Pearson BTEC Level 4 Higher Nationals in Engineering (RQF)

Unit 8: Mechanical Principles

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Dynamic Mechanical Systems

1.3 Work Done and power transmitted by a constant torque

Figure 1 shows a pulley wheel of radius r metres attached to a shaft and a force F Newton's applied to the rim at point P.

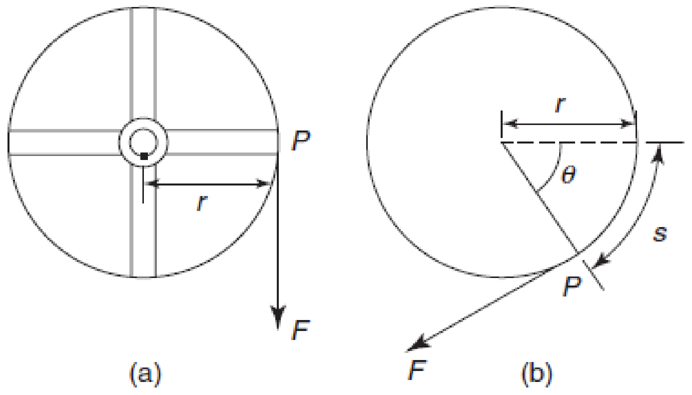

Figure 1 Pulley Wheel

Figure 1(b) shows the pulley wheel having turned through an angle θ radians as a result of the force F being applied. The force moves through a distance s, where arc length s = rθ

Work done = force × distance moved by the force = F × rθ = Frθ (N m) = Frθ (J) But, Fr is the torque T, so,

Work Done = Tθ joules

Average power = work done/time taken = = T θ/time taken for a constant torque T

However, (angle θ) / (time taken) = angular velocity, ω rad/s. Hence,

Power = Tω watts (Eq 1)

Angular velocity, ω = 2πn rad/s where n is the speed in rev/s. Hence, Power

P = 2πnT watts (Eq 2)

Sometimes power is in units of horsepower (hp), where 1 horsepower = 745.7 watts

Worked Example 1

A motor connected to a shaft develops a torque of 5 kN m. Determine the number of revolutions made by the shaft if the work done is 9 MJ.

Work Done = T θ, from which,

Angular displacement, θ = Work Done/Torque

Hence, Angular displacement, θ = 9 x 10^6 / 5000 = 1800 rad.

But 2π rad = 1 rev., hence,

<u>No of revs made by the shaft = 1800/2π = 286.5 revs</u>

1.4 Power transmission and efficiency

A common and simple method of transmitting power from one shaft to another is by means of a belt passing over pulley wheels which are keyed to the shafts, as shown in Figure 2. Typical applications include an electric motor driving a lathe or a drill, and an engine driving a pump or generator.

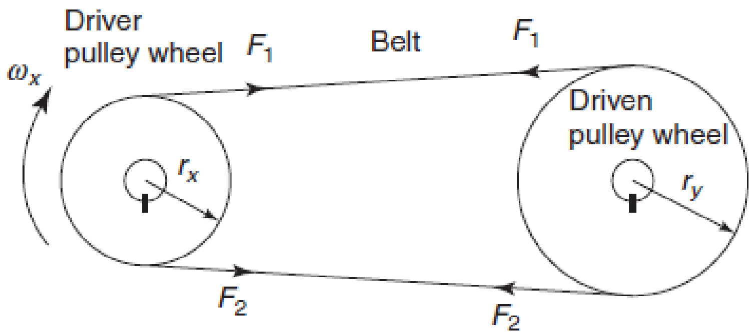

Figure 2 Pulley Belt Drive

For a belt to transmit power between two pulleys there must be a difference in tensions in the belt on either side of the driving and driven pulleys. For the direction of rotation shown in Figure 2, $F_2 > F_1$. The torque T available at the driving wheel to do work is given by:

$T = (F_2 - F_1)\, r_x$ (N m) and the available power P is given by:

$P = T\omega = (F_2 - F_1)\, r_x\omega_x$ (Watts)

The linear velocity of a point on the driver wheel is $v_x = r_x\omega_x$

Similarly, the linear velocity of a point on the driven wheel, $v_y = r_y\omega_y$.

