C = 0$$

Almost there. Earlier we had…

$$F \propto Q^A r^B m^C$$

Since we know that C = 0 then the 'm' term disappears (anything to the power zero is 1). We also know values for A and B…

$$F \propto Q^A r^B m^C$$

$$F \propto Q^A r^B m^0$$

$$F \propto Q^A r^B$$

$$F \propto Q^2 r^{-2}$$

$$\therefore \quad F \propto \frac{Q^2}{r^2}$$

An important thing to note here; we suspected that the mass of the particles may have had an influence on the force between them. Our dimensional analysis determined that this was not the case – **the mass was irrelevant**. Also, the top of our solution contains a charge squared term, and we know that we had two charges, $Q_1$ and $Q_2$, so we can confidently write:

$$F \propto \frac{Q_1 Q_2}{r^2}$$

This result looks remarkably like Coulomb's Law.

However, lets look at a more complicated example.

It is believed that pressure drop $\Delta P$ in a pipe is related to the diameter of the pipe d, the length of the pipe L, the density of the fluid $\rho$, the dynamic viscosity $\mu$, and the velocity u. Using Rayleigh's method, determine the relationship between the variables.

$$\Delta P = d^a L^b \rho^c \mu^d u^e$$

Which in dimensional form can be expressed as:

$$[MLT^{-2}] = [L]^a [L]^b [ML^{-3}]^c [ML^{-1}T^{-1}]^d [LT^{-1}]^e$$

The function can then be written as below, where k and c will be determined by experimentation

$$\frac{\Delta P}{\rho u^2} = k\left(\frac{L}{d}, \frac{\mu}{\rho du}\right)^c$$

As all the π groups are dimensionless. It could be a case that the π groups given are the inverse. It is entirely possible to write:

$$\frac{\rho u^2}{\Delta P} = k \cdot f\left(\frac{d}{L}, \frac{\rho du}{\mu}\right)^c$$

## 4.5.3  Scale Models

Dimensional analysis can be useful when developing scale models of experiments. For example, if engineers were building an experimental turbine for electrical generation for the national grid, it could cost in the realm of hundreds of thousands of pounds. So, for a purely experimental system, it's just not worth it.



Figure 4.19: Siemens Turbogenerator system

However, by using dimensional analysis to find the connection relationships between variables, it becomes easier to understand the effect of scale models. Then, by changing one variable, the value for c can be found, meaning we can figure out the effects of changing other variables.

The power output of a wheel can be given as a system involving Torque, the radius of the wheel and the tangential velocity of the wheel.

a)  The dimensions for each variable are:

$$T_w = ML^2T^{-2}$$

$$r = L$$

$$u = LT^{-1}$$

$$\rho = ML^{-3}$$

$$Q = L^3T^{-1}$$

Our recurring set will be:

$$r = L$$

$$u = LT^{-1}$$

$$\rho = ML^{-3}$$

To which the dimensions can be written as:

$$L = r$$

$$M = \rho r^3$$

$$T = \frac{r}{u}$$

So, for $T_w$:

$$\pi_1 = T_w \times M^{-1}L^{-2}T^2$$

$$\pi_1 = T_w \frac{1}{\rho r^3} \frac{1}{r^2} \frac{r^2}{u^2}$$

$$\pi_1 = \frac{T_w}{\rho r^3 u^2}$$

And for Q:

$$\pi_2 = Q \times L^{-3}T$$

$$\pi_2 = Q \frac{1}{r^3} \frac{r}{u}$$

$$\pi_2 = \frac{Q}{r^2 u}$$

And the $\pi$ groups are:

$$\frac{T_w}{\rho r^3 u^2} = k\left(\frac{Q}{r^2 u}\right)^c$$

b)  If the torque output is proportional to the square of the radius, then we can rearrange to give.

$$T_w = k\left(\frac{Q}{r^2 u}\right)^c \times \rho r^3 u^2$$

If $T_w \propto r^2$:

$$r^3 \times \left(\frac{1}{r^2}\right)^c = r^2$$

Looking at the powers only:

$$3 - 2c = 2$$