Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 8: Mechanical Principles

Unit Workbook 1

in a series of 4 for this unit

Learning Outcome 1

Static Mechanical Systems

1.1 Shafts and beams:

1.1.1 Introduction.

Supporting structures that do not bend withstand externally applied loads in either tension or compression. However, in practice, many structures are subjected to bending action. Prior to calculating bending moments and shearing forces we need to define bending moment and shearing force.

1.1.2 Revision – Moments, couples, torque, stress, and strain

A **moment** is a turning force producing a turning effect. The magnitude of this turning force depends on the size of the force applied and the perpendicular distance from the pivot or axis to the line of action of the force.

A **couple** occurs when two equal forces acting in opposite directions have their lines of action parallel.

Another important application of the couple is its **turning moment** or **torque**. Torque is the turning moment of a couple and is measured in newton-metres (Nm): torque T = force F × radius r.

If a solid is subjected to an external force (load), a resisting force is set up within the solid and the material is said to be in a state of **stress**. There are three basic types of **stress**:

• **tensile stress** – which is set up by forces tending to pull the material apart

• **compressive stress** – produced by forces tending to crush the material

• **shear stress** – resulting from forces tending to cut through the material, i.e. tending to make one part of the material slide over the other.

Stress is defined as force per unit area, i.e. $Stress\ (\sigma) = \frac{Force\ (F)}{Area\ (A)}$

A material that is altered in shape due to the action of a force acting on it is said to be strained.

Direct strain may be defined as: the ratio of change in dimension

(deformation) over the original dimension, i.e. $Direct\ Strain\ (\varepsilon) = \frac{Deformation\ (x)}{Original\ length\ (l)}$

1.1.3 Types of Beams

A beam is a structure which is loaded transversely (sideways). The loads may be point loads or Uniformly Distributed Loads (UDL). Figure 1 indicates the way that these are illustrated.

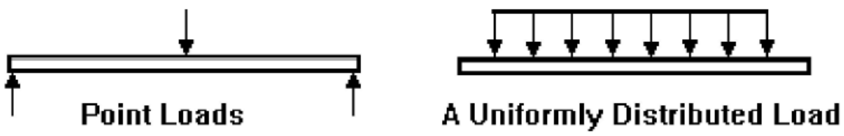

Figure 1 Point Loads and UDLs

The beam may be simply supported or built in, as illustrated in Figure 2.

Figure 2 Types of beam

Transverse loading causes bending, and bending is a very severe form of stressing a structure. The bent beam goes into tension (stretched) on one side and compression on the other as shown in Figure 3.

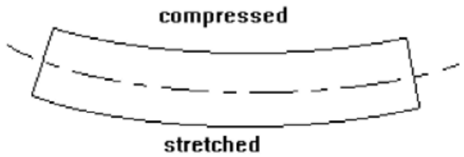

Figure 3 Stretching and compression

The complete formula which describes all aspects of bending is:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where:
M is the moment produced by a force
I is the second moment of area.
σ is the stress produced during bending,
y is the strain produced during bending
E is the Modulus of Elasticity
R is the radius of the neutral axis

1.1.4 Neutral Axis and Radius of Curvature

This is the axis along the length of the beam which remains unstressed, neither compressed nor stretched when it is bent. Normally the neutral axis passes through the centroid of the cross-sectional area. The position of the centroid is hence important.

Consider that the beam is bent into an arc of a circle through angle 0 radians. AB is on the neutral axis and is the same length before and after bending. The radius of the neutral axis is R, as indicated in Figure 4.

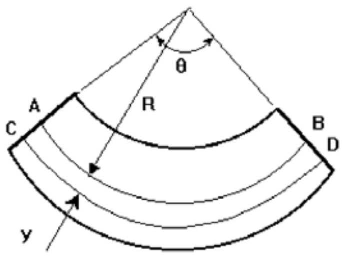

Figure 4 Neutral Axis

Normally the beam does not bend into a circular arc. However, whatever shape the beam takes under the sideways loads; it will basically form some form of curve. In mathematical terms, the radius of curvature at any point on a graph is the radius of a circle that just touches the graph and has the same tangent at that point.

