Pearson BTEC Level 4 Higher Nationals in Engineering (RQF)

Unit 11: Fluid Mechanics

Unit Workbook 1

in a series of 4 for this unit

Learning Outcome 1

Static Fluid Systems

Contents

INTRODUCTION

Determine the behavioural characteristics of static fluid systems

- Pressure and force:
  - How Pascal's laws define hydrostatic pressure.
  - Pressure with the use of manometers.
  - Transmission of force in hydraulic devices.
- Submerged surfaces:
  - Determining thrust on immersed surfaces.
  - Moments of area and parallel axis theorem.
  - Calculating centre of pressure with moments of area.

GUIDANCE

This document is prepared to break the unit material down into bite size chunks. You will see the learning outcomes above treated in their own sections. Therein you will encounter the following structures;

Purpose

Explains *why* you need to study the current section of material. Quite often learners are put off by material which does not initially seem to be relevant to a topic or profession. Once you understand the importance of new learning or theory you will embrace the concepts more readily.

Theory

Conveys new material to you in a straightforward fashion. To support the treatments in this section you are strongly advised to follow the given hyperlinks, which may be useful documents or applications on the web.

Example

The examples/worked examples are presented in a knowledge-building order. Make sure you follow them all through. If you are feeling confident then you might like to treat an example as a question, in which case cover it up and have a go yourself. Many of the examples given resemble assignment questions which will come your way, so follow them through diligently.

Question

Questions should not be avoided if you are determined to learn. Please do take the time to tackle each of the given questions, in the order in which they are presented. The order is important, as further knowledge and confidence is built upon previous knowledge and confidence. As an Online Learner it is important that the answers to questions are immediately available to you. Contact your Unit Tutor if you need help.

Challenge

You can really cement your new knowledge by undertaking the challenges. A challenge could be to download software and perform an exercise. An alternative challenge might involve a practical activity or other form of research.

Video

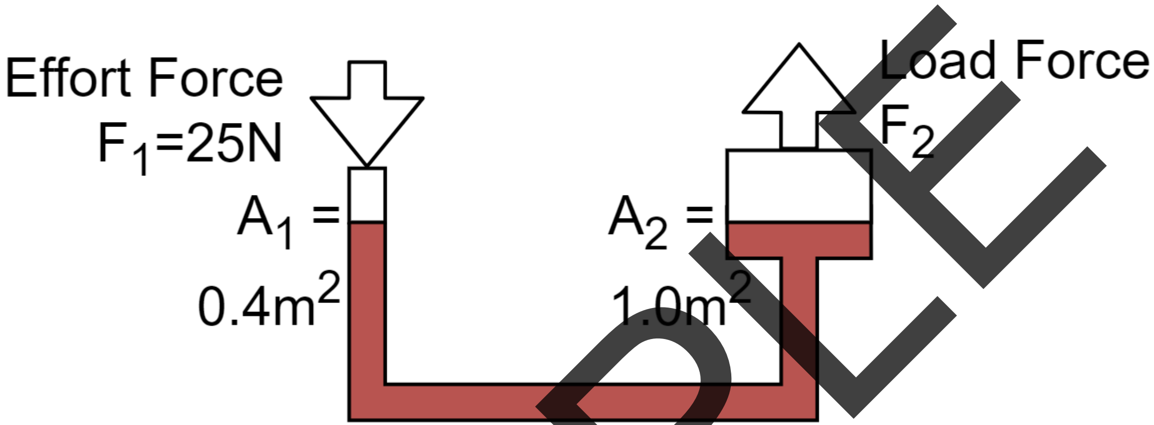
Videos on the web can be very useful supplements to your distance learning efforts. Wherever an online video(s) will help you then it will be hyperlinked at the appropriate point.

$$F_2 = F_1 \cdot \frac{A_2}{A_1} \quad (1.2)$$

Example 1

A master piston, which has a cross sectional area of 0.4m^2 receives an effort force of 25N to push a slave piston with a cross-sectional area of 1.0m^2. Calculate:

1. The pressure created on the fluid by the effort force.
2. The load force exerted.



1. The pressure is given as:

$$P = \frac{F_1}{A_1} = \frac{25}{0.4} = 62.5\ \text{Nm}^{-2}$$

2. Since this is a closed system, the resultant load force can be calculated as:

$$F_2 = P \cdot A_2 = 62.5 \cdot 1.0 = 62.5\ \text{N}$$

Hydraulics can be found in a lot of heavy equipment, such as cranes, diggers, etc. However, one of the most common uses is found in the brakes of a car. As the driver pushes the pedal, the small master cylinder pushes down on the hydraulic fluid, which will then push the slave pistons and clamp down on the brake discs, which will slow the turning speed of the wheels. A basic schematic of a car's brakes can be seen in Fig.1.2.

