Pearson BTEC Level 4 Higher Nationals in Engineering (RQF)

Unit 8: Mechanical Principles

Unit Workbook 3

in a series of 4 for this unit

Learning Outcome 3

Mechanical Power Transmissions Systems

1.2 Simple Systems

1.2.1 Belt Drive Dynamics

Purpose

Understand flat and v-section belts; limiting coefficient friction; limiting slack and tight side tensions; initial tension requirements; maximum power transmitted

Theory Revision

Consider the basic belt drive arrangement below…

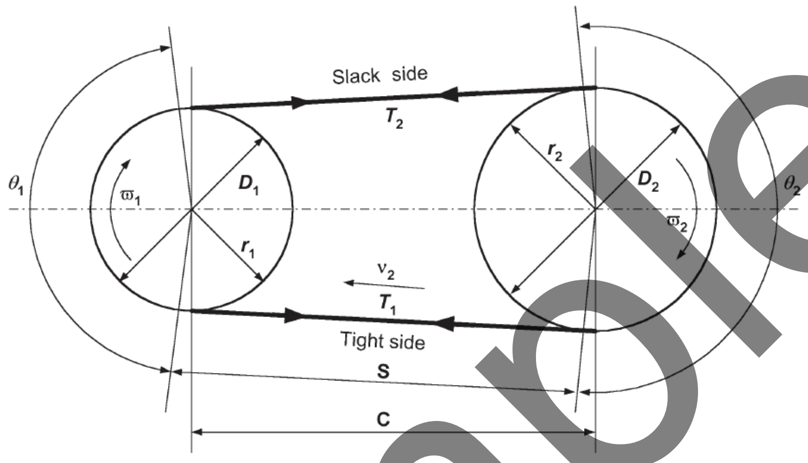

Figure 1 A basic belt drive arrangement

The belt drive arrangement in *Figure 1* has the driving side on the left and the driven side on the right. This causes the bottom of the belt to be the tight side and the top to be the slack side. Some parameters of note in this arrangement…

D_1, D_2	Respective diameters of the pulleys
r_1, r_2	Respective radii of the pulleys
ω_1, ω_2	Angular velocity of each pulley (in radians per second)
θ_1, θ_2	Lap angle (angle subtended to centre of pulley by contact length of belt with pulley surface)
T_1, T_2	Belt tensions on tight side and slack side, respectively
S	Belt length which does not touch the pulleys
C	Distance between pulley centres
v_2	Linear belt velocity

Many formulae may be derived for such a pulley system, but the most important ones are …

<u>Belt lap angles for pulleys</u>

$$\vartheta_1 = 180^o - 2\,sin^{-1}\left\{\frac{D_2 - D_1}{2C}\right\} \qquad \text{(Eq 3)}$$

$$\vartheta_2 = 180^o + 2\,sin^{-1}\left\{\frac{D_2 - D_1}{2C}\right\} \qquad \text{(Eq 4)}$$

Power transmitted by the system

$$\boldsymbol{P = T_1\left(1 - e^{-\mu\theta}\right)v} \qquad \text{(Eq 5)}$$

where: -

$v = r\omega$

μ is the coefficient of friction between the belt and the pulley

Belt pitch length

$$\boldsymbol{L = 2C + 1.57(D_2 - D_1) + \frac{(D_2 - D_1)^2}{4C}} \qquad \text{(Eq 6)}$$

Span length

$$\boldsymbol{S = \sqrt{C^2 - \left[\frac{D_2 - D_1}{2}\right]^2}} \qquad \text{(Eq 7)}$$

Worked Example 2

A flat belt drive system consists of two parallel pulleys of diameter 300 and 500 mm, which have a distance between centres of 600 mm. Given that the maximum belt tension is not to exceed 1.5 kN, the coefficient of friction between the belt and pulley is 0.3 and the larger pulley rotates at 40 rad/sec. Find;

a) the belt lap angles for the pulleys
b) the power transmitted by the system
c) the belt pitch length *L*
d) the pulley system span length between centres

ANSWERS

(a)