1.1.8 Bending Moment (M).

The Bending moment is measured in N m. When a beam is subjected to the couples shown in Figure 10 the beam will suffer flexure due to the bending moment M

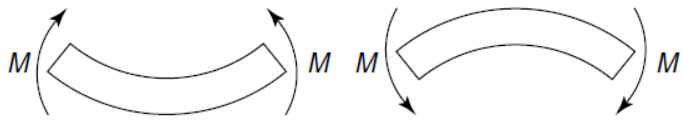

Figure 10 (a) Sagging moment (+) (b) Hogging moment (-)

If the beam is in equilibrium and it is subjected to a clockwise couple of magnitude M on the left of the section, then from equilibrium considerations, the couple on the right of the section will be of exactly equal magnitude and of opposite direction to the couple on the left of the section. Thus, when calculating the bending moment at a particular point on a beam in equilibrium, we need only calculate the magnitude of the resultant of all the couples on one side of the beam under consideration. This is because as the beam is in equilibrium, the magnitude of the resultant of all the couples on the other side of the beam is exactly equal and opposite. The beam in Figure 10(a) is said to be sagging and the beam in Figure 10(b) is said to be hogging. The sign convention adopted here is:

(a) sagging moments are said to be positive

(b) hogging moments are said to be negative

1.1.9 Shearing Force (F).

Whereas a beam can fail due to its bending moments being excessive, it can also fail due to other forces being too large, namely the shearing forces; these are shown in Figure 11.

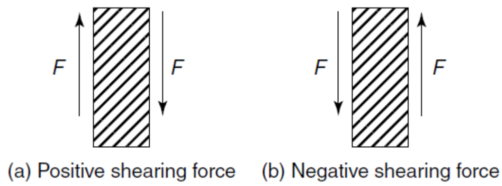

Figure 11 (a) Positive shearing force (b) Negative shearing force

The units of shearing force are N. It can be seen from Figure 11 (a) and (b) that the shearing forces F act in a manner similar to that exerted by a pair of garden shears when they are used to cut a branch through shearing action. This mode of failure is different to that caused by bending action.

In the case of the garden shears, it is necessary for the blades to be close together and sharp, so that they do not bend the branch at this point. If the garden shears are old and worn the branch can bend and may lie between the blades. Additionally, if the garden shears are not sharp, it may be more difficult to cut the branch because the shearing stress exerted by the blades will be smaller as the contact area between the blades and the branch will be larger.

The shearing action is illustrated in Figure 12

Once again, if the beam is in equilibrium, then the shearing forces either side of the point being considered will be exactly equal and opposite, as shown in Figure 11(a) and (b). The sign convention for shearing force is that it is said to be positive if the right hand is going down; see Figure 11(a). Thus, when calculating the shearing force at a particular point on a horizontal beam, we need to calculate the resultant of all the vertical forces on one side of the beam, as the resultant of all the vertical forces on the other side of the beam will be exactly equal and opposite. The calculation of bending moments and shearing forces and the plotting of their respective diagrams are demonstrated in the following worked problems.

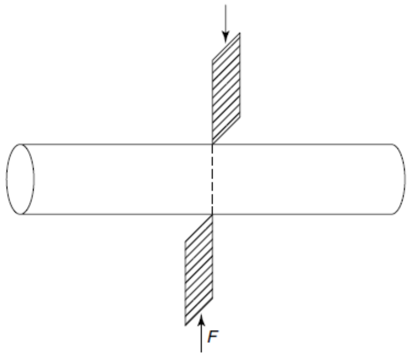

Figure 12 Shearing action

1.1.10 Ultimate Shear Stress

If a material is sheared beyond a certain limit it becomes permanently distorted and does not spring all the way back to its original shape. The elastic limit has been exceeded. If the material is stressed to the limit so that it parts into two (e.g. a guillotine or punch), the ultimate limit has been reached. The ultimate shear stress is T_u and this value is used to calculate the force needed by shears and punches.

Worked Example

Calculate the force needed to guillotine a sheet of metal 5 mm thick and 0.8 m wide given that the ultimate shear stress is 50 MPa.

