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## INTRODUCTION

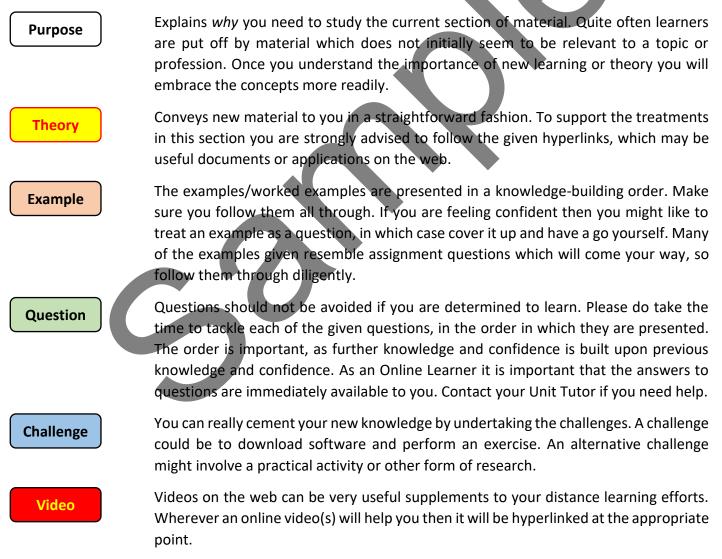
This Workbook guides you through the learning outcomes related to:

#### Identifying solutions to problems within static mechanical systems

- Shafts and beams:
  - The effect of shear forces on beams.
  - Bending moments and stress due to bending in beams.
  - Selection of appropriate beams and columns to satisfy given specifications.
  - $\circ$   $\;$  The theory of torsion in solid and hollow circular shafts.

## GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;





# 1.1 Shafts and beams:

### 1.1.1 Introduction.

Supporting structures that do not bend withstand externally applied loads in either tension or compression. However, in practice, many structures are subjected to bending action. Prior to calculating bending moments and shearing forces we need to define bending moment and shearing force.

#### 1.1.2 Revision – Moments, couples, torque, stress, and strain

A **moment** is a turning force producing a turning effect. The magnitude of this turning force depends on the size of the force applied and the perpendicular distance from the pivot or axis to the line of action of the force.

A couple occurs when two equal forces acting in opposite directions have their lines of action parallel.

Another important application of the couple is its **turning moment** or **torque**. Torque is the turning moment of a couple and is measured in newton-metres (Nm): torque T =force  $F \times$  radius r.

If a solid is subjected to an external force (load), a resisting force is set up within the solid and the material is said to be in a state of **stress**. There are three basic types of **stress**:

- tensile stress which is set up by forces tending to pull the material apart
- compressive stress produced by forces tending to crush the material

• **shear stress** – resulting from forces tending to cut through the material, i.e. tending to make one part of the material slide over the other.

Stress is defined as force per unit area, i.e. Stress ( $\sigma$ ) =  $\frac{Force(F)}{4\pi \sigma_{\sigma}(A)}$ 

A material that is altered in shape due to the action of a force acting on it is said to be strained.

Direct strain may be defined as: the ratio of change in dimension

(deformation) over the original dimension, i.e. Direct Strain ( $\varepsilon$ ) =  $\frac{Deformation(x)}{Original length(l)}$ 

### 1.1.3 Types of Beams

A beam is a structure which is loaded transversely (sideways). The loads may be point loads or Uniformly Distributed Loads (UDL). Figure 1 indicates the way that these are illustrated.

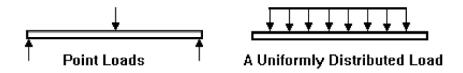


Figure 1 Point Loads and UDLs

The beam may be simply supported or built in, as illustrated in Figure 2.



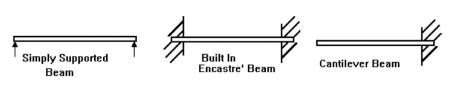


Figure 2 Types of beam

Transverse loading causes bending, and bending is a very severe form of stressing a structure. The bent beam goes into tension (stretched) on one side and compression on the other as shown in Figure 3.

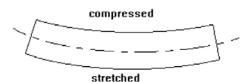


Figure 3 Stretching and compression

The complete formula which describes all aspects of bending is:

 $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ 

#### Where:

M is the moment produced by a force

I is the second moment of area.

 $\sigma$  is the stress produced during bending,

y is the strain produced during bending

E is the Modulus of Elasticity

R is the radius of the neutral axis

### 1.1.4 Neutral Axis and Radius of Curvature

This is the axis along the length of the beam which remains unstressed, neither compressed nor stretched when it is bent. Normally the neutral axis passes through the centroid of the cross-sectional area. The position of the centroid is hence important.

Consider that the beam is bent into an arc of a circle through angle 0 radians. AB is on the neutral axis and is the same length before and after bending. The radius of the neutral axis is R, as indicated in Figure 4.

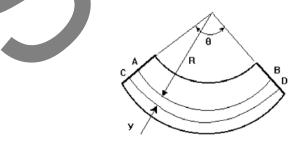


Figure 4 Neutral Axis

Normally the beam does not bend into a circular arc. However, whatever shape the beam takes under the sideways loads; it will basically form some form of curve. In mathematical terms, the radius of curvature at any point on a graph is the radius of a circle that just touches the graph and has the same tangent at that point.