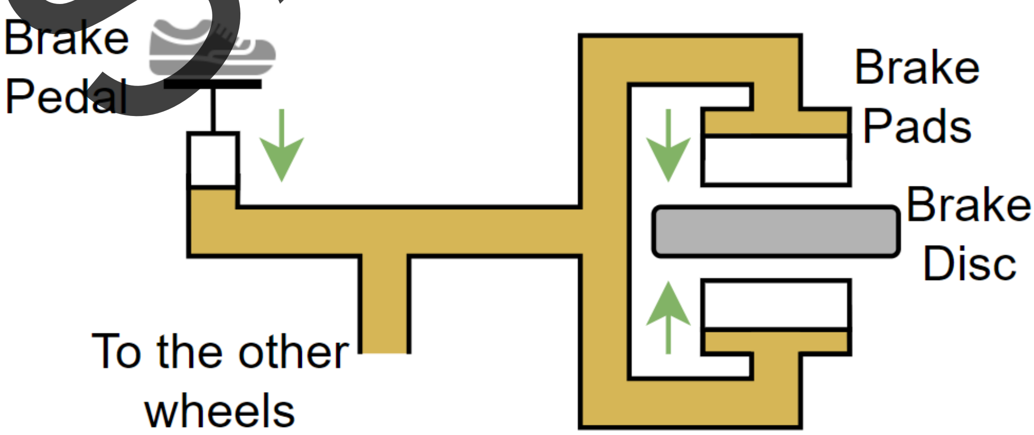


Figure 1.2: A schematic of a vehicle's braking system

Example 2

A manometer is showing a height difference of $+3\ \text{cm}$, the working fluid is mercury ($13{,}560\ \text{kg/m}^3$). Calculate the absolute pressure of the system if:

a) The manometer is sealed off to a vacuum
b) The manometer is open to atmospheric pressure ($101.3\ \text{kPa}$)

You are to assume that acceleration due to gravity is $9.81\ \text{m/s}^2$

Answer:

a) If the manometer is sealed off to a vacuum, then the absolute pressure is given as:

$$P_{abs} = \rho gh = 13560 \times 9.81 \times 0.03 = 3990.7\ \text{Pa}$$

b) The manometer is open to atmospheric pressure, so absolute pressure can be given as:

$$P_{abs} = P_{atm} + \rho gh = 101.3 \times 10^3 + 3990.7\text{Pa} = 105291\ \text{Pa}$$

1.2 Submerged Surfaces

The human body is constantly under pressure from the atmosphere, however, since the body is so accustomed to it, there is no feeling, no problem with breathing. However, once the human body is submerged in water there is a noticeable difference, the force on the body is greater from the increased hydrostatic pressure of the water, meaning it becomes harder to move, and harder to breathe.

A similar situation can be seen when a car falls into a deep body of water. The people in the car are left with two options, they can get out immediately, or they can wait until the hydrostatic pressure balances on the inside and outside of the car, and open the door with little to no problem; this does mean, however, that the inside of the car has to be completely full of water before trying to open the door.

Hydrostatic pressure increases almost linearly as depth increases. Every 10 metres submerged increase the pressure by 1bar.

1.2.1 Submarines

Submarines are incredible engineering systems; the hull of a submarine is under an incredible pressure imbalance. The inside of a submarine is required to be at atmospheric pressure, as increasing the air pressure will result in a toxic amount of oxygen for the passengers.

Submarines are typically given four classifications as a measure for their hull strength:

- Design depth: This is the depth set by the design specification and is the depth that is used in calculations for the hull's depth, size and other design considerations.
- Test Depth: This is the depth that submarines are tested to in order to test hull integrity. This is also the deepest permissible depth allowed during peacetime.
- Operating depth: The maximum allowable depth in any conditions during war time.
- Collapse depth: This is the depth that the submarine will begin to crumble under the hydrostatic pressure. Nuclear submarines are predicted to have a collapse depth of roughly 700 metres, which will be a pressure of 71 bar.