The area to be cut is a rectangle 800 mm x 5 mm

A= 800 x 5 = 4000 mm^2. The ultimate shear stress is 50 N/mm^2

$T_u = \frac{F}{A}$ So, $F = T_u A =$ 50 x 4000 = 200,000N or **200kN**

1.1.11 Bending moments and shearing force diagrams.

Worked Example

Calculate and sketch the bending moment and shearing force diagrams for the horizontal beam shown in Figure 13, which is simply supported at its ends.

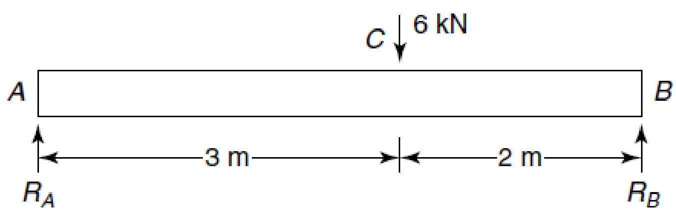

Figure 13 Horizontal beam worked example

Firstly, calculate the magnitude of reactions R_A and R_B.

Taking moments about B gives:

Clockwise moments about B = ant-clockwise moments about B

Thus, R_A x 5m = 6 kN x 2m = 12 kN m, ∴ **R_A = 2.4 kN**

Resolving forces vertically gives:

Upward forces = downward forces

Thus, $R_A + R_B$ = 6 kN, ∴ **R_B = 3.6 kN**

As there is a discontinuity at point C in Figure 13, due to the concentrated load of 6 kN, it will be necessary to consider the length of AC separately from the length of CB. The reason for this is that the equations for bending moment and shearing force for span AC are different to the equations for the span CB; this is caused by the concentrated load of 6 kN.

For the present problem, to demonstrate the nature of bending moment and shearing force, these values will be calculated on both sides of the point of the beam under consideration. It should be noted that normally, the bending moment and shearing force at any point on the beam, are calculated only due to the resultant couples or forces, respectively, on one side of the beam.

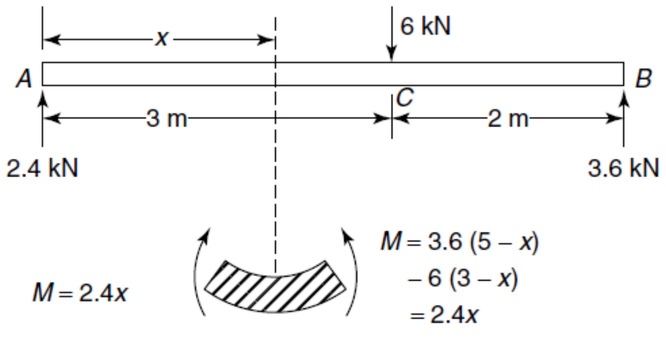

Figure 14 Horizontal beam bending moment span AC

<u>Consider span AC</u>

Bending moment

Consider a section of the beam at a distance x from the left end A, where the value of x lies between A and C, as shown in Figure 14. It can be seen that the reaction RA causes a clockwise moment of magnitude RA ×x = 2.4x on the left of this section and as shown in the lower diagram of Figure 14. It can also be seen from the upper diagram of Figure 14, that the forces on the right of this section on the beam causes an anti-clockwise moment equal to RB X (5-x) or 3.6(5 − x) and a clockwise moment of 6 × (3 − x), resulting in an anti-clockwise moment of:

$3.6(5 - x) - 6(3 - x) = 3.6 \times 5 - 3.6x - 6 \times 3 + 6x$

$= 18 - 3.6x - 18 + 6x$

= 2.4x

Thus, the left side of the beam at this section is subjected to a clockwise moment of magnitude 2.4x and the right side of this section is subjected to an anti-clockwise moment of 2.4x, as shown by the lower diagram of Figure 14. As the two moments are of equal magnitude but of opposite direction, they cause the beam to be subjected to a bending moment M = 2.4x. As this bending moment causes the beam to sag between A and C, the bending moment is assumed to be positive, or at any distance x between A and C:

Bending moment = M = +2.4x (Eq 1)

Shearing force

Here again, because there is a discontinuity at C, due to the concentrated load of 6 kN at C, we must consider a section of the beam at a distance x from the left end A, where x varies between A and C, as shown in Figure 15.