Assuming no slipping, $vx = vy$ and therefore, $rx\omega_x = r_y\omega_y$

Hence $r_x(2\pi n_x) = r_y\,(2\pi n_y)$ from which,

$r_x/r_y = n_y/n_x$

Percentage efficiency = useful work output / energy output $\times$ 100 or,

efficiency = power output / power input $\times$ 100%

2 Linear and angular motion.

2.1 Linear and Angular velocity

Linear velocity v is defined as the rate of change of linear displacement s with respect to time t, and for motion in a straight line:

Linear velocity = change of displacement / change of time i.e. $\mathbf{v = s/t}$ (ms^{-1}) (Eq 3)

Angular velocity is defined as the rate of change of angular displacement θ, with respect to time t, and for an object rotating about a fixed axis at a constant speed:

Angular velocity = angle turned through / time taken i.e. $\boldsymbol{\omega = \theta/t}$ (rads-1) (Eq 4)

An object rotating at a constant speed of n revolutions per second subtends an angle of 2πn radians in one second.

Hence, its angular velocity, $\boldsymbol{\omega = 2\pi n}$ $rads^{-1}$ (Eq 5)

From **s (arc length) = r (radius) θ (angle) and ω = θ/t**

$s = r\,\omega\,t$ or $s/t = \omega r$ and $v = s/t$

Hence, $\mathbf{v = \omega r}$ (Eq 6)

This equation gives the relationship between linear velocity, v and angular velocity, ω.

2.2 Linear and Angular acceleration

Linear acceleration, a, is defined as the rate of change of linear velocity with respect to time. For an object whose linear velocity is increasing uniformly:

linear acceleration = change of linear velocity/time taken i.e. $\mathbf{a = (v_2 - v_1)/t}$ (Eq 7)

The unit of linear acceleration is metres per second squared (m/s^2).

Rewriting equation (Eq 7) with v_2 as the subject of the formula gives:

$\mathbf{v_2 = v_1 + at}$ (Eq 8)

where v_2 = final velocity and v_1 = initial velocity.

Angular acceleration, α, is defined as the rate of change of angular velocity with respect to time. For an object whose angular velocity is increasing uniformly:

Angular acceleration = change of angular velocity/time taken i.e. $\boldsymbol{\alpha = (\omega_2 - \omega_1)/t}$ (Eq 9)

The unit of angular acceleration is radians per second squared (rad/s^2). Rewriting equation (Eq 9) with ω_2 as the subject of the formula gives:

$\boldsymbol{\omega_2 = \omega_1 + \alpha t}$ (Eq 10)

where ω_2 = final angular velocity and ω_1 = initial angular velocity. From equation (Eq 6), $v = \omega r$. For motion in a circle having a constant radius r, $v_2 = \omega_2 r$ and $v_1 = \omega_1 r$, hence equation (Eq 9) can be rewritten as:

$a = (\omega_2 r - \omega_1 r)/t = r\,(\omega_2 - \omega_1)/t$

But from equation (Eq 9), $(\omega_2 - \omega_1)/t = \alpha$, Hence $a = r\,\alpha$ (Eq 11)

2.3 Relative Velocity

Quantities used in engineering and science can be divided into two groups:

Scalar quantities have a size or magnitude only and need no other information to specify them. Thus 20 centimetres, 5 seconds, 3 litres and 4 kilograms are all examples of scalar quantities.

Vector quantities have both a size (or magnitude), and a direction, called the line of action of the quantity. Thus, a velocity of 30 km/h due west, and an acceleration of 7 m/s2 acting vertically downwards, are both vector quantities.

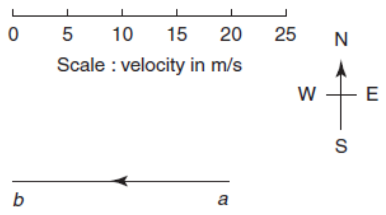

Figure 3 Vector quantity

A vector quantity is represented by a straight line lying along the line of action of the quantity, and having a length that is proportional to the size of the quantity. Thus, ab in Figure 3. represents a velocity of 20m/s, whose line of action is due west. The bold letters, ab, indicate a vector quantity and the order of the letters indicate that the line of action is from a to b.

