Pearson BTEC Levels 4 Higher Nationals in Engineering (RQF)

Unit 29: Electro, Pneumatic and Hydraulic Systems

Unit Workbook 1

in a series of 4 for this unit

Learning Outcome 1

Parameters of Pneumatic and Hydraulic Systems

Pneumatic and hydraulic theory:

Combined and ideal gas laws: Boyle's Law, Charles' Law and Gay-Lussac's Law

Boyle’s Law

Boyle’s Law describes, quite simply, how gas pressure is inversely proportional to the volume of the container which it occupies. Put mathematically…

$$P \propto \frac{1}{V}$$

where P is gas pressure and V is volume of the container. Temperature must remain constant (isothermal process)

This is a very intuitive relationship. If you blow up a balloon and then sit on it, well, you are squashing (reducing the volume) of the balloon and therefore expect the air pressure inside to increase. The balloon is very likely to burst as a result.

Let’s take away the proportionality term in the middle and form our first equation…

$$PV = k \quad [1]$$

k is a constant, so it represents the product of pressure and volume.

If we have the same mass of gas (air in this case, same number of molecules) but place it into a second much larger balloon, we will still have the same product of pressure and volume…

$$P_1V_1 = P_2V_2 \quad [2]$$

There could be a gas supply feeding your home via a large pipe (subscript 1 in equation 2). As the gas enters the property it is usual for it to enter a gas meter and then exit the meter via a gas pipe of smaller radius (subscript 2 in equation 2). Boyle's law (equation 1) tells us that the product of pressure and volume in the larger pipe is equal to the product of the pressure and volume in the smaller pipe. The smaller pipe has lower volume; therefore, the smaller pipe has a larger gas pressure.

Worked Example 1

A gas in a piston occupies a volume of 0.2 m^3 at a pressure of 1.6 MPa. Determine;

a) The gas pressure if the volume is changed to 0.08 m^3 at constant temperature.
b) The volume if the gas pressure is changed to 3.2 MPa at constant temperature.

ANSWERS

a)

$$P_1V_1 = P_2V_2$$

$$P_1 = 1.6 \times 10^6;\ V_1 = 0.2;\ P_2 = ?;\ V_2 = 0.08$$

$$P_2 = \frac{P_1V_1}{V_2} = \frac{(1.6 \times 10^6)(0.2)}{(0.08)} = \mathbf{4\ MPa}$$

b)

$$P_1V_1 = P_2V_2$$

$$P_1 = 1.6 \times 10^6;\ V_1 = 0.2;\ P_2 = 3.2\ MPa;\ V_2 = ?$$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{(1.6 \times 10^6)(0.2)}{(3.2 \times 10^6)} = \mathbf{0.1\ m^3}$$

Charles' Law

Now that we appreciate Boyle's Law, let's wonder what might happen if we were to heat the gas (air). Well, blow up your balloon and place it in the fridge for a while. Afterwards you will notice that the volume of the balloon has decreased. The opposite applies if you place the balloon in the oven for a while; it increases in volume. What is going on here? Atomically, all of those air molecules become very active when in the oven and bounce off each other like crazy, causing collisions and expansion. This is effectively what Charles' Law says…

$$V \propto T \quad [3]$$

This says that the volume of a gas is proportional to its temperature. OK, let's strip away the constant of proportionality again and write down Charles' Law…

$$\frac{V}{T} \propto k \quad [4]$$

This tells us that gas volume divided by temperature is constant. For the same mass of gas (air) this relationship holds good under different conditions. We can now write Charles' Law…

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad or, more\ simply: V_1T_2 = V_2T_1 \quad [5]$$

Gay-Lussac's Law

This law is analogous to Charles' Law, but says that the pressure of a gas is proportional to the gas temperature – **IF** we hold the volume and mass of the gas constant.

$$P \propto T \quad [6]$$

Now we strip-away the constant of proportionality once more…

$$\frac{P}{T} = k \quad [7]$$

We now use previous analogies to arrive at…

$$P_1T_2 = P_2T_1 \quad [8]$$

Fluid mechanics deals with both gases and liquids. Where liquids are concerned we are looking at water, or, more probably, oils, for hydraulic systems. Gases are compressible, of course, as seen in the previous balloon experiment. For ease of calculation we normally prefer to treat liquids as incompressible, but, in practice, this is not the case.

We need to remember how a liquid is formed; molecules closely arranged, basically. And, what is a molecule? Well, it is simply a combination of atoms. In the case of water, its chemical symbol is H_2O and this means that two hydrogen atoms are bonded with one oxygen atom to form a water molecule. Most of the volume occupied by an atom is empty space (almost 100% of it, in fact) so it seems logical that this space can be squeezed (pressurised) to shrink in volume somewhat.