The belt lap angles for the pulleys are given by equations (Eq 3) and (Eq 4) …

$$\vartheta_1 = 180^o - 2\,sin^{-1}\left\{\frac{D_2 - D_1}{2C}\right\} = 180^o - 2\,sin^{-1}\left\{\frac{0.5 - 0.3}{2(0.6)}\right\} = \mathbf{160.8^o}$$

$$\vartheta_2 = 180^o + 2\,sin^{-1}\left\{\frac{D_2 - D_1}{2C}\right\} = 180^o + 2\,sin^{-1}\left\{\frac{0.5 - 0.3}{2(0.6)}\right\} = \mathbf{199.2^o}$$

(b)

The power transmitted by the system is given by equation (Eq 5) …

$$P = T_1\left(1 - e^{-\mu\theta}\right)v$$

where: -

$$v = r\omega = (0.25)(40) = 10\ m.s^{-1}$$

The angle θ_1 is expressed in degrees in part (a) but we must convert this to radians to be compatible with equation (Eq 5) …

$$160.8^o \equiv \left(160.8 \times \frac{\pi}{180}\right) \; radians = 2.81 \; rads.$$

$$\therefore \quad P = T_1\left(1 - e^{-\mu\theta}\right)v = 1500\left(1 - e^{-(0.3)(2.81)}\right)(10) = \mathbf{8.54 \; kW}$$

(c)

The belt pitch length is given by equation (Eq 6) …

$$L = 2C + 1.57(D_2 - D_1) + \frac{(D_2 - D_1)^2}{4C} = 2(0.6) + 1.57(0.5 - 0.3) + \frac{(0.5 - 0.3)^2}{4(0.6)} = \mathbf{1.53 \; m}$$

(d)

Pulley system span length is given by equation (Eq 7) …

$$S = \sqrt{C^2 - \left[\frac{D_2 - D_1}{2}\right]^2} = \sqrt{0.6^2 - \left[\frac{0.5 - 0.3}{2}\right]^2} = \mathbf{0.59 \; m}$$

Additional worked examples are available in the eBooks section on Moodle.

1.2.2 Friction Clutches

Purpose

Understand flat single and multi-plate clutches; conical clutches; coefficient of friction; spring force requirements; maximum power transmitted by constant wear and constant pressure theories; validity of theories

Theory Revision

Consider the friction clutch shown below …

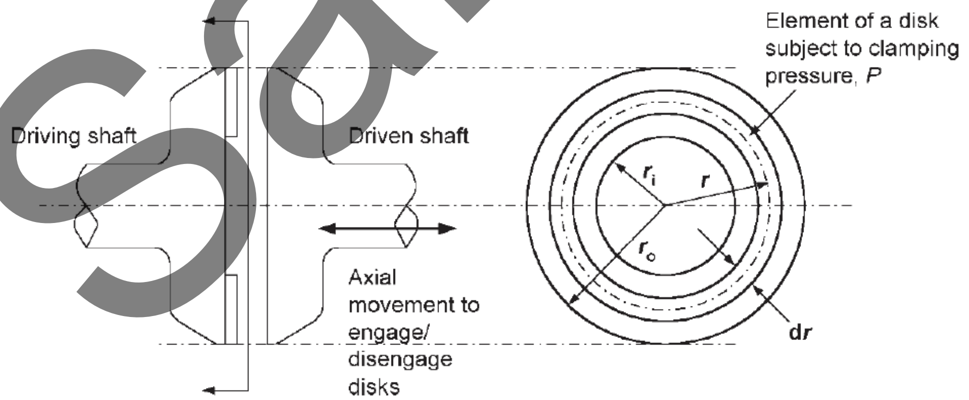

Figure 2 A basic friction clutch

The total number of discs required (N) on a multiple-plate clutch is given by …

$$N = \frac{T}{\pi r_i p_{max} \mu ({r_0}^2 - {r_i}^2)} \qquad \text{(Eq 8)}$$

where: -

N	number of clutch discs required (rounded up to nearest integer)
T	transmitted torque
p_{max}	maximum allowable pressure for the friction surface
μ	coefficient of friction between rubbing surfaces
r_0	external radius of friction plate
r_1	internal radius of friction plate

Also, the axial clamping force (W) is given by …

$$W = \frac{2T}{\mu N (r_o - r_i)} \qquad \text{(Eq 9)}$$

Worked Example 3

A multiple-plate clutch needs to be able to transmit a torque of 150 Nm. The external and internal diameters of the friction plates are 110 mm and 56 mm, respectively. If the friction coefficient between rubbing surfaces is 0.3 and $p_{\max}$ = 1.3 MPa, determine;

a) the total number of discs required,
b) the axial clamping force.