Consider two aircraft A and B flying at a constant altitude, A travelling due north at 200 m/s and B travelling 30° east of north, written N 30° E, at 300 m/s, as shown in Figure 4. Relative to a fixed-point o, **oa** represents the velocity of A and **ob** the velocity of B. The velocity of B relative to A, that is the velocity at which B seems to be travelling to an observer on A, is given by **ab**, and by measurement is 160 m/s in a direction E 22° N. The velocity of A relative to B, that is, the velocity at which A seems to be travelling to an observer on B, is given by **ba** and by measurement is 160 m/s in a direction W 22° S.

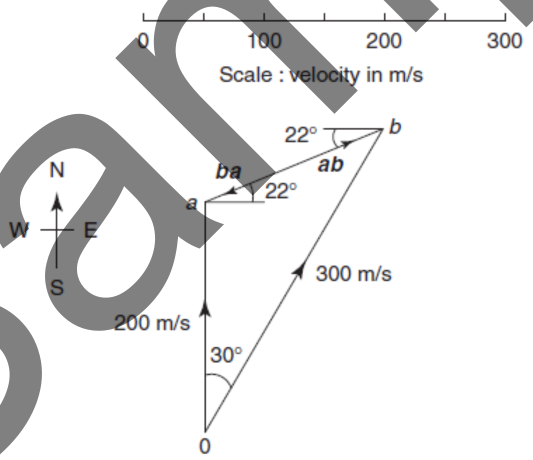

Figure 4 Relative velocities

Worked Example 6

Two cars are travelling on horizontal roads in straight lines, car A at 70 km/h at N 10° E and car B at 50 km/h at W 60° N. Determine, by drawing a vector diagram to scale, the velocity of car A relative to car B.

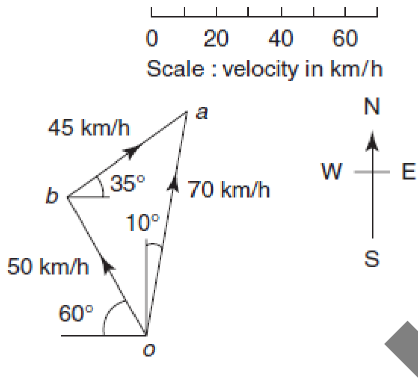

Figure 5 Vector solution to problem (a)

With reference to Figure 5, **oa** represents the velocity of car A relative to a fixed-point o, and **ob** represents the velocity of car B relative to a fixed-point o. The velocity of car A relative to car B is given by **ba** and by measurement is 45 km/h in a direction of E 35° N.

Worked Example 7

Verify the result obtained in the previous worked example by calculation.

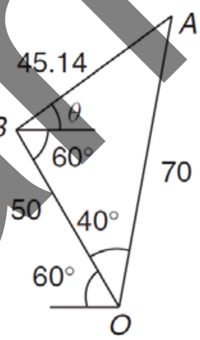

Figure 6 Vector solution to problem (b)

The triangle shown in Figure 6 is similar to the vector diagram shown in Figure 9. Angle BOA is 40°.
Using the cosine rule:
$BA^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \times \cos 40°$
from which, **BA = 45.14**
Using the sine rule:
50/sin (angle BAO) = 45.14/sin 40°
from which, sin (angle BAO) = 50 sin 40°/45.14 = 0.7120
Hence, angle BA0 = 45.40°
thus, angle ABO = 180° – (40° + 45.40°) = 94.60°, and
angle θ = 94.60° – 60° = 34.60°.
Thus, **ba is 45.14 km/h in a direction E 34.60° N by calculation**.