Strange things can happen to atoms (molecules) when they are subjected to intense pressures. The majority of the inner core of the planet Jupiter consists of hydrogen atoms. These atoms are under such extreme pressure that the hydrogen behaves like a metal; hence the massive magnetic field which Jupiter exerts (the largest magnetic field of all the planets in our solar system). Let's look at a formula which governs the change in pressure and volume relationship for a liquid then…

$$\frac{\Delta V}{V} = -\frac{\Delta P}{E} \qquad [9]$$

where;

$\Delta V \textit{ is the reduction (hence the minus sign) in the volume of liquid}$

$V \textit{ is the original liquid volume}$

$\Delta P \textit{ is the increase in pressure}$

$E \textit{ is the elasticity of the liquid}$

We may now rearrange equation [9] …

$$\Delta V = -\frac{V\ \Delta P}{E} \qquad [10]$$

and

$$\Delta P = -\frac{E\ \Delta V}{V} \qquad [11]$$

Let's take a look at a worked example…

Worked Example 2

Water, with an elasticity, E, of 24000 bar, and a volume of 0.1 m³ rests in a hydraulic pipe, Determine;

c) The change in water pressure if the volume is reduced by 0.01 m³ at constant temperature.
d) The change in water volume if the water pressure is changed by 1.2 kPa at constant temperature.

ANSWERS

a)

$$\frac{\Delta V}{V} = -\frac{\Delta P}{E}$$

$$\therefore \quad \Delta P = -\frac{E\,\Delta V}{V} = -\frac{24000 \times (-0.01)}{0.1} = \mathbf{+2400\ Pa}$$

b)

$$\frac{\Delta V}{V} = -\frac{\Delta P}{E}$$

$$\therefore \quad \Delta V = -\frac{V\,\Delta P}{E} = -\frac{0.1 \times 1.2 \times 10^3}{24000} = \mathbf{-0.005\ m^3}$$

Fluid flow, calculation of pressure and velocity using Bernoulli's Equation for Newtonian fluids

Most fluids are (practically) incompressible (you cannot easily squash them into a smaller volume). Try filling a plastic bottle full to the top and replacing the lid tightly, no air allowed. Then try compressing the bottle; you cannot easily compress the fluid. If it was a bottle full of air, then it would be easy to compress.

This principle of constancy in an incompressible fluid, whether static, or in motion, is borne out by Bernoulli's equation…

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = constant \quad [12]$$

where;

v is fluid velocity

g is the acceleration due to gravity

z is the elevation of the centre of the fluid above a reference plane

p is the fluid pressure at the chosen point

ρ is the density of the fluid at all points

We need to account for friction losses when the fluid flows from one section of a pipe (section 1) to another section of a pipe (section 2) as follows…

$$\frac{P_1}{\rho} + \frac{{v_1}^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{{v_2}^2}{2} + g\left(z_2 + h_f\right) \quad [13]$$

where;

$h_f\ are\ the\ frictional\ losses\ (in\ metres)\ due\ to\ flow\ from\ section\ 1\ to\ 2$

Let's see how we might use Bernoulli's equation in a practical example…

Worked Example 3

An oil storage tank contains an outlet pipe at the bottom. The tank is filled with oil to a depth (head) of 5 metres above the outlet pipe.

Assuming a tap is opened at the outlet pipe, determine the velocity of oil through this pipe, given that the losses at the entry to the pipe, due to friction, are $gh_f = 0.4v^2$

ANSWER

Assume v_2 is the oil velocity through the outlet pipe

Using Bernoulli's equation…

$$\frac{P_1}{\rho} + \frac{{v_1}^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{{v_2}^2}{2} + g(z_2 + h_f)$$

$$\therefore \quad \frac{P_1}{\rho} + \frac{{v_1}^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{{v_2}^2}{2} + gz_2 + 0.4{v_2}^2$$

$$\therefore \quad 0 + 0 + g(5) = 0 + \frac{{v_2}^2}{2} + 0 + 0.4{v_2}^2$$

$$\therefore \quad 5g = \frac{{v_2}^2}{2} + 0.4{v_2}^2$$

$$\therefore \quad {v_2}^2 = \frac{5g}{\frac{1}{2} + 0.4}$$

$$\therefore \quad {v_2}^2 = \frac{5 \times 9.81}{0.5 + 0.4} = \frac{49.05}{0.9} = 54.5$$

$$\therefore \quad v_2 = \sqrt{54.5} = \mathbf{7.38\ m/s}$$