Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 8: Mechanical Principles

Unit Workbook 4

in a series of 4 for this unit

Learning Outcome 4

Translational and Rotational Mass-Spring Systems

Contents

INTRODUCTION

Analyse natural and damped vibrations within translational and rotational mass-spring systems

- *Types of motion:*
  - Simple harmonic motion.
  - Natural frequency of vibration in mass-spring systems.
- *Damped systems:*
  - Frequency of damped vibrations in mass-spring-damper systems.
  - The conditions for an external force to produce resonance.

GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

Purpose

Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

Theory

Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

Example

The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself. Many of the examples given resemble assignment questions which will come your way, so follow them through diligently.

Question

Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence. As an Online Learner it is important that the answers to questions are immediately available to you. Contact your Unit Tutor if you need help.

Challenge

You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

Video

Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

1 Types of motion:

1.1 Simple harmonic motion.

A particle is said to be under Simple Harmonic Motion (SHM) if its acceleration along a line is directly proportional to its displacement from a fixed point on that line.
Consider the motion of a particle A, rotating in a circle with a constant angular velocity ω, as shown in Figure 1 (a).

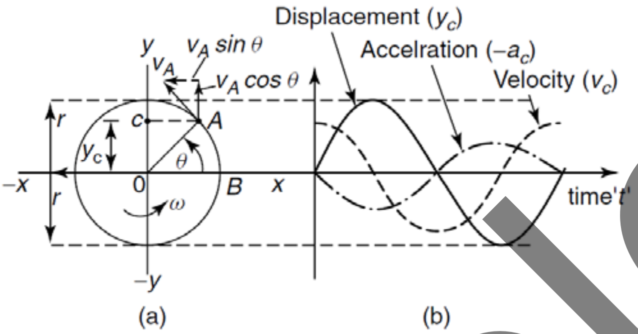

Figure 1 Simple Harmonic Motion

Consider now the vertical displacement of A from the x-axis, as shown by the distance y_C. If P is rotating at a constant angular velocity ω then the periodic time τ to travel an angular distance of 2π, is given by:

$$\boldsymbol{\tau = \frac{2\pi}{\omega}} \qquad \text{(Eq 1)}$$

Let f = frequency of motion C (in Hertz), where

$$\boldsymbol{f = \frac{1}{t} = \frac{\omega}{2\pi}} \qquad \text{(Eq 2)}$$

To determine whether SHM is taking place, consider the motion of A in the vertical direction (y-axis).
Now y_C = OA sin ωt, i.e.,

$$\boldsymbol{y = r\sin\omega t,} \text{ where t = time in seconds} \qquad \text{(Eq 3)}$$

Plotting of equation (Eq 3) against t results in the sinusoidal variation for displacement, as shown in Figure 1 (b).
We know that $\mathbf{v_A = \omega r}$, which is the tangential velocity of the particle A. From the velocity vector diagram, at the point A on the circle of Figure 1 (a),

$$\boldsymbol{v_c = v_A\cos\theta = v_A\cos\omega t} \qquad \text{(Eq 4)}$$

Plotting of equation (Eq 4) against t results in the sinusoidal variation for the velocity v_C, as shown in Figure 1 (b).
The centripetal acceleration of A $= a_A = \omega^2 r$

Now $\quad a_C = -a_A\sin\theta, \qquad \therefore \boldsymbol{a_C = -\omega^2 r\sin\omega t}$ (Eq 5)

Plotting equation (Eq 5) against t results in the sinusoidal variation for the acceleration at C, a_C, as shown in Figure 1 (b).

Substituting equation (Eq 3) into equation (Eq 5) gives:

$$\boldsymbol{a_C = -\omega^2 y_C} \qquad \text{(Eq 6)}$$

Equation (Eq 6) shows that the acceleration along the y-axis is directly proportional to the displacement along this line, therefore the point C is moving with SHM. Now,

$T = \frac{2\pi}{\omega}$, but from equation (Eq 6) $\quad a_C = -\omega^2 y_C \quad$ i.e. $\quad \omega^2 = \frac{a}{y}$

Therefore, $\quad T = \frac{2\pi}{\sqrt{\frac{a}{y}}} \quad$ or $\quad T = 2\pi\sqrt{\frac{y}{a}} \quad$ i.e. $\quad T = 2\pi\sqrt{\frac{displacemet}{acceleration}}$

In general, from equation (Eq 6) $\quad \boldsymbol{a + \omega^2 y = 0} \quad$ (Eq 7)

1.2 Natural frequency of vibration in mass-spring systems.

1.2.1 The Spring-Mass System vibrating horizontally

Consider a mass m resting on a smooth surface and attached to a spring of stiffness k, as shown in Figure 2.

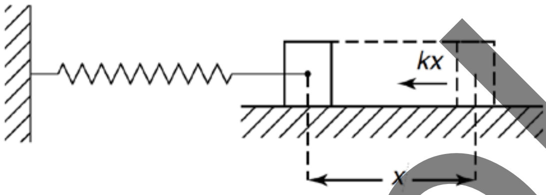

Figure 2 Horizontal Spring-Mass System

If the mass is given a small displacement x, the spring will exert a resisting force of kx,

i.e. $\quad F = -kx \quad$ But $\quad F = ma \quad$ Hence, $\quad ma = -kx$

or, $\quad ma + kx = 0 \quad$ or, $\quad \boldsymbol{a + \frac{k}{m}x = 0} \quad$ (Eq 8)

Equation (Eq 8) shows that this mass is oscillating (or vibrating) in SHM, or according to equation (Eq 7).

Comparing (Eq 7) with (Eq 8), we see that; $\quad \omega^2 = \frac{k}{m} \quad$ from which $\quad \omega = \sqrt{\frac{k}{m}}$

Now $\quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad$ and f = frequency of oscillation or vibration.

i.e. $\quad \boldsymbol{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}} \quad$ (Eq 9)

1.2.1 The Spring-Mass System vibrating horizontally

Consider a mass m, supported by a vertical spring of stiffness k, as shown in Figure 3. In this equilibrium position, the mass has an initial downward static deflection of y_o. If the mass is given an additional downward displacement of y and then released, it will vibrate vertically.