2.4 Gyroscopic motion.

A Gyroscope is a spinning disc mounted in gimbals such that it may pivot in the x, y and z axis, as indicated in Figure 7

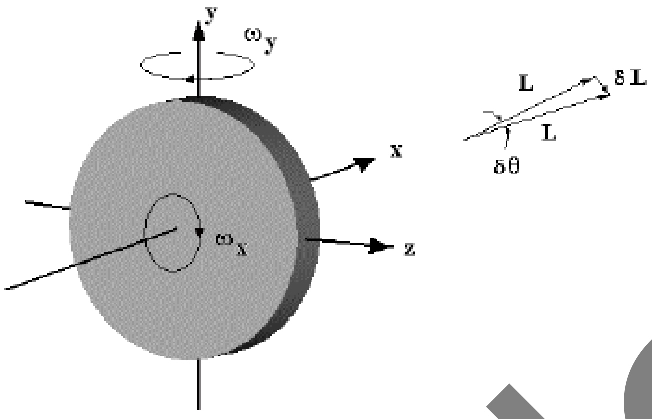

Figure 7 Gyroscopic motion

Now consider a disc spinning about the x-axis with velocity ω_x as shown. The angular momentum of the disc is $L = I\,\omega_x$. This is a vector quantity and the vector is drawn with a direction conforming to the corkscrew rule (point the index finger of your right hand in the direction of the vector, as shown in Figure 8.

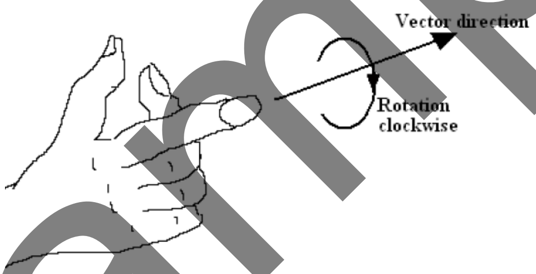

Figure 8 Corkscrew rule

Suppose the disc also rotates about the y-axis as shown in Figure 9 through a small angle δθ. The vector for L changes direction but mot magnitude. This produces a change in the angular momentum of $\delta L = \delta\,(I\,\omega_x)$.

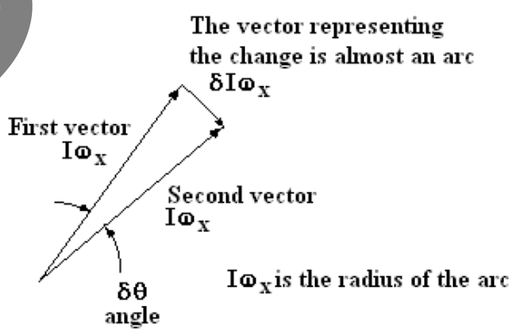

Figure 9 Vector for change in angular momentum

The vector diagram conforms to the vector rule (*final vector = first vector + change*). The change is the arrow going from the tip of the first to the tip of the second, so the direction is as shown.

The vector representing the change is almost an arc of radius I ω_x and angle $\delta\theta$. The length of the arc is the product of radius and angle. Taking the radius as I ω_x and the angle as $\delta\theta$, the change is:

$\delta (I\ \omega_x) = I\ \omega_x\ \delta\theta = L\ \delta\theta$

Newton's second law of motion applied to rotating bodies tells us that the change in momentum can only be brought about by applying a torque.

Torque = rate of change of angular momentum.

If the rotation occurred in time δt seconds, the rate of change of momentum is L ($\delta\theta$/ δt).

$\delta\theta$/ δt is the angular velocity ω_y so the rate $\mathbf{T = L\ \omega_y = I\ \omega_x\ \omega_y}$

This is the torque that must be applied to produce the change in angle and the direction of the vector is the same as the change in momentum. The applied torque may hence be deduced in magnitude and direction. Examining the vector for this torque we can deduce that it applied in the z-axis.

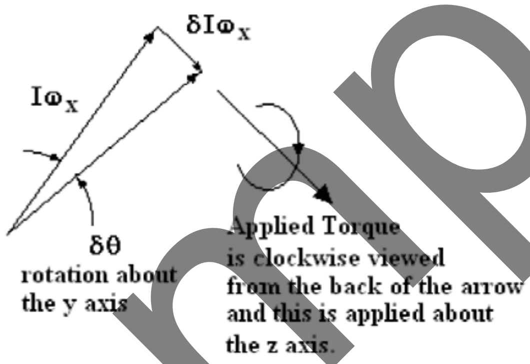

Figure 10 Applied torque and precession

If the torque is applied about the z-axis, the result will be rotation about the z-axis and this is called **precession**.