1.3 The Centre of Pressure

So far, the average force has acted on rectangular shapes, however in practical applications they are not the only shapes to consider. It's important to know where the centre of pressure acts on a vessel, otherwise there is a distinct possibility of capsizing or sinking. Most nuclear submarines are a cylindrical shape, but what about the Dockwise Vanguard shown in the link below? The semi-submersible ship will slowly sink itself to allow oil rigs or even other ships onto its deck, so care needs to be taken to ensure that there isn't an imbalance while billions of pounds of equipment are at risk. The average force that acts on a submerged surface is located at a point that is known as the centre of pressure, which needs to be calculated for non-uniform shapes.

https://www.youtube.com/watch?v=o0HfhBDmUbE (Dockwise Vanguard)

The centre of pressure is not to be confused with the centre of mass, the centre of pressure is the point where the fluid can be considered as a concentrated force. The centre of mass is the concentrated point force of the object, and they can both be in different locations, such as the rocket shown in Fig.1.5. In this case, there is a moment imbalance, explaining why rockets need constant adjustments mid-flight, controlled by their guidance systems.

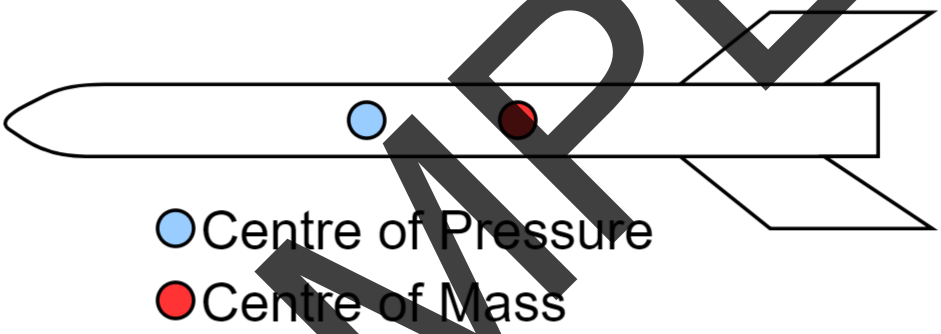


Fig.1.5: A rocket's centre of mass and centre of pressure

1.3.1 The First Moment of Area

To calculate the centre of pressure, the first and second moments of area need to be calculated, which are noted as S and I, respectively. The first moment of area is calculated by integrating the position of the centroid (arithmetic mean position of all the points drawn on the object) with respect to the area. The general formula for the first moment of area S is shown as Eq.1.7:

$$S = \int \bar{y} dA = \iint \bar{y} dy dz \quad (1.7)$$

Where A is the area, and $\bar{y}$ is the distance between the "centroid" (the centre of pressure) and a reference point. This reference point is defined usually as one of the edges of the shape. Eq.1.7 can be simplified further to Eq.1.8:

$$S = A\bar{y}$$

Which is a more useful form when dealing with complex shapes.

The first moments of area for the given shapes are given as Table 1.1:

Table 1.1: General first moment of area equations for different shapes

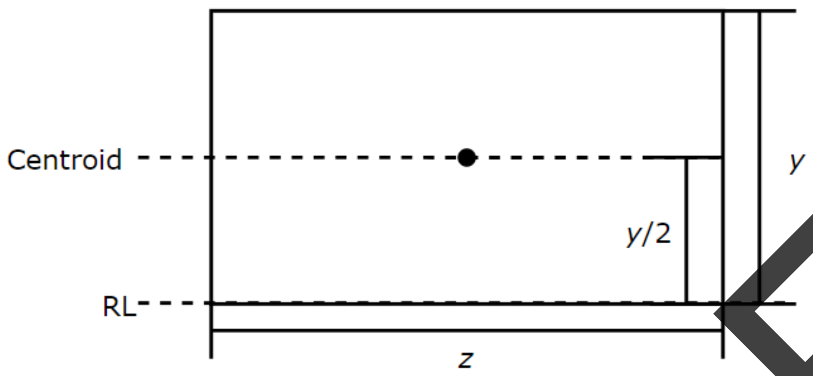
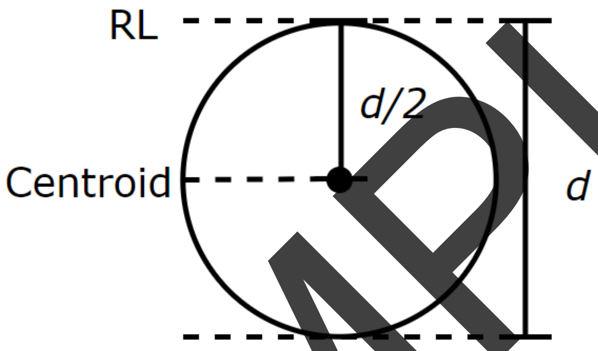
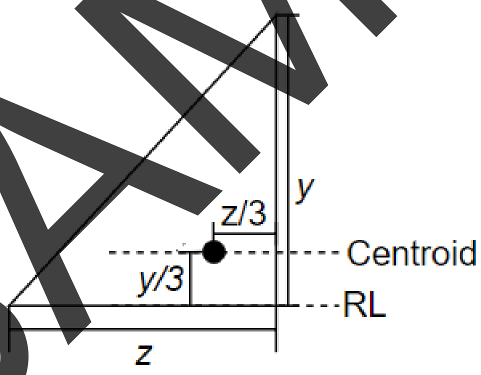
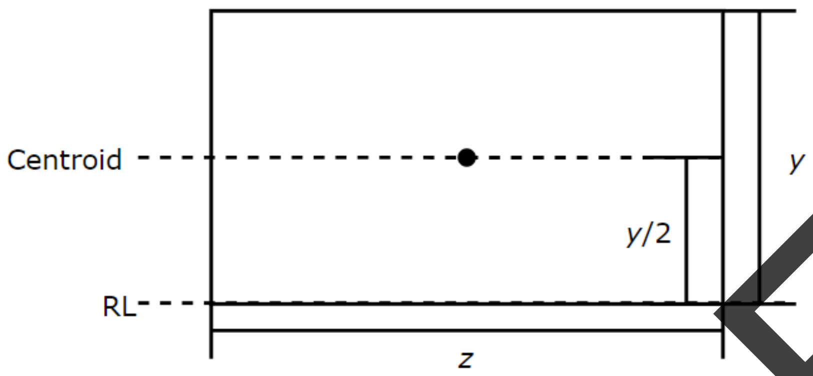
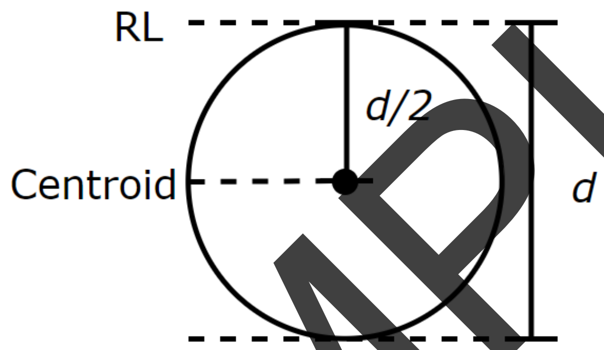
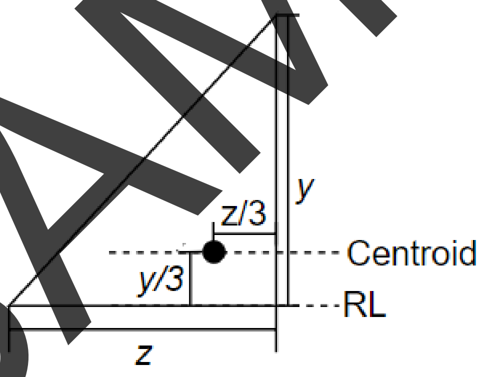
Shape	**Diagram**	**Equation**
Rectangle		$S = \frac{zy^2}{4}$ (1.8)
Circle		$S = \frac{\pi d^3}{8}$ (1.9)
Right-Angled Triangle		$S = \frac{zy^2}{6}$ (1.11)