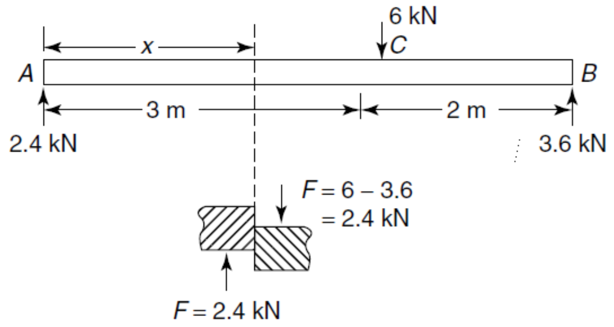

Figure 15 Horizontal beam shearing force span AC

It can be seen that the resultant of the vertical forces on the left of the section at x are 2.4 kN acting upwards. This force causes the left of the section at x to slide upwards, as shown in the lower diagram of Figure 15. Similarly, if the vertical forces on the right of the section at x are considered, it can be seen that the 6 kN acts downwards and that RB = 3.6 kN acts upwards, giving a resultant of 2.4 kN acting downwards. The effect of the two shearing forces acting on the left and the right of the section at x, causes the shearing action shown in the lower diagram of Figure 15. As this shearing action causes the right side of the section to glide downwards, it is said to be a positive shearing force.

Summarising, at any distance x between A and C:

Shearing force = F = +2.4 kN (Eq 2)

Consider span CB:

Bending moment

At any distance x between C and B, the resultant moment caused by the forces on the left of x is given by:

$M = R_A \times x - 6(x-3) = 2.4x - 6(x-3) = 2.4x - 6x + 18 \therefore$ **M = 18 − 3.6x (clockwise)** (Eq 3)

The effect of this resultant moment on the left of x is shown in the lower diagram of Figure 16.

It can be seen that on the right side of x, there is an anti-clockwise moment of:

$M = R_B \times (5-x) = 3.6(5-x) = 18 - 3.6x \therefore$ **M = 18 − 3.6x (anti-clockwise)** (Eq 4)

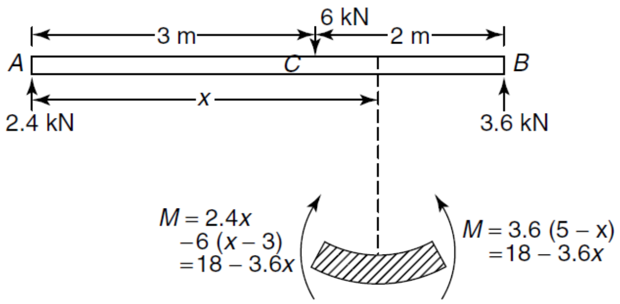

Figure 16 Horizontal beam bending moment span CB

The effect of these bending moments is to cause the beam to sag at this point as shown by the lower diagram of Figure 16, i.e. M is positive between C and B, and ∴ **<u>M = +18 − 3.6x</u>** (Eq 5)

Shearing force

Consider a distance x between C and B, as shown in Figure 6.8.

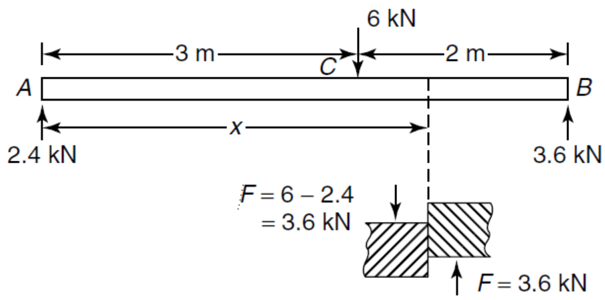

Figure 17 Horizontal beam shearing force span CB

It can be seen that at x, there are two vertical forces to the left of this section, namely, the 6 kN load acting downwards and the 2.4 kN load acting upwards, resulting in a net value of 3.6 kN acting downwards, as shown by the lower diagram of Figure 17. Similarly, by considering the vertical forces acting on the beam to the right of x, it can be seen that there is one vertical force, namely the 3.6 kN load acting upwards, as shown by the lower diagram of Figure 17. Thus, as the right hand of the section is tending to slide upwards, the shearing force is said to be negative, i.e. between C and B, ∴ **<u>F = −3.6 kN</u>** (Eq 6)

It should be noted that at C, there is a discontinuity in the value of the shearing force, where, over an infinitesimal length the shearing force changes from +2.4 kN to −3.6 kN, from left to right.