Torque-induced precession (gyroscopic precession) is the phenomenon in which the axis of a spinning object (e.g., a gyroscope) describes a cone in space when an external torque is applied to it. The phenomenon is commonly seen in a spinning toy top, but all rotating objects can undergo precession.

If the torque is not applied and the rotation is made to happen (applied), a reaction torque will be produced (Newton's 3rd law) and the disc will respond to the reaction torque.

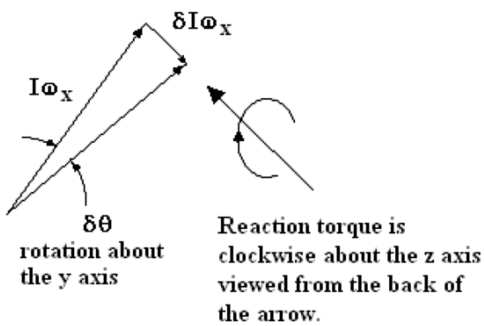

Figure 11 Reaction torque

A gyroscopic torque may occur in any machine with rotating parts if a change in the direction of the x-axis occurs. Examples include any vehicle where a gyroscopic torque is produced by the engines when a course alteration is made.

You can watch a video of gyroscopic torque at

Video
https://www.youtube.com/watch?v=ty9QSiVC2g0

2.5 Spinning tops.

A top is a spinning body that is symmetrical about its axis of spin. This could be a cylinder or a cone rotating about its axis.

If the axis is inclined at angle ϕ as shown in Figure 12 and the top is spinning at ω rad/s, it will have angular momentum $L = I\omega$ in the direction shown. Figure 12 shows the top precessing about the vertical axis that intercepts with the axis of spin at O. Point O is a fixed-point, so the axis of spin forms a surface of a cone as it precesses.

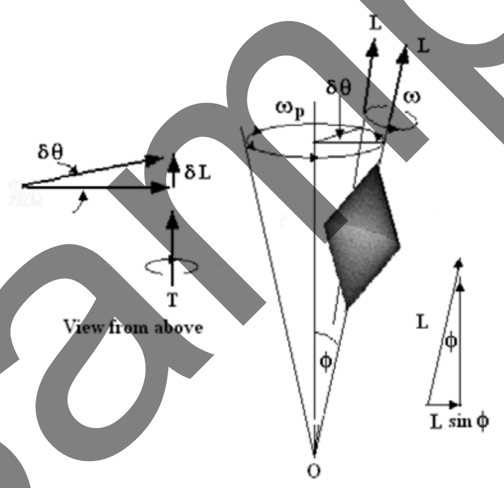

Figure 12 Precession in a spinning top

Suppose the axis of spin rotates through an angle $\delta\theta$ in time δt.

The change in angular momentum is $\delta L = L\,\delta\theta$, but this is the change for the axis of spin.

The change in the horizontal plane is $\delta L = L \sin\phi\,\delta\theta$.

Divide through by the time it takes to happen. Therefore $\delta L / \delta t = L \sin\phi\,\delta\theta / \delta t$

In the limit when the change is infinitesimally small $dL / dt = L \sin\phi\, d\theta /dt = L \sin\phi\,\omega_p$

The rate of change of angular momentum must be produced by a torque T and equal to $T = L \sin\phi\,\omega_p$ and the direction may be deduced from the corkscrew rule from the direction of the change. If the axis of spin is vertical $\phi = 0$, $T = 0$, and $\omega_p = 0$.

At any other angle, there must be applied torque, and this could be due to the weight, for example.

Worked Example 8

A top consists of a spinning disc of radius 50 mm and mass 0.8 kg mounted at the end of a light rod as shown in Figure 13. If the disc rests on a pivot with its axis of spin horizontal as shown, and the distance X is 30 mm, calculate the velocity of the precession when it spins at 40 rev/min.

