Pearson BTEC Level 4 Higher Nationals in Engineering (RQF)

Unit 8: Mechanical Principles

Unit Workbook 2

in a series of 4 for this unit

Learning Outcome 2

Dynamic Mechanical Systems

Contents

INTRODUCTION

Illustrate the effects that constraints have on the performance of a dynamic mechanical system.

- *Energy and work:*
    - The principle of conservation of energy and work-energy transfer in systems.
    - Linear and angular velocity and acceleration.
    - Velocity and acceleration diagrams of planar mechanisms.
    - Gyroscopic motion.

GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

Purpose

Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

Theory

Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

Example

The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself. Many of the examples given resemble assignment questions which will come your way, so follow them through diligently.

Question

Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence. As an Online Learner it is important that the answers to questions are immediately available to you. Contact your Unit Tutor if you need help.

Challenge

You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

Video

Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

1.3 Work Done and power transmitted by a constant torque

Figure 1 shows a pulley wheel of radius r metres attached to a shaft and a force F Newton's applied to the rim at point P.

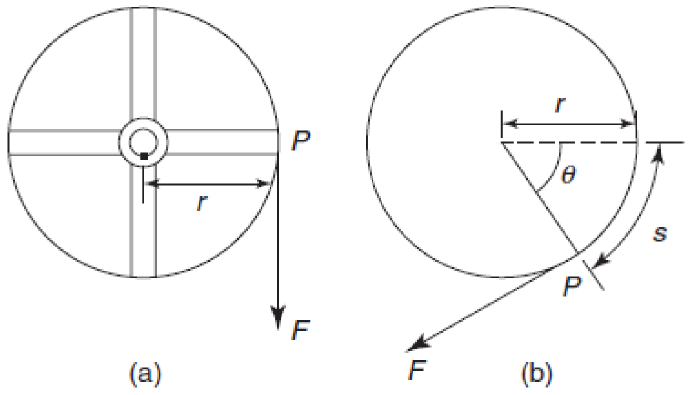

Figure 1 Pulley Wheel

Figure 1(b) shows the pulley wheel having turned through an angle θ radians as a result of the force F being applied. The force moves through a distance s, where arc length s = rθ

Work done = force × distance moved by the force = F × rθ = Frθ (N m) = Frθ (J) But, Fr is the torque T, so,

Work Done = Tθ joules

Average power = work done/time taken = = T θ/time taken for a constant torque T

However, (angle θ) / (time taken) = angular velocity, ω rad/s. Hence,

Power = Tω watts (Eq 1)

Angular velocity, ω = 2πn rad/s where n is the speed in rev/s. Hence, Power

P = 2πnT watts (Eq 2)

Sometimes power is in units of horsepower (hp), where 1 horsepower = 745.7 watts

Worked Example 1

A motor connected to a shaft develops a torque of 5 kN m. Determine the number of revolutions made by the shaft if the work done is 9 MJ.

Work Done = T θ, from which,

Angular displacement, θ = Work Done/Torque

Hence, Angular displacement, θ = 9 x 10^6 / 5000 = 1800 rad.

But 2π rad = 1 rev., hence,

No of revs made by the shaft = 1800/2π = 286.5 revs

1.4 Power transmission and efficiency

A common and simple method of transmitting power from one shaft to another is by means of a belt passing over pulley wheels which are keyed to the shafts, as shown in Figure 2. Typical applications include an electric motor driving a lathe or a drill, and an engine driving a pump or generator.

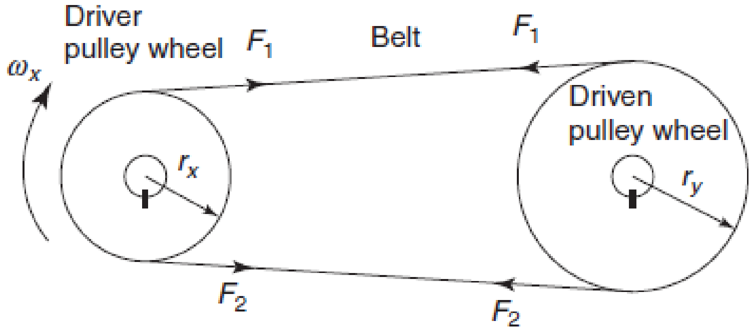

Figure 2 Pulley Belt Drive

For a belt to transmit power between two pulleys there must be a difference in tensions in the belt on either side of the driving and driven pulleys. For the direction of rotation shown in Figure 2, $F_2 > F_1$. The torque T available at the driving wheel to do work is given by:

$T = (F_2 - F_1)\, r_x$ (N m) and the available power P is given by:

$P = T\omega = (F_2 - F_1)\, r_x\omega_x$ (Watts)

The linear velocity of a point on the driver wheel is $v_x = r_x\omega_x$

Similarly, the linear velocity of a point on the driven wheel, $v_y = r_y\omega_y$.

Assuming no slipping, vx = vy and therefore, $rx\omega_x = r_y\omega_y$

Hence $r_x(2\pi n_x) = r_y\,(2\pi n_y)$ from which,

$r_x/r_y = n_y/n_x$

Percentage efficiency = useful work output / energy output $\times$ 100 or,

efficiency = power output / power input $\times$ 100%

2 Linear and angular motion.

2.1 Linear and Angular velocity

Linear velocity v is defined as the rate of change of linear displacement s with respect to time t, and for motion in a straight line:

Linear velocity = change of displacement / change of time i.e. $\mathbf{v = s/t}$ (ms^{-1}) (Eq 3)

Angular velocity is defined as the rate of change of angular displacement θ, with respect to time t, and for an object rotating about a fixed axis at a constant speed:

Angular velocity = angle turned through / time taken i.e. $\boldsymbol{\omega = \theta/t}$ (rads-1) (Eq 4)

An object rotating at a constant speed of n revolutions per second subtends an angle of 2πn radians in one second.

Hence, its angular velocity, $\boldsymbol{\omega = 2\pi n}$ $rads^{-1}$ (Eq 5)

From **s (arc length) = r (radius) θ (angle) and ω = θ/t**

$s = r\,\omega\,t$ or $s/t = \omega r$ and $v = s/t$

Hence, $\mathbf{v = \omega r}$ (Eq 6)

This equation gives the relationship between linear velocity, v and angular velocity, ω.

2.2 Linear and Angular acceleration

Linear acceleration, a, is defined as the rate of change of linear velocity with respect to time. For an object whose linear velocity is increasing uniformly:

linear acceleration = change of linear velocity/time taken i.e. $\mathbf{a = (v_2 - v_1)/t}$ (Eq 7)

The unit of linear acceleration is metres per second squared (m/s^2).

Rewriting equation (Eq 7) with v_2 as the subject of the formula gives:

$\mathbf{v_2 = v_1 + at}$ (Eq 8)

where v_2 = final velocity and v_1 = initial velocity.

Angular acceleration, α, is defined as the rate of change of angular velocity with respect to time. For an object whose angular velocity is increasing uniformly:

Angular acceleration = change of angular velocity/time taken i.e. $\boldsymbol{\alpha = (\omega_2 - \omega_1)/t}$ (Eq 9)

The unit of angular acceleration is radians per second squared (rad/s^2). Rewriting equation (Eq 9) with ω_2 as the subject of the formula gives:

$\boldsymbol{\omega_2 = \omega_1 + \alpha t}$ (Eq 10)

where ω_2 = final angular velocity and ω_1 = initial angular velocity. From equation (Eq 6), $v = \omega r$. For motion in a circle having a constant radius r, $v_2 = \omega_2 r$ and $v_1 = \omega_1 r$, hence equation (Eq 9) can be rewritten as:

$a = (\omega_2 r - \omega_1 r)/t = r\,(\omega_2 - \omega_1)/t$

But from equation (Eq 9), $(\omega_2 - \omega_1)/t = \alpha$, Hence $a = r\,\alpha$ (Eq 11)

2.3 Relative Velocity

Quantities used in engineering and science can be divided into two groups:

Scalar quantities have a size or magnitude only and need no other information to specify them. Thus 20 centimetres, 5 seconds, 3 litres and 4 kilograms are all examples of scalar quantities.