1.1.12 Bending moment and shearing force diagrams

The bending moment and shearing force diagrams are simply diagrams representing the variation of bending moment and shearing force, respectively, along the length of the beam. In the bending moment and shearing force diagrams, the values of the bending moments and shearing forces are plotted vertically and the value of x is plotted horizontally, where x = 0 at the left end of the beam and x = the whole length of the beam at its right end.

In the case of the beam of Figure 13, bending moment distribution between A and C is given by (Eq 1), (M = 2.4x), where the value of x varies between A and C.

At A, x = 0, therefore $M_A = 2.4 \times 0 = 0$

and at C, x = 3 m, therefore $M_C = 2.4 \times 3 = 7.2$ kN.

Additionally, as the equation M = 2.4x is a straight line, the bending moment distribution between A and C will be as shown by the left side of Figure 18,

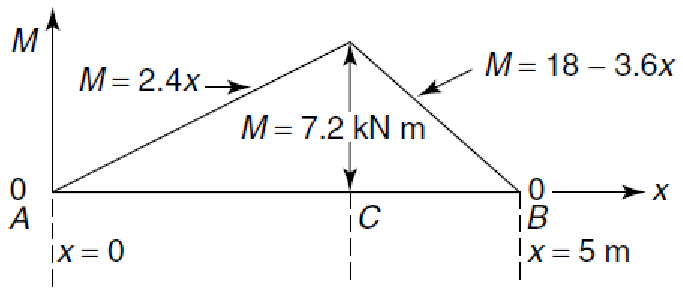

Figure 18 Bending moment diagram

Similarly, the expression for the variation of bending moment between C and B is given by equation (Eq 3) (M = 18–3.6x), where the value of x varies between C and B. The equation can be seen to be a straight line between C and B.

At C, x = 3 m, therefore $M_C = 18 - 3.6 \times 3 = 18 - 10.8 = 7.2$ kN m

At B, x = 5 m, therefore $M_B = 18 - 3.6 \times 5 = 18 - 18 = 0$

Therefore, plotting of the equation M = 18 − 3.6x between C and B results in the straight line on the right of Figure 18, i.e. the bending moment diagram for this beam has now been drawn.

In the case of the beam of Figure 13, the shearing force distribution along its length from A to C is given by (Eq 2) (F = 2.4 kN), i.e. F is constant between A and C. Thus, the shearing force diagram between A and C is given by the horizontal line shown on the left of C in Figure 19.

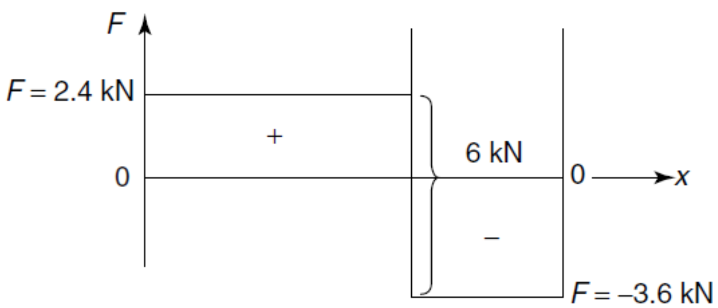

Figure 19 Shearing force diagram

Similarly, the shearing force distribution to the right of C is given by (Eq 6) (F = −3.6 kN), i.e. F is a constant between C and B, as shown by the horizontal line to the right of C in Figure 19. At the point C, the shearing force is indeterminate and changes from +2.4 kN to −3.6 kN over an infinitesimal length.