ANSWERS

(a)

The total number of discs required is given by equation (Eq 8) …

$$N = \frac{T}{\pi r_i p_{max} \mu ({r_0}^2 - {r_i}^2)} = \frac{150}{\pi (0.028)(1.3\times 10^6)(0.3)(0.055^2 - 0.028^2)} = 1.95$$

Since N must be an integer, we round up, giving **N = 2**

(b)

The axial clamping force is given by equation (Eq 9) …

$$W = \frac{2T}{\mu N (r_o - r_i)} = \frac{2(150)}{(0.3)(2)(0.055 - 0.028)} = \mathbf{18.5\ kN}$$

Additional worked examples are available in the eBooks section on Moodle.

1.3 Gear Trains

Purpose

Understand compound and epicycle gear trains; velocity ratios; torque, speed and power relationships; efficiency; fixing torques

1.3.1 Simple Gear Trains

A simple gear train is used to transmit rotary motion and can change both the magnitude and the line of action of a force, hence is a simple machine. The gear train shown in Figure 3 consists of spur gears and has an effort applied to one gear, called the driver, and a load applied to the other gear, called the follower.

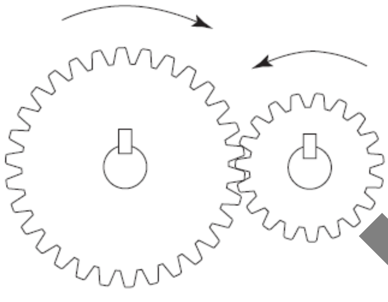

Figure 3 Simple Spur Gears

In such a system, the teeth on the wheels are so spaced that they exactly fill the circumference with a whole number of identical teeth, and the teeth on the driver and follower mesh without interference.
Under these conditions, the number of teeth on the driver and follower are in direct proportion to the circumference of these wheels, i.e.
(no of teeth on driver) / (no of teeth on follower) = (driver circumference) / (follower circumference)
When the same direction of rotation is required on both the driver and the follower an idler wheel is used as shown in Figure 4.

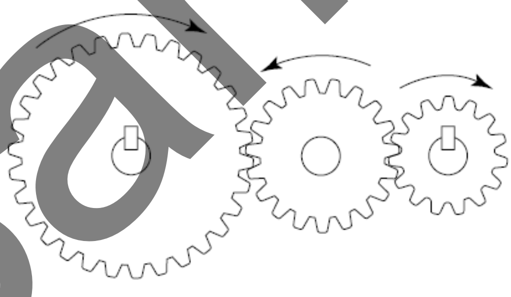

Figure 4 Simple Gear Train with Idler Wheel

1.3.2 Compound Gear Trains

A compound gear train has gear wheels fixed to the same shaft.

Theory Revision

Consider the compound gear train given below.

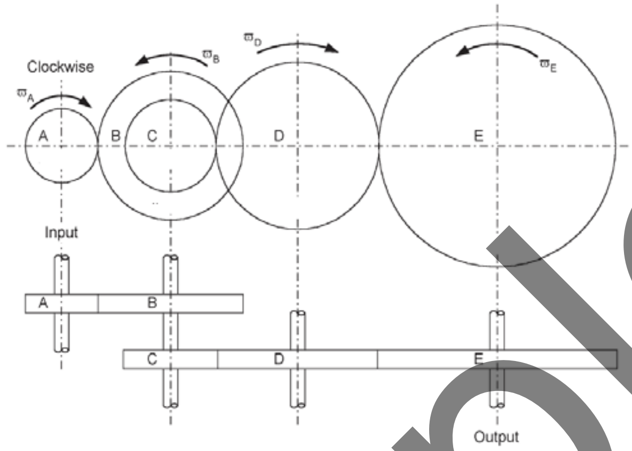

Figure 5 A compound gear train

For such a system of gears, we may determine the overall gear ratio as follows: -

$$\boldsymbol{G = \frac{\omega_A}{\omega_E} = \frac{\omega_A}{\omega_B} \times \frac{\omega_B}{\omega_C} \times \frac{\omega_C}{\omega_D} \times \frac{\omega_D}{\omega_E}} \qquad \text{(Eq 10)}$$

$$now, since\ \omega_B = \omega_C\ (same\ shaft) \ldots$$

$$G = \frac{\omega_A}{\omega_E} = \frac{\omega_A}{\omega_B} \times 1 \times \frac{\omega_C}{\omega_D} \times \frac{\omega_D}{\omega_E}$$

$$\therefore \quad \frac{\omega_A}{\omega_E} = \frac{\omega_A}{\omega_B} \times \frac{\omega_C}{\omega_D} \times \frac{\omega_D}{\omega_E}$$