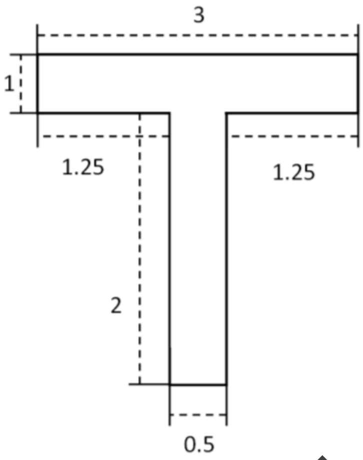
Table 1.2 gives the second moments of area for certain shapes:

Table 1.2: General second moments of area for different shapes

Shape	Diagram	Equation
Rectangle		$I = \frac{zy^3}{12}$ (1.13)
Circle		$I = \frac{\pi d^4}{64}$ (1.14)
Right-Angled Triangle		$I = \frac{zy^3}{36}$ (1.15)

Example 5

Calculate the second moment of area for the T-beam below



The beam can be broken down into a 3×1 rectangle at the top (which we call element 1), and a 2×0.5 rectangle at the bottom (which we call element 2). The beam has a vertical line of symmetry, meaning the horizontal coordinate of the centroid ($\bar{x}$) is 1.5 (i.e. half the width of the diagram), so we only need to find the vertical coordinate, $\bar{y}$.

Let the area of the element 1 be a_1 and that of element 2 be a_2.

$$\therefore \quad a_1 = 1 \times 3 = 3\, m^2$$

$$\therefore \quad a_2 = 0.5 \times 2 = 1\, m^2$$

$$\therefore \quad \sum a = a_1 + a_2 = 3 + 1 = 4\, m^2$$

Let y_1 be the vertical distance of the centroid in element 1 to the base (bottom edge of the 'T').

Let y_2 be the vertical distance of the centroid in element 2 to the base (bottom edge of the 'T').

$$\therefore \quad y_1 = 3 - 0.5 = 2.5\, m$$

$$\therefore \quad y_2 = 2 - 1 = 1\, m$$

Now let's determine the ay products…

$$a_1 y_1 = 3 \times 2.5 = 7.5\, m^3$$

$$a_2 y_2 = 1 \times 1 = 1\, m^3$$

$$\therefore \quad \sum ay = 7.5 + 1 = 8.5\, m^3$$

Now we determine the ay^2 products…

$$a_1 {y_1}^2 = (a_1 y_1)(y_1) = (7.5)(2.5) = 18.75\, m^4$$

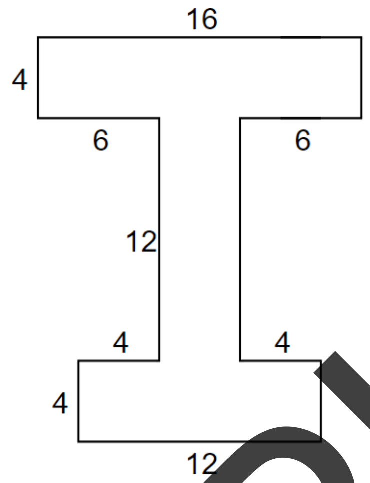
$$a_2 {y_2}^2 = (a_2 y_2)(y_2) = (1)(1) = 1\, m^4$$

$$\therefore \quad \sum ay^2 = 18.75 + 1 = 19.75\, m^4$$

$$C_P = \frac{I}{S} \quad (1.18)$$

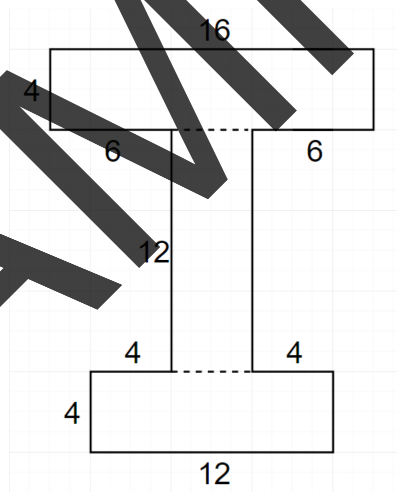
Example 6

Find the centre of pressure for the beam below.



Answer:

The shape can be broken down into three rectangles, shown below:



S is found using Eq.1.16, which will then be used to find the centroid.

$$S = A\bar{y} = \sum \bar{y}_c A_c$$