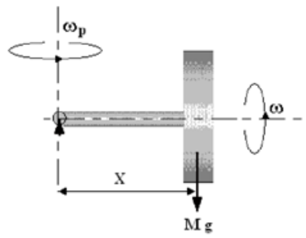

Figure 13 Diagram for worked example 4

The angle ϕ is 90^o so $T = L \sin \phi \, \omega_p$
For a plain disc $I = MR^2 / 2 = 0.8 \times (0.05)^2 / 2 = 0.004$ kg m^2
$\omega = 2\pi \times 50 = 100\pi$ rad/s
$L = I \omega = 0.004 \times 100\pi = 1.2566$ kg m^2 /s
T is due to the weight acting at 30 mm from the rest point.
$T = mgX = 0.8 \times 9.81 \times 0.03 = 0.235$ Nm.
$\omega_p = T/L = 0.235 / 1.2566 =$ **0187 rad/s**

Worked Example 9

A top consists of a spinning disc of radius 40 mm and mass 0.5 kg mounted at the end of a light rod as shown. The distance from the tip to the centre of gravity is 100 mm. Calculate the velocity of the precession when it spins at 30 rev/min.

Figure 14 Diagram for worked example 5

The angle ϕ is 30^o so $T = L \sin \phi \, \omega_p = 0.5 \, L \, \omega_p$
For a plain disc $I = MR^2 / 2 = 0.5 \times (0.04)^2 / 2 = 0.0004$ kg m^2
$\omega = 2\pi \times 30 = 60\pi$ rad/s
$L = I \omega = 0.0004 \times 60\pi = 0.075$ kg m^2 /s
T is due to the weight acting at $100 \cos 30^o = 86.6$ mm from the rest point.
$T = mg \times 0.0866 = 0.5 \times 9.81 \times 0.0866 = 0.424$ Nm

$\omega_p = T / L = 0.424 / 0.0754 =$ **<u>5.63 rad/s</u>**

Worked Example 10

A cycle takes a right-hand bend at a velocity of v m/s and radius R. Show that the cyclist must lean into the bend in order to go around it.

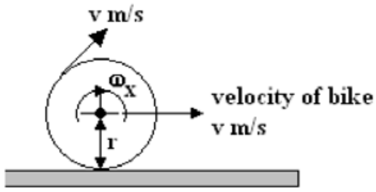

Figure 15 Diagram (a) for worked example 6

First, remember that the velocity of the edge of the wheel must be the same as the velocity of the bike. So, $\omega_x = v/r$ where r is the radius of the wheel.

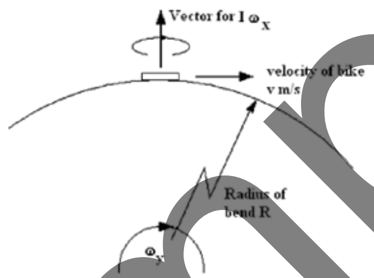

Figure 16 Diagram (b) for worked example 6

Now draw the vector diagrams to determine the change. As the wheel goes around a right hand bend the direction of the vector for ω_x changes as shown in Figure 17 (c). The applied torque is a vector in the same direction as the change, so we deduce that the torque must act clockwise viewed from behind the cyclist. This torque may be produced by leaning into the bend and letting gravity do the job. By applying this torque, the wheels must precess in the correct direction to go around the bend.

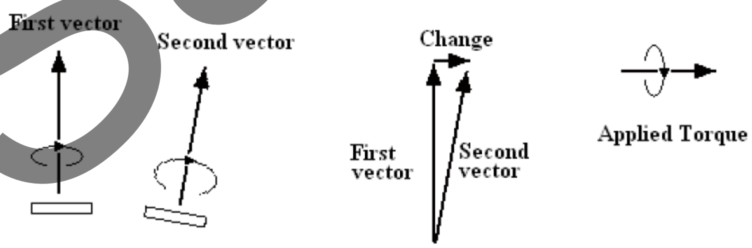

Figure 17 Diagram (c) for worked example 6

If the cyclist steers around the bend by turning the handlebars, then the reaction torque would throw him over outwards (anticlockwise as viewed from the back).