For two neighbouring (touching) gears we may say that the product of 'number of teeth' (N) and angular velocity (ω) are equal. For example, consider gear 1 and gear 2 …

$$N_1\omega_1 = N_2\omega_2$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1}$$

$$\therefore \quad \frac{\omega_A}{\omega_E} = \frac{N_B}{N_A} \times \frac{N_D}{N_C} \times \frac{N_E}{N_D}$$

Thos two N_D terms cancel, and, transposing, gives;

$$\boldsymbol{\omega_E = \frac{N_A N_C}{N_B N_E}\omega_A} \qquad \text{(Eq 11)}$$

Worked Example 4

Consider the compound gear train given below …

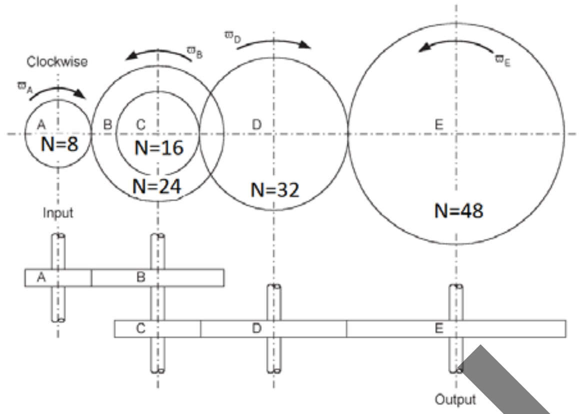

Figure 6 An example compound gear train (Worked Example 3)

The system consists of five gears, two of which, gears B and C, are on the same shaft. If gear A drives the system at 300 rpm (clockwise) and the number of teeth (N) on each gear are as shown, determine;

a) The angular velocity of output gear E.
b) The system gear ratio.

ANSWERS

(a)

The angular velocity of output gear E is given by equation (Eq 11) …

$$\omega_E = \frac{N_A N_C}{N_B N_E}\omega_A = \frac{(8)(16)}{(24)(48)}(300) = \mathbf{33.\dot{3}\ rpm}$$

(b)

The gear ratio (G) is given by …

$$G = \frac{\omega_A}{\omega_E} = \frac{300}{33.\dot{3}} = \mathbf{9}$$

Worked Example 5

A gear box has an input speed of 1500 rev/min clockwise and an output speed of 300 rev/min ant-iclockwise. The input power is 20 kW and the efficiency is 70%. Determine the following.

a) The gear ratio
b) The input torque.
c) The output power.
d) The output torque.
e) The holding torque.

a) **Gear Ratio** = Input Speed / Output Speed = N_1 / N_2 = 1500 / 300 = **5**

b) Power in = $(2\pi N_1 T_1) / 60$ where N_1 is the Input Speed and T_1 the **Input Torque**
T_1 = (60 x Power In) / $(2\pi N_1)$ = (60 x 20,000) / (2π x 1500) = **127.3 Nm (clockwise)**

c) Efficiency η = 0.7 = Power Out / Power In,
Power Out = 0.7 x Power In = 0.7 x 20 = **14 kW**

d) Power Out = $(2\pi N_2 T_2) / 60$ so **Output Torque** T2 = (Power Out x 60) / $(2\pi N_2)$
= (60 x 14000) / (2π x 300) = **445.6 Nm (anticlockwise)**

e) Because the torque in and out are different, this gear box must be clamped to stop the casing rotating, that is, a holding torque T_3 must be applied to the body through clamps.
The Total Torque must add up to zero; $T_1 + T_2 + T_3 = 0$
If we use the convention that anti-clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.
Here $T_1 + T_2 + T_3 = 0$
So, +127.3 +(-445.6) +T_3 = 0, So **Holding Torque**, T_3 = **318.3 Nm i.e 318.3 Nm clockwise**

Additional worked examples are available in the eBooks section on Moodle.

1.3.2 Epicyclic Gear Trains

Theory Revision

Epicyclic gears are allowed to physically rotate around other gears. These include …

- A **sun gear**
- A **link arm**
- A **planet gear**(s) – one or more of these are possible
- An **annulus** – a ring with internal gear teeth, meshing with the planet teeth

Video 1

Worked Example 5

A single-stage epicyclic gearbox has the following properties …

- 100% efficiency
- Input shaft – rotating at 1000 rpm and a torque of 150 Nm
- Sun gear – with 60 teeth (driven by the input shaft)
- Planet gear – with 20 teeth
- Annulus gear – with 100 teeth
- Link arm – attached to the planet
- Output shaft – attached to the link arm

Find;

a) The output shaft rpm
b) The torque on the output shaft
c) The power produced at the output

ANSWERS

(a) The sequence of operations needed to find the output shaft rpm are most easily described by using a table …

Steps Needed	Sun	Link	Annulus
Rotate all parts by 'a' revolutions	**+a**	**+a**	**+a**
Hold the link steady and turn the Sun by '+b' revolutions	**+b**	**0**	$-b\left(\frac{t_{sun}}{t_{planet}} \cdot \frac{t_{planet}}{t_{annulus}}\right)$ $= -b\left(\frac{t_{sun}}{t_{annulus}}\right)$ $= -b\left(\frac{60}{100}\right)$ $= \mathbf{-0.6\,b}$
Add	a + b	a	$a - 0.6\,b$
Real	1000	rpm for the link is calculated below.	0 (fixed)

Watch the video below for a detailed explanation of the above steps.

Video 2

Calculating the link rpm …

$a + b = 1000\ (noted\ from\ the\ Sun\ column)$ (Eq 12)

$a - 0.6\,b = 0\ (noted\ from\ the\ Annulus\ column)$ (Eq 13)

From equation (Eq 13): $b = \frac{a}{0.6}$

Substitute this result into [10]: $a + b = 1000 \quad \therefore \quad a + \frac{a}{0.6} = 1000 \quad \therefore \quad a\left(1 + \frac{1}{0.6}\right) = 1000$

$\therefore \quad a = \frac{1000}{\left(1+\frac{1}{0.6}\right)} = 375\ rpm$

So, since the link is connected to the output shaft, **the output shaft rotates at 375 rpm**.

(b)

Since the system is quoted as 100% efficient, the input power must equal the output power, i.e.

$$P_{IN} = P_{OUT}$$

We must convert the rpm figures into seconds by dividing by 60, of course. Shaft power is given by the product of angular velocity and torque, therefore …

$$\left(2\pi \cdot \frac{1000}{60}\right)(150) = \left(2\pi \cdot \frac{375}{60}\right)(Torque_{out})$$

Dividing both sides by 2π and multiplying both sides by 60 will simplify the expression to …

$$(1000)(150) = (375)(Torque_{out})$$

$$\therefore Torque_{out} = \frac{(1000)(150)}{375} = 400\ Nm$$

(c)

The output power is easy, since the system is 100% efficient, it's the same as the input power …

$$Output\ power = \left(2\pi \cdot \frac{375}{60}\right)(400) = \mathbf{15.7\ kW}$$

Additional worked examples are available in the eBooks section on Moodle.

1.4 Lead Screws and Screw Jacks

1.4.1 Efficiency of lead screws and screw jacks.

A simple screw-jack is shown in Figure 7 and is a simple machine since it changes both the magnitude and the line of action of a force.

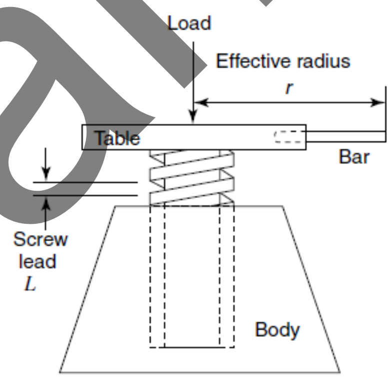

Figure 7 A Simple Screw Jack

The screw of the table of the jack is located in a fixed nut in the body of the jack. As the table is rotated by means of a bar, it raises or lowers a load placed on the table. For a single-start thread, as shown, for one complete revolution of the table, the effort moves through a distance 2πr, and the load moves through a distance equal to the lead of the screw, say, L.

Movement Ratio = (2πr) / L (Eq 14)